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# Algebraic Applications on l-Fuzzy Soft Group

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**Abstract:** In this research, we extend the notion of l-fuzzy soft function and l-fuzzy soft homomorphism on l-fuzzy soft group. Furthermore, we discuss some theorems of homomorphic image and homomorphic pre-image of an l-fuzzy soft group under an l-fuzzy soft function. And we study some properties of l-fuzzy soft groups.

**Key words:** l-fuzzy soft group, l-fuzzy soft function, l-fuzzy soft homomorphism, properties, homomorphic, extend

## INTRODUCTION

The lattice theory has implemented to algebra, creation of geometry and other various fields. It was first proposed by Richard Dedikind in 1930's but Birkhoff (1967) investigated the theory of lattices in 1940's. A fuzzy set is defined by a membership function which allocates a membership grade ranging between 0 and 1 to each object. In 1965, Zadeh (1965) defined the concept of fuzzy set and its operations. In 1999, Molodtsov (1999) initiated the notion of soft set theory and soft function. In 2006, Aktas and Cagman (2007) proposed soft group and soft homomorphism and derived some of its fundamental properties by applying Molodsov's definition of soft sets. Fuzzy group (Rosenfeld, 1991) was propounded by Rosenfeld. Fuzzy soft set (Cagman *et al*., 2010; Maji *et al*., 2001) was investigated by Maji *et al*. (2011) and extended by Cagman *et al*. (2010) and also fuzzy soft ring (Feng *et al*., 2008; Ghosh *et al*., 2011; Pazar *et al*., 2012) has been estabished. In 2009, Aygunoglu and Aygun (2009) in-troduced the notion of fuzzy soft group and also established fuzzy soft function and fuzzy soft homomorphism. In 2013, Celik *et al*. (2013) explored the notion of ring to the algebraic structures of fuzzy soft sets and defined fuzzy soft function and fuzzy softring homomorphism. Also, he investigated a different approach to group theory through soft sets and l-fuzzy soft sets (Celik, 2015). In 2015, Ali *et al*. (2015) studied a new concept of soft sets with some order among attributes and described its properties. In 2016, Vimala (2016) studied the concept of homomorphism on fuzzy l-ideal. Also, Vimala and Reeta (2016) defined l-fuzzy soft group and discussed some pertinent properties. Distributive and modular l-fuzzy soft group and its duality was proposed by Vimala and Reeta. In this research, we study the concept of the l-fuzzy soft homo-morphism and analyse some of its algebraic properties.

## MATERIALS AND METHODS

**Preliminaries:** In this study, we have presented the fundamental definitions and results of fuzzy sets, soft sets, fuzzy soft group, fuzzy soft function and fuzzy soft homomorphism which will be very useful for subsequent exploration.

A poset $(L, \leq)$ is said to form a lattice if for every a, $b \in L$, Max$\{a, b\}$ and Min$\{a, b\}$ exist in L. Then, we write Max$\{a, b\} = a \vee b$ and Min$\{a, b\} = a \wedge b$. A function $f: L \rightarrow M$ from a lattice L to a lattice M is called join and meet homomorphism of lattices when for all $x, y \in L$, $f(x \vee y) = f(x) \vee f(y)$ and $f(x \wedge y) = f(x) \wedge f(y)$, respectively.

Let G be a group. A fuzzy subset $\mu$ of a group G is called a fuzzy subgroup of the group G if (i) $\mu(xy) \geq$ Min $\{\mu(x), \mu(y)\}$ for every $x, y \in G$ and (ii) $\mu(x^{-1}) = \mu(x)$, for every $x \in G$. A fuzzy poset (X, A) is a fuzzy lattice iff $x \vee y$ and $x \wedge y$ exist for all $x, y \in X$. Throughout this work, I refers a unit closed interval, i.e., I = [0, 1].

**Definition 2.1:** Let X be a non-empty set, then a fuzzy set $\mu$ over X is a function from X into I = [0, 1], i.e., $\mu: X \rightarrow I$ (Zadeh, 1965).

**Definition 2.2:** Let X be an initial universe set and E a set of parameters with respect to X. Let P(X) denote the power set of X and $A \subseteq E$. A pair (F, A) is called a soft set over X, where F is a mapping given by $F: A \rightarrow P(X)$. A soft set over X is a parameterized family of subsets of the universe X.

**Definition 2.3:** Let $I^X$ denote the set of all fuzzy sets on X and $A \subset E$ (Aygunoglu and Aygun, 2009). A pair (f, A) is called a fuzzy soft set over X, where f is a mapping from A into $I^X$. That is, for each $a \in A$, $f(a) = f_a: X \rightarrow I$ is a fuzzy set on X.

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**Definition 2.4:** For two fuzzy soft sets (f, A) and (g, B) over a common universe X, we say that (f, A) is a fuzzy soft subset of (g, B) and write $(f, A)\subseteq(g, B)$ if:

- $A\subset B$
- For each $a\in A$, $f_a\leq g_a$ that is $f_a$ is fuzzy subset of $g_a$

Note that for all $a\in A$, $f_a$ and $g_a$ are identical approximations.

**Definition 2.5:** Two fuzzy soft sets (f, A) and (g, B) over a common universe X are said to be equal if $(f, A)\subseteq v(g, B)$ and $(g, B)\subseteq(f, A)$ (Aygunoglu and Aygun, 2009).

**Definition 2.6:** Let X be a group and (F, A) be a soft set over X (Aktas and Cagman, 2007). Then (F, A) is said to be a soft group over X iff F(a) is a subgroup of X, for each $a\in A$. A soft group is a parameterized family of subgroups of X.

**Definition 2.7:** Let X be a group and (f, A) be a fuzzy soft set over X (Aygunoglu and Aygun, 2009). Then (f, A) is said to be a fuzzy soft group over X iff for each $a\in A$ and $x, y\in X$:

- $f_a(x, y)\geq\min\{f_a(x), f_a(y)\}$
- $f_a(x^{-1})\geq f_a(x)$

That is for each $a\in A$, $f_a$ is a fuzzy subgroup.

