

A Comparative Study on Software Reliability Models with Shape Parameter of Type-2 Gumble Life Distribution

Hee-Cheul Kim

Department of Industrial and Management Engineering,
Namsseoul University, Cheonan, Republic of Korea

Abstract: Software reliability is one of the most basic and essential problems in software development. In order to detect the software failure phenomenon, the intensity function which is the instantaneous failure rate in the non-homogeneous Poisson process can have the property that it is constant, non-increasing or non-decreasing independently at the failure time. In this study, was compared the reliability performance of the software reliability model using the Type-2 Gumble life distribution with the intensity function from increasing to decreasing pattern in the software product testing process. In order to identify the software failure phenomenon, the parametric estimation was applied to the maximum likelihood estimation method. Therefore, in this study, was compared and evaluated software reliability using software failure time data. As a result because of the smaller the shape parameter of the Type-2 Gumble distribution is the lower the mean square error, the model of the smaller the shape parameter relatively efficient properties appears. However, from the reliability point of view, the reliability of is higher than that of and about shape parameter of the Type-2 Gumble life distribution. Through this study, the software design department will be able to help the software design by applying various life distribution and shape parameters and providing basic knowledge using software failure analysis.

Key words: Type-2 Gumble life distribution, mean square error, non-homogeneous Poisson process, box plot, mission time, software reliability

INTRODUCTION

Software reliability is a fundamental and essential factor affecting the reliability of computer systems. In terms of design attributes, it is different from hardware reliability. Thus, the failure of a computer system due to a software defect factor can cause tremendous property damage to software users. In conclusion, software reliability analysis techniques to reduce software defects during software development are fundamental and essential. In this situation, the reliability requirements of the software operator and the minimum testing cost must be met. In order to manage the minimum cost in the execution of software testing, it can be an efficient development process if the reliability pattern of software can be predicted in advance.

Therefore, the process of implementing software development using reliability, minimum cost and estimated delivery time for software delivery time is a basic and essential issue. Therefore, it is indispensable to develop a model for predicting the defects of software products. Many software reliability analysis models have been proposed to minimize this situation. Among the proposed models, the model using the Non-Homogeneous Poisson

Process (NHPP) (Gokhale and Trivedi, 1999) is a reliable model in terms of defect search analysis, it is known that the model assumes that if a fault occurs, it is removed immediately and no other faults occur during the debugging process. Also, was proposed an exponential software reliability growth model using the mean value function with S-shaped or exponential trends of software cumulative defects (Goel and Okumoto, 1979). By extending this model was proposed a technique to analyze the software reliability by applying generalized logistic testing effort function and change point parameter (Huang, 2005). In terms of fault observation, the S-shape model presents a learning process technique that software operators can use in software failure inspection tools (Chiu *et al.*, 2008). Based on this preliminary study, have studied the reliability performance of the software reliability model using the shape parameter of Gumble life distribution in software product testing.

MATERIALS AND METHODS

Type-2 Gumbel distribution: The Type-2 Gumbel distribution is a distribution that can represent various reliability features. The probability density function and

the cumulative distribution according to the shape parameter and are known as the following properties (https://en.wikipedia.org/wiki/Type-2_Gumbel_distribution):

$$f(t) = abt^{-a-1} e^{-bt^a}, a>0, b>0, t>0 \quad (1)$$

$$F(t) = e^{-bt^a} \quad (2)$$

Parameter estimation using finite-failure NHPP based on Type-2 Gumbel distribution: In the finite-fault NHPP Model, the intensity function and the mean value function (Kwon and Kim, 2016) of the NHPP of the Type-2 Gumbel distribution model using the Eq. 1 and 2:

$$\lambda(t|\theta, a, b) = \theta f(t) = \theta abt^{-a-1} e^{-bt^a} \quad (3)$$

$$m(t|\theta, a, b) = \theta F(t) = \theta e^{-bt^a} \quad (4)$$

In Eq. 3 and 4, θ is expected value of the defect that can be observed given finite failure time. If the time truncated model is used to the observation time (0, t) the likelihood function (Kim, 2016a, b) can be derived by the following relation using the Eq. 3 and 4:

$$L_{NHPP}(\Theta|\underline{x}) = \prod_{i=1}^n \lambda(x_i) \exp[-m(x_n)] = \prod_{i=1}^n \theta \left[abx_i^{-a-1} e^{-bx_i^a} \right] \times \exp[-\theta e^{-bx_n^a}] \quad (5)$$

