

New Tuning Approach of Fuzzy Logic System using Proportional Integral Observer for Tracking a Nonlinear System

Mustefa Jibril

School of Electrical and Computer Engineering, Dire Dawa Institute of Technology, Dire Dawa, Ethiopia

Key words: Proportional integral observer, fuzzy proportional integral observer, active suspension system

Corresponding Author: Mustefa Jibril School of Electrical and Computer Engineering, Dire Dawa Institute of Technology, Dire Dawa, Ethiopia

Page No.: 127-133 Volume: 16, Issue 6, 2021 ISSN: 1816-9503 International Journal of Soft Computing Copy Right: Medwell Publications

INTRODUCTION

In-state feedback compels systems, all denote state variables are requisite for feedback to set the system unchangeability. Proportionate Integral Observer (PIO) was foremost designed by S. Beale and B. Shafai to design observer-based controller formatting less sensitivity to the incessant fluctuation of the system by adding an integral law to the observer which provides the extra extent of freedom^[1].

The essential look of the FLC is that old to modify the interaction changing based on the entropy of the manipulator into correction front practical to the design under control. A standard mechanism much as PID Abstract: Proportional Integral Observer (PIO) for tracking a nonlinear method has a lower sentiency to cipher the state and output variables. So, a more nonlinear controller has to be else to control to activity. In this study, a fuzzy logic (FLC) controller has been added to the PIO to meliorate the calculation transmute. A Fuzzy Proportional Integral Observer (FPIO) for following a nonlinear system has been premeditated to decimate the susceptibleness to cipher the tell and turnout variables with the existent posit and product variables. The FPIO controller has been tested for improving the estimation control using a nonlinear quarter vehicle active suspension system with a nonlinear hydraulic actuator. A comparison simulation of the proposed nonlinear system for estimating the state variables and tracking the output (suspension deflection) with a set point bump road disturbance using FPIO and PIO. The comparison simulation result shows that the estimated state variables and system output match the actual ones perfectly using a fuzzy PIO controller.

individual is efficacious and offers a regnant technique to linear systems. In the debate of nonlinear systems classical supervisor does not yield copasetic value due to the nonlinearities of these methods^[2]. Therefore, FLC may be an effectual stillness to moderate these nonlinear systems.

In this study, a fuzzy proportional integral observer-based controller has been designed to eliminate the delicateness to parameter change of the system completely by using the tracking system method. This system is experimentation to prove the perfectness of the estimated state with the actual estate and the precision computations of the output variable.

MATERIALS AND METHODS

Proportional integral observer control: The state-space method of an nth inflict, p input and q output complex with Eq. 1 independent of invariable parameter reckon becomes as:

$$\begin{array}{l} \mathbf{x}^{*} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{d} \\ \mathbf{d}^{*} = \mathbf{0} \end{array} \right\}$$
 (1)

The schemes state vector x is $n \times 1$, the outline input vector u is $p \times 1$ and the independent disturbance d is an 1×1 vector. In the situation, of the unwanted disturbance, mold is unknown, the matrix form E can be assumed to be an identity matrix with the same order of the system. The output y, a q×1 vector, of the design is:

$$y = Cx$$
 (2)

The proportional integral observer is constructed to approximation state variables using the system input and output. The state-space model of the proportional-integral observer becomes:

$$\hat{\mathbf{x}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}})$$
(3)

where, x° is the estimated state variables. Subtract Eq. 3 from (Eq. 1), letting $e = x \cdot x^{\circ}$ where the error between actual and estimated variables and disturbances, so that:

$$\dot{\mathbf{e}} = \dot{\mathbf{x}} - \hat{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{E}\mathbf{d} - (\mathbf{A} - \mathbf{L}\mathbf{C})$$
$$\hat{\mathbf{x}} - \mathbf{L}\mathbf{y} = (\mathbf{A} - \mathbf{L}\mathbf{C})\mathbf{e} + \mathbf{E}\mathbf{d}$$
(4)

If there is no disturbance in intrigue (d = 0), a proportional-integral observer has the ability to approximation the state variables, if (A-LC) is Routh-Hurwitz stable in which all eigenvalues of (A-LC) have negative actuality parts^[3]. However, if d is a non-zero constant, their testament is a constant steady-state error between the estimated and actual state variables. To eliminate this error in estimation, Disturbance Observer (DO) is described as follows:

$$\dot{\hat{x}} = A\hat{x} + Bu + L_{P}(y - C\hat{x}) + E\hat{d}$$

$$\dot{\hat{d}} = L_{I}(y - C\hat{x})$$

$$(5)$$

The state-space model of the proportional-integral observer becomes:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{E}\mathbf{v} + \mathbf{G}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}})$$

$$\dot{\mathbf{v}} = \mathbf{F}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}})$$

$$(6)$$

Comparison of Eq. (5) and (6) shows that it is obvious that disturbance bystanders can be regarded as a



Fig. 1: Block diagram of proportional-integral observer tracking system

proportional-integral observer in a special case. The system block of proportional-integral observer for the tracing method is shown in Fig. 1.