**Example 2.8:** Let N be the set of all natural numbers and define $f: N\rightarrow I^R$ by $f(n) = f_n$, $R\rightarrow I$, for each $n\in N$ where Aygunoglu and Aygun (2009):

$$f_n(x) = \begin{Bmatrix} 1/n & \text{if } x=k2^n, \exists k\in Z \\ 0 & \text{otherwise} \end{Bmatrix}$$

where, Z is the set of all integers. Then, the pair (f, N) forms a fuzzy soft set over R and the fuzzy soft set (f, N) is a fuzzy soft group over R.

**Definition 2.9:** Let X be a non-empty set and P(X) be bounded lattice with respect to operations of intersection and union and set inclusion as a partial order (Ali *et al.*, 2015). If the set of parameters E is also a lattice with respect to certain binary operations or partial order then a non-empty subset A of E also inherits the partial order from the set E. A soft set (F, A) is called an l-soft set if for the mapping $F: A\rightarrow P(X)$, $x\leq y$ implies $F(x)\subseteq F(y)$, for each $x, y\in A\subseteq E$.

**Definition 2.10:** Union of two fuzzy soft sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A\cup B$ and:

$$h(c) = \begin{Bmatrix} f_c & \text{if } c\in A\text{-}B \\ g_c & \text{if } c\in B\text{-}A, \text{ for all } c\in C \\ f_c\vee g_c & \text{if } c\in A\cap B \end{Bmatrix}$$

We write $(f, A)\cup(g, B) = (h, C)$.

**Definition 2.11:** Intersection of two fuzzy soft sets (f, A) and (g, B) over a common universe X is the fuzzy soft set (h, C), where $C = A\cap B$ and $h_c = f_c\wedge g_c$, for all $c\in C$ (Maji *et al.*, 2001). We write $(f, A)\cap(g, B) = (h, C)$.

**Definition 2.12:** If (f, A) and (g, B) are two fuzzy soft sets, then (f, A) and (g, B) is denoted as (f, A) f (g, B) (Maji *et al.*, 2001). (f, A) f (g, B) is defined as $(h, A\times B)$ where $h(a, b) = h_{a,b} = f_a\wedge g_b$, for all $(a, b)\in A\times B$.

**Definition 2.13:** If (f, A) and (g, B) are two fuzzy soft sets, then (f, A) or (g, B) is denoted as (f, A) g (g, B). (f, A) g (g, B) is defined as $(k, A\times B)$, where $k(a, b) = k_{a,b} = f_a\vee g_b$, for all $(a, b)\in A\times B$.

**Theorem 2.14:** Let (f, A) and (g, B) be two fuzzy soft groups over X (Aygunoglu and Aygun, 2009). Then, their intersection $(f, A)\cap(g, B)$ is a fuzzy soft sub-group over X.

**Theorem 2.15:** Let (f, A) and (g, B) be two fuzzy soft groups over X (Aygunoglu and Aygun, 2009). If $A\cap B = \Phi$, then $(f, A)\cup(g, B)$ is a fuzzy soft subgroup over X.

**Theorem 2.16:** Let (f, A) and (g, B) be two fuzzy soft groups over X (Aygunoglu and Aygun, 2009). Then (f, A) f (g, B) is a fuzzy soft subgroup over X.

**Definition 2.17:** Let (F, A) and (G, B) be two fuzzy soft sets over $R_1$ and $R_2$, respectively (Celik *et al.*, 2013 ). Let $\phi: R_1\rightarrow R_2$, $\psi: A\rightarrow B$ be two functions. Then, the pair $(\phi, \psi)$ is a fuzzy soft function from (F, A) to (G, B) denoted by $(\phi, \psi)$, $(F, A)\rightarrow(G, B)$ if $\phi(F(x)) = G(\psi(x))$ for all $x\in A$. If $\phi$ and $\psi$ are injective (resp. surjective, bijective) then $(\phi, \psi)$ is said to be injective (resp. surjective, bijective).

In this definition, if $\phi$ is a ring homomorphism from $R_1$ to $R_2$ then $(\phi, \psi)$ is said to be a fuzzy soft ring homomorphism and that (F, A) is fuzzy soft homomorphic to (G, B). The later is denoted by $(F, A)\sim(G, B)$. If $\phi$ is an isomorphism from $R_1$ to $R_2$ and $\psi$ is a bijection mapping from A onto B, then $(\phi, \psi)$ is a fuzzy soft ring isomorphism and that (F, A) is fuzzy soft isomorphic to (G, B). The later is denoted by $(F, A)\simeq(G, B)$.

**Definition 2.18:** Let (F, A) and (G, B) be two fuzzy soft sets over $R_1$ and $R_2$, respectively (Celik *et al*., 2013 ). Let (ϕ, ψ) be a fuzzy soft function from (F, A) to (G, B). Then:

- The image of (F, A) under the fuzzy soft function (ϕ, ψ) is defined as the fuzzy soft set (ϕ, ψ) (F, A) = (ϕ(F), B) over $R_2$ where:

$$\phi(F)(y) = \begin{cases} V_{\psi(x)=y}\ \phi(F(X)) & \text{if } y\in \mathrm{Im}\,\psi \\ 0_{R_2} & \text{otherwise} \end{cases}$$

for all $y\in B$

- The pre-image of (G, B) under the fuzzy soft function (ϕ, ψ) is defined as the fuzzy soft set $(\phi, \psi)^{-1}$ (G, B) = $(\phi^{-1}(G), A)$ over $R_1$ where:

$$\phi^{-1}(G)(x) = \phi^{-1}(G(\psi(x))), \text{ for all } x\in A$$

**Proposition 2.19:** Let (f, A) be a fuzzy soft group over X (ϕ, ψ) be a fuzzy soft homomorphism from X to Y and T be a contonuous t-norm. Then (ϕ, ψ) (f, A) is a fuzzy soft group over Y (Aygunoglu and Aygun, 2009).