Where:

$$\underline{x} = (x_1 \leq x_2 \leq x_3, \dots, \leq x_n)$$

$$i = 1, 2, \dots, n,$$

$$\Theta = \{\theta, a, b\}$$

Indicates parameter space. When shape parameter a is fixed, using the Eq. 5, the estimator $\hat{\theta}_{MLE}$ and \hat{b}_{MLE} must be meted the following conditions for the maximum likelihood estimation about each parameters:

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\theta} e^{-bx_n^a} = 0 \quad (6)$$

In Eq. 6, solving for θ :

$$\hat{\theta}_{MLE} = \frac{n}{e^{-bx_n^a}}$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{b} \sum_{i=1}^n x_i^{-a} + \theta x_n^{-a} e^{-bx_n^a} = 0 \quad (7)$$

RESULTS AND DISCUSSION

Type-2 performance analysis of the software failure time:

In this study, using the failure time structure, the property for software reliability model considering the shape parameter of the Type-2 Gumbel distribution that controls the several life style distribution were studied. Table 1 is information of the software failure time (Kuo and Yang, 1995). In order to confirm the reliability of the data in terms of data, a trend test should be preceded (Kim, 2015). In this study, the trend analysis was used box plot. In Fig. 1, the result of the box figure trend test shows that the 31st data is the extreme value, so in this study, only up to the 30th data was used for the parameter estimation except the 31st data.

The estimation approximation value of the parameters for the projected model was used the maximum likelihood method. In this study, the numerical conversion (Failure time (hours)×0.01) for the parameter estimation was used. A consequence of the parameter estimation was attained from Table 2. These calculations to estimate the parameter, solving mathematically because the initial values were given 0.001 and 5.000 and tolerance value for the measurement of interval (10^{-5}) were specified were accomplished repetition of 100 times using C-Language checking acceptable convergent.

The mathematical formula of Mean Square Error (MSE) (Chiu *et al.*, 2008) indicates measure that is often used to tell the difference between the actual value (observed value) and the estimated value that is the residual value. It was obtained as next measure:

$$MSE = \frac{\sum_{i=1}^n [m(x_i) - \hat{m}(x_i)]^2}{n-k} \quad (8)$$

Also, R^2 (coefficient of determination) (Chiu *et al.*, 2008) indicates the predictive measure of the difference among the forecasting values:

$$R^2 = 1 - \frac{\sum_{i=1}^n [m(x_i) - \hat{m}(x_i)]^2}{\sum_{i=1}^n \left(m(x_i) - \sum_{j=1}^n m(x_j)/n \right)^2} \quad (9)$$

Where:

$m(x_i)$ = The cumulated number of the faults detected in (0, x_i)

$\hat{m}(x_i)$ = Estimating cumulated number of the faults detected in (0, x_i) specifies the number of realizing values

k = The number of the parameter (Tae-Hyun, 2015)