**Proposition 2.20:** Let (g, B) be a fuzzy soft group over Y and (ϕ, ψ) be a fuzzy soft homomorphism from X to Y. (Aygunoglu and Aygun, 2009). Then $(\phi, \psi)^{-1}$ (g, B) is a fuzzy soft group over X.

**Proposition 2.21:** Let (F, A), (G, B) and (H, C) be fuzzy softsets over $R_1$, $R_2$ and $R_3$, respectively. Let (ϕ, ψ), (F, A)→(G, B) and (β, γ): (G, B→(H, C) be two fuzzy soft functions. Then (β∘ϕ, γ∘ψ): (F, A)→(H, C) is a fuzzy soft function.

## RESULTS AND DISCUSSION

**Algebraic applications on l-fuzzy soft group:** In this study, we study the definition of l-fuzzy soft function and l-fuzzy soft homomorphism and discuss some of its related properties. Reeta and Vimala (2016) throughout this study, let X be a group and P(X) be the power set of X. If the set of parameters E is also a lattice with respect to certain binary operations or partial order, then a non-empty subset A of E also inherits the partial order from the set E and we use ∨ for maximum and ∧ for minimum.

**Definition 3.1:** Let X be a group and (f, A) be a fuzzy soft set over X (Reeta and Vimala, 2016). Then (f, A) is said to be a l-fuzzy soft group (lattice ordered fuzzy soft group) over X if for each a∈A and x, y∈X:

- $f_a(x: y)\geq \min \{f_a(x), f_a(y)\}$ (Aygunoglu and Aygun, 2009)
- $f_a(x^{-1})\geq f_a(x)$ (Aygunoglu and Aygun, 2009)
- $a\leq b$ implies $f_a\subseteq f_b$ for all a, b∈A, i.e., for all a, b∈A, $f_a\vee f_b$ and $f_a\wedge f_b$ exist in (f, A)

**Example 3.2:** Let N be the set of all natural numbers and (N, ≤) be a lattice (Reeta and Vimala, 2016). If a, b∈N, then a∨b = max {a, b} and a∧b = min {a, b}. Define f: $N\to I^R$ by f(n) = $f_n$: R→I for each n∈N where:

$$fn(x) = \begin{cases} 1-1/n & \text{if } x=k2^{-n}, k\in Z \\ 0 & \text{otherwise} \end{cases}$$

where, Z is the set of all integers. Here, for each $n_1$, $n_2$∈N, $n_1\leq n_2$ implies $f_{n1}\subseteq f_{n2}$. Then, the pair ((f, N), ∨, ∧, ⊆) forms an l-fuzzy soft group over R.

**Proposition 3.3:** Let ((f, A), ∨, ∧, ⊆) be an l-fuzzy soft group over X. Then for all a, b∈A and x∈X, $f_{a1\vee a2}(x) = f_{a1}(x)\vee f_{a2}(x)$ and $f_{a1\vee a2}(x) = f_{a1}(x)\wedge f_{a2}(x)$.

**Proof:** Let ((f, A), ∨, ∧, ⊆) be an l-fuzzy soft group over X. Then for all $a_1$, $a_2$∈A and x∈X, $a_1\leq a_2$ implies $f_{a1}\subseteq f_{a2}$. Then, we get $a_1\vee a_2 = a_2$, $a_1\wedge a_2 = a_1$ and $f_{a1}\vee f_{a2} = f_{a2}$, $f_{a1}\wedge f_{a2} = f_{a1}$, i.e., $f_{a1}(x)\vee f_{a2}(x) = f_{a2}(x)$, $f_{a1}(x)\wedge f_{a2}(x) = f_{a1}(x)$. Therefore, $f_{a1\vee a2}(x) = f_{a2}(x) = f_{a1}(x)\vee f_{a2}(x)$. Similarly, we get $f_{a1\wedge a2}(x) = f_{a1}(x)\wedge f_{a2}(x)$.

**Definition 3.4:** Let X and Y be groups. Let ((f, A), ∨, ∧, ⊆) and ((g, B), ∨, ∧, ⊆) be two l-fuzzy soft groups over X and Y, respectively. Let ϕ: X→Y, ψ: A→B be two functions where A and B are parameter sets (A, B⊆E) for the crisp sets X and Y, respctively. Then, the pair (ϕ, ψ) is an l-fuzzy soft function from (f, A) to (g, B) denoted by (ϕ, ψ): (f, A)→(g, B) if $\phi(f_a(x)) = g_\psi(a)(y)$ for all a∈A, x∈X and y∈Y.

**Definition 3.5:** Let ((f, A), ∨, ∧, ⊆) and ((g, B), ∨, ∧, ⊆) be two l-fuzzy soft groups over X and Y, respectively. Let (ϕ, ψ)) be an l-fuzzy soft function from (f, A) to (g, B). Then:

- The image of (f, A) under the fuzzy soft function (ϕ, ψ) is defined as the fuzzy soft set (ϕ, ψ) (f, A) = (ϕ(f), ψ(A)) over Y where:

$$\phi(f)_b(y) = \begin{cases} V_{\psi(a)=b}\phi(f_a(x)) & \text{if } x\in \phi^{-1}(y) \\ 0 & \text{otherwise} \end{cases}$$

$\forall b\in \psi(A), \forall y\in Y$

- The pre-image of (g, B) under the fuzzy soft function (ϕ, ψ) is defined as the fuzzy soft set $(\phi, \psi)^{-1}$ (g, B) = $(\phi^{-1}(g), \psi^{-1}(B))$ over X where:

$\phi^{-1}(g)_a(x) = \phi^{-1}(g_{\psi(a)}(y))$, $\forall a \in \psi^{-1}(B)$, $\forall x \in X$ and $y \in Y$

**Proposition 3.6:** Let $((f, A), \vee, \wedge, \subseteq)$ and $((g, B), \vee, \wedge, \subseteq)$ be two l-fuzzy soft groups over X and Y, respectively. Let $(\phi, \psi)$ be an l-fuzzy soft function from (f, A) to (g, B). If the mapping $\psi$: A→B is homomorphism then for all $a_1, a_2 \in A$ and $x \in X$, $\phi(f_{a1\vee a2}(x)) = \phi(f_{a1}(x)) \vee \phi(f_{a2}(x))$ and $\phi(f_{a1\wedge a2}(x)) = \phi(f_{a1}(x)) \wedge \phi(f_{a2}(x))$.

**Proof:** Let $((f, A), \vee, \wedge, \subseteq)$ and $((g, B), \vee, \wedge, \subseteq)$ be two l-fuzzy soft groups over X and Y, respectively. Let $(\phi, \psi)$ be an l-fuzzy soft function from (f, A) to (g, B). Then, we have $\phi(f_a(x)) = g_{\psi(a)}(y)$. For each $b_1, b_2 \in \psi(A)$, there exist $a_1, a_2 \in A$ such that $\psi(a_1) = b_1$ and $\psi(a_2) = b_2$. Since, the mapping $\psi$: A→B is homomorphism then for all $a_1, a_2 \in A$, $\psi(a_1 \vee a_2) = \psi(a_1) \vee \psi(a_2)$ and $\psi(a_1 \wedge a_2) = \psi(a_1) \wedge \psi$ $a_2)$. Therefore:

$$\phi(f_{a1\vee a2}(x)) = g\psi_{(a1\vee a2)}(y) = g\psi(a_1) \vee \psi(a_2)(y) =$$
$$gb_1 \vee b_2(y) = gb_1(y) \vee gb_2(y) \text{ (from pro 3.3) } =$$
$$g\psi(a_1)(y) \vee g\psi(a_2)(y)\ \phi(f_{a1\vee a2})(x) = \phi(f_{a1}(x)) \vee \phi(f_{a2})(x))$$

Similarly, we get, $\phi(f_{a1\wedge a2}(x)) = \phi(f_{a1}(x)) \wedge \phi(f_{a2}(x))$.

**Proposition 3.7:** Let $((f, A), \vee, \wedge, \subseteq)$ and $((g, B), \vee, \wedge, \subseteq)$ be two l-fuzzy soft groups over X and Y, respectively. Let $(\phi, \psi)$ be a l-fuzzy soft function from (f, A) to (g, B). Then for all $a_1, a_2 \in A$ and $x \in X$, $\phi^{-1}(g)_{a1\vee a2}(x) = \phi^{-1}(g)a_1(x) \vee \phi^{-1}(g)a_2(x)$ and $\phi^{-1}(g)_{a1\wedge a2}(x) = \phi^{-1}(g)a_1(x) \wedge \phi^{-1}(g)a_2(x)$.

**Proof:** Let $((f, A), \vee, \wedge, \subseteq)$ and $((g, B), \vee, \wedge, \subseteq)$ be two l-fuzzy soft groups over X and Y, respectively. Let $(\phi, \psi)$ be an l-fuzzy soft function from (f, A) to (g, B). Then, we have $\phi(f_a(x)) = g_{\psi(a)}(y)$. Therefore, $f_a(x) = \phi^{-1}(g_{\psi(a)}(y))$ which implies $f_{a1\vee a2}(x) = \phi^{-1}(g_{\psi(a1\vee a2)}(y))$, for all $a_1, a_2, a_2 \in A$. The pre-image of (g, B) is defined as $\phi^{-1}(g)_a(x) = \phi^{-1}(g_{\psi(a)}(y))$, $\forall a \in \phi^{-1}(B)$, $\forall_x \in X$ and $y \in Y$. Therefore:

$$\phi^{-1}(g)_{a_1\vee a_2}(x) = \phi^{-1}(g\psi_{(a_1\vee a_2)}(y)) = f_{a_1\vee a_2}(x) =$$
$$f_{a_1}(x) \vee f_{a_2}(x) \text{ (from pro 3.3) } = \phi^{-1}(g\psi_{(a_1)}(y)) \vee$$
$$\phi^{-1}(g\psi_{(a_2)}(y))\ \phi^{-1}(g)_{a_1\vee a_2}(x) = \phi^{-1}(g)_{a_1}(x) \vee \phi^{-1}(g)_{a_2}(x)$$

Similarly, we get, $\phi^{-1}(g)_{a1\wedge a2}(x) = \phi^{-1}(g)_{a1}(x) \wedge \phi^{-1}(g)_{a2}(x)$.

**Definition 3.8:** Let X and Y be groups. Let $((f, A), \vee, \wedge, \subseteq)$ and $((g, B), \vee, \wedge, \subseteq)$ be two l-fuzzy soft groups over X and Y, respectively. Let $\phi$: X→Y, $\psi$: A→B be two functions where A and B are parameter sets for the crisp sets X and Y, respctively. The pair $(\phi, \psi)$ is an l-fuzzy soft function from (f, A) to (g, B) denoted by $(\phi, \psi)$: (f, A)→(g, B). Then $(\phi, \psi)$ is an l-fuzzy soft homomorphism if the following conditions are satisfied:

- $\phi$ is homomorphism(group, meet and join)
- $\psi$ is meet and join homomorphism
- For all a1, $a_2 \in A$ and $x \in X$, $f_{a1\vee a2}(x) = f_{a1}(x) \vee f_{a2}(x)$ and $f_{a1\wedge a2}(x) = f_{a1}(x) \wedge f_{a2}(x)$
- $\phi(f)_{b1\vee b2}(y) = \phi(f)b_1(y) \vee \phi(f)b_2(y)$, $\forall x \in \phi^{-1}(y)$; $\forall b \in \psi(A)$, $\forall y \in Y$ and $\phi(f)_{b1\wedge b2}(y) = \phi(f)_{b1}(y) \wedge \phi(f)_{b2}(y)$, $\forall x \in \phi^{-1}(y)$, $\forall b \in \psi(A)$, $\forall y \in Y$

**Theorem 3.9:** Let $((f, A), \vee, \wedge, \subseteq)$ be l-fuzzy soft group over X and $(\phi, \psi)$ be an l-fuzzy soft homomorphism from X to Y. Then $(\phi, \psi)$ (f, A) is an l-fuzzy soft group over Y.