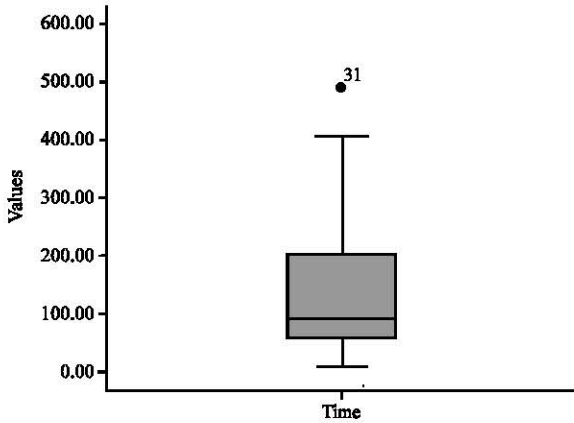


Fig. 1: Results of box plot failure time data

Table 1: Failure time data (Kuo and Yang, 1995)

Failure No.	Failure time (h)	Failure interval time	Failure time $\times 10^{-2}$
1	9	9	0.09
2	21	12	0.21
3	32	11	0.32
4	36	4	0.36
5	43	7	0.43
6	45	2	0.45
7	50	5	0.50
8	58	8	0.58
9	63	5	0.63
10	70	7	0.70
11	71	1	0.71
12	77	6	0.77
13	78	1	0.78
14	87	9	0.87
15	91	4	0.91
16	92	1	0.92
17	95	3	0.95
18	98	3	0.98
19	104	6	1.04
20	105	1	1.05
21	116	11	1.16
22	149	33	1.49
23	156	7	1.56
24	247	91	2.47
25	249	2	2.49
26	250	1	2.50
27	337	87	3.37
28	384	47	3.84
29	396	12	3.96
30	405	9	4.05
31	540	135	5.40

Table 2: MLE, MSE and R² for the each model

Models	MLE	Model comparison	
		MSE	R ²
a = 1	$\hat{b}_{MLE} = 7.247 \times 10^{-1}$ $\hat{\theta}_{MLE} = 35.876$	2.5588	0.9773
a = 2	$\hat{b}_{MLE} = 1.507 \times 10^{-1}$ $\hat{\theta}_{MLE} = 30.277$	148.01	0.5752
a = 3	$\hat{b}_{MLE} = 1.885 \times 10^{-2}$ $\hat{\theta}_{MLE} = 30.001$	176.49	0.4608

Maximum likelihood estimation, MSE: Mean Square Error, R²: Coefficient of determination

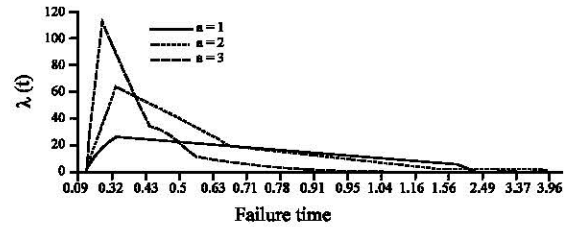


Fig. 2: Types of intensity function; intensity function vs. failure time

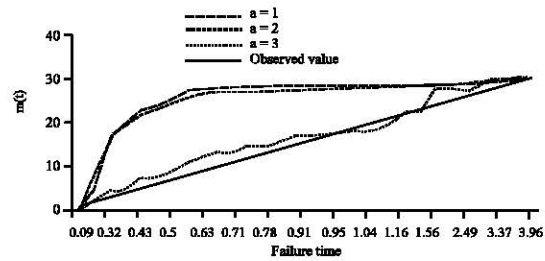


Fig. 3: Types of mean value function; mean value function vs. failure time

For the software model valuation in Table 2 because of the smaller the shape parameter of the Type-2 Gumbel distribution is the lower about the mean square error, the model of the smaller the shape parameter relatively efficient properties appears. Also, in terms of R² because of the smaller the shape parameter of the Type-2 Gumbel distribution is the higher about the coefficient of determination, the model of the smaller the shape parameter relatively helpfulness model performs.

The consequence of intensity functions was itemized in Fig. 2. Since, this Fig. 2, the case of Type-2 Gumbel distribution model, the pattern of intensity functions shows decreasing after increase as the failure time passes. Additionally, the case of the higher shape parameter, the higher appears the estimated value of the intensity function.