**Proof:** By proposition 2.19 we get $(\phi, \psi)$(f, A) is a fuzzy soft group over Y. Since $((f, A), \vee, \wedge, \subseteq)$ be an l-fuzzy soft group over X, for all $a_1, a_2 \in A$, $a_1 \leq a_2$ implies $f_{a1} \subseteq f_{a2}$. Then, we get $a_1 \vee a_2 = a_2$, $a_1 \wedge a_2 = a_1$ and $f_{a1} \vee f_{a2} = f_{a2}$, $f_{a1} \wedge f_{a2} = f_{a1}$, i.e., $f_{a1}(x) \vee f_{a2}(x) = f_{a2}(x)$, $f_{a1}(x) \wedge f_{a2}(x) = f_{a1}(x)$. And for each $b_1$, $b_2 \in \psi(A)$, there exist $a_1, a_2 \in A$ such that $\psi(a_1) = b_1$ and $\psi(a_2) = b_2$:

$$\phi(f)_{b1\vee b2}(y) = \vee_{\psi(a1\vee a2)=b1\vee b2}\ \phi(f_{a1\vee a2}(x)) =$$
$$\vee_{\psi(a1\vee a2)=b1\vee b2}\ \phi(f_{a1}(x) \vee f_{a2}(x)) =$$
$$\vee_{\psi(a2)=b2}\ \phi(f_{a2}(x)) = \phi(f)_{b2}(y)$$
$$\therefore \phi(f)_{b1\vee b2}(y) = \phi(f)_{b1}(y) \vee \phi(f)_{b2}(y) =$$
$$\phi(f)_{b2}(y),\ \forall x \in \phi^{-1}(y),\ \forall b \in \psi(A),\ \forall y \in Y$$

Similarly, we get $\phi(f)_{b1}(y) \wedge \phi(f)_{b2}(y) = \phi(f)b_1(y)$, $\forall x \in \phi^{-1}(y)$, $\forall b \in \psi(A)$, $\forall_y \in Y$. Hence, we get, $b_1 \leq b_2$ implies $\phi(f)_{b1}(y) \subseteq \phi(f)_{b2}(y)$, $\forall_{b1,\, b2} \in \psi(A)$ $\therefore (\phi, \psi)$(f, A) is an l-fuzzy soft group over Y.

**Theorem 3.10:** Let $((g, B), \vee, \wedge, \subseteq)$ be l-fuzzy soft group over Y and $(\phi, \psi)$ be an l-fuzzy soft homomorphism from X to Y. Then $(\phi, \psi)^{-1}$(g, B) is an l-fuzzy soft group over X.

**Proof:** By proposition 2.20 we get $(\phi, \psi)^{-1}$(g, B) is a fuzzy soft group over X. Since $((g, B), \vee, \wedge, \subseteq)$ be an l-fuzzy soft group over Y, for all $b_1, b_2 \in B$, $b_1 \leq b_2$ implies $g_{b1} \subseteq g_{b2}$. Then, we get $b_1 \vee b_2 = b_2$, $b_1 \wedge b_2 = b_1$ and $g_{b1} \vee g_{b2} = g_{b2}$, $g_{b1} \wedge g_{b2} = g_{b1}$, i.e., $g_{b1}(y) \vee g_{b2}(y) = g_{b2}(y)$, $g_{b1}(y) \wedge g_{b2}(y) = g_{b1}(y)$. And for each $a_1, a_2 \in \psi^{-1}(B)$, there exist $b_1, b_2 \in B$ such that $\psi(a_1) = b_1$ and $\psi(a_2) = b_2$:

$$\phi^{-1}(g)_{a1a2}(x) = \phi^{-1}(g_{\psi(a1\vee a2)}(y)),\ \forall a \in \psi^{-1}(B),$$
$$\forall x \in X \text{ and } y \in Y = \phi^{-1}(g_{\psi(a1)\vee\psi(a2)}(y)) =$$
$$\phi^{-1}(g_{b1\vee b2}(y)) = \phi^{-1}(g_{b1}(y) \vee g_{b2}(y)) =$$
$$\phi^{-1}(g_{b2}(y)) = \phi^{-1}(g)_{a2}(x)$$

Therefore, $\phi^{-1}(g)_{a1\vee a2}(x) = \phi^{-1}(g)_{a1}(x)\vee\phi^{-1}(g)_{a2}(x) = \phi^{-1}(g)_{a2}(x)$. Similarly, we get $\phi^{-1}(g)_{a1}(x)\wedge\phi^{-1}(g)_{a2}(x) = \phi^{-1}(g)_{a1}(x)$. Hence, we get, $a_1 \le a_2$ implies $\phi^{-1}(g)a_1(x)\subseteq\phi^{-1}(g)a_2(x)$, $\forall a_1, a_2\in\psi^{-1}(B)$. Therefore $(\phi, \psi)^{-1}(g, B)$ is an l-fuzzy soft group over X.

**Example 3.11:** Let $X = \{k4^{-n1}/k\in Z, n_1\in N\}$ and $Y = \{k2^{-n1}/k\in Z, n_2\in N\}$ be two groups. Let E = N be the set of all natural numbers and $A, B\subseteq N$. Consider $\phi$: X→Y defined by $\phi(x) = x$, for all $x\in X$ and $\psi$: A→B defined by $\psi(n_1) = 2n_1$, for all $n_1\in A$. Define two l-fuzzy soft groups $f: N\to I^R$ by $f(n_1) = fn_1$: R→I for each $n_1\in N$ and $g: N\to I^R$ by $g(n_2) = gn_2$: R→I for each $n_2\in\psi(n_1)$ over X, Y, respectively where:

$$f_{n_1}(x) = \begin{cases} 1-1/(n_1+1) & \text{if } x = k4^{-n_1} \\ 0 & \text{otherwise} \end{cases}$$
$$k\in Z,\ n_1\in N$$

where, Z is the set of all integers:

$$g_{n_2}(y) = \begin{cases} 1-1/(n_2/2+1) & \text{if } y = k^{2-n_2} \\ 0 & \text{otherwise} \end{cases}$$
$$k\in Z,\ \forall n_2\in \psi(n_1)$$

where, Z is the set of all integers. Hence, we get, $\phi(f_{n1}(x)) = g\psi(n_1)(y)$. Then, the pair $(\phi, \psi)$ is an l-fuzzy soft function over R. Since, $\phi$, $\psi$ are homomorphism and $(\phi, \psi)$ satisfies (iii), (iv) conditions of definition 3.8 $(\phi, \psi)$ is an l-fuzzy soft homomorphism from X to Y.

**Theorem 3.12:** Let ((f, A), $\vee$, $\wedge$, $\subseteq$), ((g, B), $\vee$, $\wedge$, $\subseteq$) and ((h, C), $\vee$, $\wedge$, $\subseteq$) be l-fuzzy soft groups over X, Y and Z, respectively. Let $(\phi, \psi)$: (f, A)→(g, B) and $(\varphi, \chi)$: (g, B)→(h, C) be l-fuzzy soft homomorphism. Then $(\varphi\circ\phi, \chi\circ\psi)$: (f, A)→(h, C) is an l-fuzzy soft homomorphism from X to Y.

**Proof:** Let $(\phi, \psi)$: (f, A)→(g, B) and $(\varphi, \chi)$: (g, B)→(h, C) be l-fuzzy soft function. By proposition 2.21, we get $(\varphi\circ\phi, \chi\circ\psi)$: (f, A)→(h, C) is an l-fuzzy soft function over Z.

Since $(\phi, \psi)$: (f, A)→(g, B) and $(\varphi, \chi)$: (g, B)→(h, C) be l-fuzzy soft homomorphism, $\phi$ and $\varphi$ are (group, join and meet) homomorphism and also $\psi$, $\chi$ are join and meet homomorphism. For each $a_1, a_2\in A$ and for all $x\in X$, $z\in Z$ we get $\varphi(\phi(f_{a1\vee a2}(x))) = h_{\chi(\psi((a1\vee a2))}(z) = h_{\chi(\psi(a1))}(z)\vee h_{\chi(\psi(a1))}(z)$ [from pro 3.3]. Therefore, $\varphi(\phi(f_{a1\vee a2}(x))) = \varphi(\Phi(f_{a1}(x)))\vee\varphi(\phi(f_{a2}(x)))$. Similarly, we get $\varphi(\phi(f_{a1\wedge a2}(x))) = \varphi(\Phi(f_{a1}(x)))\wedge\varphi(\phi(f_{a2}(x)))$:

$$\varphi^{\circ}\phi(f_{a1\vee a1}(x)) = \varphi(\phi(f_{a1\vee a2}(x))) =$$
$$\varphi(\phi(f_{a1}(x)\vee f_{a2}(x))) = \varphi(\phi(f_{a1}(x)\vee$$
$$\phi(f_{a1}(x))) = \varphi(\phi(f_{a1}(x)))\vee\varphi(\phi(f_{a1}(x)))$$

Hence $(\varphi\circ\phi, \chi\circ\psi)$ is an l-fuzzy soft homomorphism from X to Y.

**Example 3.13:** Let $X = \{k16^{-n1}/k\in Z, n_1\in N\}$, $Y = \{k4^{-n2}/k\in Z, n_2\in N\}$ and $Z = \{k2^{-n3}/k\in Z, n_3\in N\}$, be three groups. Let E = N be the set of all natural numbers and $A, B, C\subseteq N$. Consider $\phi$: X→Y defined by $\phi(x) = x$, for all $x\in X$, $\varphi$: Y→Z defined by $\varphi(y) = y$, for all $y\in Y$ and $\psi$: A→B defined by $\psi(n_1) = 2n_1$, for all $n_1\in A$, $\chi$: B→C defined by $\chi(n_2) = 2n_2$, for all $n_2\in N$. Define three l-fuzzy soft groups $f: N\to I^R$ by $f(n_1) = fn_1$: R→I, for each $n_1\in N$, $g: N\to I^R$ by $g(n_2) = g_{n2}$: R→I, for each $n_2\in\psi(n_1)$ and $h: N\to I^R$ by $h(n_3) = h_{n3}$: R→I, for each $n_3\in\chi(n_2)$ over X, Y and Z, respectively where:

$$f_{n1}(x) = \begin{cases} 1-1/(n_1+1) & \text{if } x = k16^{-n_1} \\ 0 & \text{otherwise} \end{cases}$$
$$k\in Z,\ n_1\in N$$

where, Z is the set of all integers:

$$g_{n_2}(y) = \begin{cases} 1-1/(n_2/2+1) & \text{if } y = k4^{-n_2} \\ 0 & \text{otherwise} \end{cases}$$
$$k\in Z,\ \forall n_2\in \psi(n_1)$$

where, Z is the set of all integers:

$$h_{n_3}(z) = \begin{cases} 1-1/(n_3/4+1) & \text{if } z = k2^{-n_3} \\ 0 & \text{otherwise} \end{cases}$$
$$k\in Z,\ \forall n_3\in \chi(n_2)$$

where, Z is the set of all integers. Hence, we get, $\varphi(\phi(f_a(x))) = h_\chi(\psi(a))(z)$. Then, the pair $(\varphi\circ\phi, \chi\circ\psi)$: (f, A)→(h, C) is an l-fuzzy soft function over R and the pair $(\varphi\circ\phi, \chi\circ\psi)$: (f, A)→(h, C) is an l-fuzzy soft homomorphism from X to Y.