The outcome of mean value functions was enumerated in Fig. 3. From Fig. 3, the shapes of mean value function have the tendency of the non-decreasing form. Additionally, from contrast about the mean value function as compared to the true value, can be seen that the smaller the shape parameter, the smaller the difference between the estimated value of the mean value function and the true value.

In Fig. 4, in terms of judgment of reliability, from the reliability point of view, the reliability of a = 3 is higher than that of a = 1 and a = 2 about shape parameter of the Type-2 Gumbel life distribution as mission time goes on.

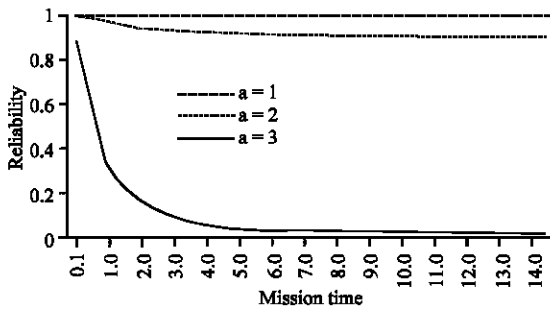


Fig. 4: Types of reliability function; reliability vs. mission time

CONCLUSION

Software reliability is one of the most basic and essential problems in software development. In this study, the reliability software reliability model, depend on the shape parameter of the Type-2 Gumbel distribution that controls the distribution of several life style from the administration of software product testing was planned. The resulting decisions were obtained.

Eventually, the model of the smaller the shape parameter of the Type-2 Gumbel distribution relatively efficient properties appears. Also, in terms of R^2 , the smaller the shape parameter of the Type-2 Gumbel distribution is the higher about the coefficient of determination, the model of the smaller the shape parameter relatively helpfulness model performs.

Finally, in terms of judgment of reliability, from the reliability point of view, the reliability of $a = 3$ is higher than that of $a = 1$ and $a = 2$ about shape parameter of the Type-2 Gumbel life distribution as mission time goes on. In this study, software designers can be helpful when used as basic material information in software production.

ACKNOWLEDGEMENT

Funding for this study was provided by Namseoul University.

REFERENCES

- Chiu, K.C., Y.S. Huang and T.Z. Lee, 2008. A study of software reliability growth from the perspective of learning effects. *Reliab. Eng. Syst. Safety*, 93: 1410-1421.
- Goel, A.L. and K. Okumoto, 1979. Time-dependent error-detection rate model for software reliability and other performance measure. *IEEE Trans. Reliab.*, R-28: 206-211.
- Gokhale, S.S. and K.S. Trivedi, 1999. A time/structure based software reliability model. *Ann. Software Eng.*, 8: 85-121.
- Huang, C.Y., 2005. Performance analysis of software reliability growth models with testing-effort and change-point. *J. Syst. Software*, 76: 181-194.
- Kim, H.C., 2015. The property of learning effect based on delayed software S-Shaped reliability model using Finite NHPP Software cost model. *Indian J. Sci. Technol.*, 8: 1-7.
- Kim, H.C., 2016a. A performance analysis of software reliability model using Lomax and Gompertz distribution property. *Indian J. Sci. Technol.*, 9: 1-6.
- Kim, H.C., 2016b. The comparative study for statistical process control of software reliability model based on finite and infinite NHPP Using rayleigh distribution. *Intl. J. Soft Comput.*, 11: 165-171.
- Kuo, L. and T. Yang, 1995. Bayesian computation of software reliability. *J. Comput. Graphical Stat.*, 4: 65-82.
- Kwon, H.K. and H.C. Kim, 2016. A comparative software development cost model based on hazard function of Lindley distribution. *Intl. Inf. Inst. Tokyo Inf.*, 19: 5137-5144.
- Tae-Hyun, Y., 2015. The infinite NHPP software reliability model based on monotonic intensity function. *Indian J. Sci. Technol.*, 8: 1-7.