**Theorem 3.14:** Let ((f, A), $\vee$, $\wedge$, $\subseteq$), ((g, B), $\vee$, $\wedge$, $\subseteq$) and ((h, C), $\vee$, $\wedge$, $\subseteq$) be l-fuzzy soft groups over X, Y and Z, respectively. Let $(\varphi\circ\phi, \chi\circ\psi)$: (f, A)→(h, C) be an l-fuzzy soft homomorphism from X to Y. Then, the image of $(\varphi\circ\phi, \chi\circ\psi)$: (f, A)→(h, C) is an l-fuzzy soft group over Z.

**Proof:** Let $c\in\chi(\psi(A))$ and $z_1, z_2\in Z$. If there exist $x_1, x_2\in X$ such that $\varphi(\phi(x_1)) = z_1$, $\varphi(\phi(x_2)) = z_2$. Let:

$$\varphi^{\circ}\phi(f)_c(z_1.z_2^{-1}) = \varphi(\phi(f_a(x_1.x_2^{-1}))) = h_{\chi(\psi(a))}(z_1.z_2^{-1})$$
$$[\text{since } \varphi(\phi(f_a(x))) = h_{\chi(\psi(a))}(z)] \ge \min\{h_{\chi(\psi(a))}(z_1),$$
$$h_{\chi(\psi(a))}(z_2^{-1})\} \ge \min\{h_{\chi(\psi(a))}(z_1), h_{\chi(\psi(a))}(z_2)\} =$$
$$(\varphi(\phi(f_a(x_1)))\wedge\varphi(\phi(f_a(x_2)))) = \varphi(\phi(f_a(x_1)))\wedge$$
$$\varphi(\phi(f_a(x_2))) = \varphi^{\circ}\phi(f_a(x_1))\wedge\varphi^{\circ}\phi(f_a(x_2)))$$

Therefore, $\varphi\circ\phi(f)_c(z_1.z_2^{-1})\geq\min\{\varphi\circ\phi(f)_c(z_1), \varphi\circ\phi(f)_c(z_2)\}$. Then $(\varphi\circ\phi, \chi\circ\psi)$ (f, A) is a fuzzy soft group over Z. Since, ((f, A), $\vee$, $\wedge$, $\subseteq$) is an l-fuzzy soft group over X for all $a_1, a_2\in A$, $a_1\leq a_2$ implies $f_{a1}\subseteq f_{a2}$. Then, we get $a_1\vee a_2 = a_2$, $a_1\wedge a_2 = a_1$ and $f_{a1}\vee f_{a2} = f_{a2}$, $f_{a1}\wedge f_{a2} = f_{a1}$, i.e., $f_{a1}(x)\vee f_{a2}(x) = f_{a2}(x)$, $f_{a1}(x)\wedge f_{a2}(x) = f_{a1}(x)$. And for each $c_1, c_2\in\chi(\psi(A))$, there exist $a_1, a_2\in A$ such that $\chi(\psi(a_1)) = c_1$ and $\chi(\psi(a_2)) = c_2$:

$$\begin{aligned}&\varphi\circ\phi(f)_{c1\vee c2}(z) = \varphi(\phi(f_{a1\vee a2}(x))) =\\ &\varphi(\phi(f_{a1}(x)\vee f_{a2}(x))) = \varphi(\phi(f_{a2}(x))) =\\ &\varphi\circ\phi(f)_{c2}(z) \therefore \varphi\circ\phi(f)_{c1\vee c2}(z) = \varphi\circ\phi(f)_{c1}(z)\vee\\ &\varphi\circ\phi(f)_{c2}(z) = \varphi\circ\phi(f)c_2(z),\ \forall x\in\phi^{-1}(y),\\ &\forall c\in\chi(\Psi(A)),\ \forall z\in Z\end{aligned}$$

Similarly, $\varphi\circ\phi(f)_{c1\wedge c2}(z) = \varphi\circ\phi(f)_{c1}(z)\wedge\varphi\circ\phi(f)_{c2}(z) = \varphi\circ\phi(f)_{c1}(z)$, $\forall_x\in\phi^{-1}(y)$, $\forall_c\in\chi(\psi(A))$, $\forall_z\in Z$. Hence, we get $c_1\leq c_2$ implies $\varphi\circ\phi(f)c_1(z)\subseteq\varphi\circ\phi(f)c_2(z)$, $\forall c_1, c_2\in\chi(\psi(A))\therefore(\varphi\circ\phi, \chi o\psi)$(f, A) is an l-fuzzy soft group over Z.

**Properties of l-fuzzy soft groups:** In this study, we present some properties of l-fuzzy soft groups.

**Theorem 4.1:** Union of two l-fuzzy soft groups over X is also an l-fuzzy soft subgroup over X.

**Proof:** Let ((f, A), $\vee$, $\wedge$, $\subseteq$) and ((g, B), $\vee$, $\wedge$, $\subseteq$) be two l-fuzzy soft groups over X. Then for all $a, b\in A$, $a\leq b$ implies $f_a\subseteq f_b$ and for all $a, b\in B$, $a\leq b$ implies $g_a\subseteq g_b$. Let $(f, A)\sqcup(g, B) = (h, C)$ where $C = A\cup B$. Let $h_c = f_c\vee g_c$, for all $c\in C$. For all $a\in A$, $f_a$ is a fuzzy subgroup of X and for all $b\in B$, $g_b$ is a fuzzy subgroup of X and $f_a\vee g_a = f_a$ or $g_a$ is a fuzzy subgroup of X. From this we get, for all $a, b\in A\cap B$, $a\leq b$ implies $f_a\vee g_a\subseteq f_b\vee g_b$. Then:

$$(h, C) = \begin{cases} f_a\subseteq f_b, & \text{for } a\leq b \quad \text{if } a, b\in A-B\\ g_a\subseteq g_b, & \text{for } a\leq b \quad \text{if } a, b\in B-A\\ f_a\vee g_a\subseteq f_b\vee g_b, & \text{for } a\leq b \quad \text{if } a, b\in A\cap B\end{cases}$$

Since, C is a lattice, for all $a, b\in C$, $a\leq b$ implies $h_a\subseteq h_b$. Then ((h, C), $\vee$, $\wedge$, $\subseteq$) is an l-fuzzy soft group over X.

**Theorem 4.2:** Intersection of two l-fuzzy soft groups over X is also an l-fuzzy soft subgroup over X.

**Proof:** Let ((f, A), $\vee$, $\wedge$, $\subseteq$) and ((g, B), $\vee$, $\wedge$, $\subseteq$) be two l-fuzzy soft groups over X. Then for all $a, b\in A$, $a\leq b$ implies $f_a\subseteq f_b$ and for all $a, b\in B$, $a\leq b$ implies $g_a\subseteq g_b$. Let $(h, C) = (f, A)\sqcap(g, B)$ where $C = A\cap B$. Let $h_c = f_c\wedge g_c$, for all $c\in C$. From this we get, for all $a, b\in A\cap B$, $a\leq b\Rightarrow f_a\wedge g_a\subseteq f_b\wedge g_b\Rightarrow h_a\subseteq h_b$. Hence by the theorem 2.14 ((h, C), $\vee$, $\wedge$, $\subseteq$ )) is an l-fuzzy soft subgroup over X.

**Theorem 4.3:** If ((f, A), $\vee$, $\wedge$, $\subseteq$) and ((g, B), $\vee$, $\wedge$, $\subseteq$) are two l-fuzzy soft groups over X, then $(f, A)\wedge(g, B)$ is also an l-fuzzy soft subgroup over X.

**Proof:** Let $(h, C) = (f, A)\wedge(g, B)$, where $C = A\times B$. Let $h_{a1, b1} = f_{a1}\wedge g_{b1}$, for all $(a_1, b_1)\in A\times B$. Let ((f, A), $\vee$, $\wedge$, $\subseteq$) and ((g, B), $\vee$, $\wedge$, $\subseteq$) be two l-fuzzy soft groups over X. Then for all $a_1, a_2\in A$, $a_1\leq a_2$ implies $f_{a1}\subseteq f_{a2}$ and for all $b_1, b_2\in B$, $b_1\leq b_2$ implies $g_{b1}\subseteq g_{b2}$. Partial orders on A and B induce the partial order on C, for any $(a_1, b_1), (a_2, b_2)\in C$. If $(a_1, b_1)\leq(a_2, b_2)$ then $f_{a1}\subseteq f_{a2}$ and $g_{b1}\subseteq g_{b2}\Rightarrow f_{a1}\wedge g_{b1}\subseteq f_{a2}\wedge g_{b2}$. Then $(a_1, b_1)\leq(a_2, b_2)\Rightarrow h_{a1, b1}\subseteq h_{a2}, b_2$ for all $(a_1, b_1)\in A\times B$. Hence by the theorem 2.16 ((h, C), $\vee$, $\wedge$, $\subseteq$) is an l-fuzzy soft subgroup over X.

**Theorem 4.4:** If ((f, A), $\vee$, $\wedge$, $\subseteq$) and ((g, B), $\vee$, $\wedge$, $\subseteq$) are two l-fuzzy soft groups over X, then $(f, A)\wedge\Upsilon(g, B)$ is also an l-fuzzy soft subgroup over X.

**Proof:** Let $(k, C) = (f, A)\Upsilon(g, B)$, where $C = A\times B$. Let $k_{a1, b1} = f_{a1}\vee g_{b1}$, for all $(a_1, b_1)\in A\times B$. Let ((f, A), $\vee$, $\wedge$, $\subseteq$) and ((g, B), $\vee$, $\wedge$, $\subseteq$) be two l-fuzzy soft groups over X. Then for all $a_1, a_2\in A$, $a_1\leq a_2$ implies $f_{a1}\subseteq f_{a2}$ and for all $b_1, b_2\in B$, $b_1\leq b_2$ implies $g_{b1}\subseteq g_{b2}$. Partial orders on A and B induce the partial order on C, for any $(a_1, b_1), (a_2, b_2)\in C$. If $(a_1, b_1)\leq(a_2, b_2)$ then $f_{a1}\subseteq f_{a2}$ and $g_{b1}\subseteq g_{b2}\Rightarrow f_{a1}\vee g_{b1}\subseteq f_{a2}\vee g_{b2}$. Then $(a_1, b_1)\leq(a_2, b_2)$ $k_{a1, b1}\subseteq k_{a2, b2}$, for all $(a_1, b_1)\in A\times B$. Hence by the theorem 4.1 ((k, C), $\vee$, $\wedge$, $\subseteq$) is an l-fuzzy soft sub-group over X.

## CONCLUSION

In the present study, concept of l-fuzzy soft homomorphism has been scrutinized. This work focused on properties of l-fuzzy soft group. To extend this work, one can investigate the other algebraic structures such as modules, rings and fields and can be examined some approach of l-fuzzy soft group to decision making problem.

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