The state-space model of the system with the unwanted disturbance is shown in Eq. 1. The 2 equations can be combined and defined as $z = \begin{bmatrix} x \\ x \end{bmatrix}$, so that:

$$z = \begin{bmatrix} d \end{bmatrix}$$
, so that

$$\dot{z} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u = A_z z + B_z u$$
(7)

The output of the system is given by the above equations yields:

$$y = Cx = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} = C_z z$$
(8)

The state-space model of the undesired disturbance observer given by Eq. 5 can also be rewritten into the above equations by defining $\hat{z} = \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}$:

$$\dot{\hat{z}} = \begin{bmatrix} A - L_p C & E \\ -L_1 C & 0 \end{bmatrix} \hat{z} + \begin{bmatrix} L_p \\ L_1 \end{bmatrix} y + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$
(9)

This implies:

$$\begin{split} \mathbf{A}_{\mathbf{Z}(n+l\times n+l)} &= \begin{bmatrix} \mathbf{A} & \mathbf{E} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \\ \mathbf{B}_{\mathbf{Z}(n+l\times p)} &= \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} \\ \mathbf{C}_{\mathbf{Z}(q\times n+l)} &= \begin{bmatrix} \mathbf{C} & \mathbf{0} \end{bmatrix} \\ \mathbf{L}_{\mathbf{Z}(n+l\times q)} &= \begin{bmatrix} \mathbf{L}_p \\ \mathbf{L}_I \end{bmatrix} \end{split}$$



Fig. 2: Error membership function

. .

Table .	1: Fuzzy rule ba	ase										
u	Δe											
	-	NVB	NB	NM	NS	Z	PS	PM	PB	PVB		
e	NVB	PVB	PVB	PVB	PB	PM	PM	PS	Z	Ζ		
	NB	PVB	PVB	PB	PM	PS	PS	PS	Z	Ζ		
	NM	PVB	PB	PM	PS	PS	Z	Z	Z	NS		
	NS	PB	PM	PM	PS	PS	Z	Z	NS	NS		
	Z	PM	PM	PS	Z	Z	Z	NS	NS	NM		
	PS	PM	PS	PS	Z	NS	NS	NM	NM	NB		
	PM	PS	PS	Z	NS	NS	NM	NB	NB	NB		
	PB	PS	Z	Z	NS	NM	NM	NB	NVB	NVB		
	PVB	Z	Z	NS	NM	NM	NB	NB	NVB	NVB		

Equation 9 becomes:

$$\dot{\hat{z}} = \left(\begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} L_p \\ L_1 \end{bmatrix} \begin{bmatrix} C & 0 \end{bmatrix} \right) \hat{z} + \begin{bmatrix} L_p \\ L_1 \end{bmatrix} y + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

$$= (A_z - L_z C_z) \hat{z} + L_z y + B_z u$$
(10)

Subtracting Eq. 7 from Eq. 10, letting $e = z-z^{*}$ which is the error between actual and estimated variables and the undesired disturbances, so that: Noticing that y = Czz in Eq. 8, the equation above can be described as:

$$\dot{e}_{z} = A_{z}z - L_{z}C_{z}z - (A_{z} - L_{z}C_{z})\hat{z} = (A_{z} - L_{z}C_{z})e$$
 (11)

Whereas long as:

$$\mathbf{A}_{\mathrm{Z}} - \mathbf{L}_{\mathrm{Z}}\mathbf{C}_{\mathrm{Z}} = \begin{bmatrix} \mathbf{A} - \mathbf{L}_{\mathrm{p}}\mathbf{C} & \mathbf{E} \\ -\mathbf{L}_{\mathrm{I}}\mathbf{C} & \mathbf{0} \end{bmatrix}$$

is Routh-Hurwitz stable, the error between actual and estimated variables will become zero as $t\rightarrow\infty$. Recall Eq. 8 and 10:

$$\dot{\hat{z}} = (A_z - L_z C_z) \hat{z} + L_z y + B_z u$$

$$y = C_z z$$

$$(12)$$

Comparisons of the above equations with the state-space model of proportional-integral observer described by Eq 2 and 3:

$$\dot{\hat{x}} = (A - LC)\hat{x} + Ly + Bu$$

$$y = Cx$$

$$(13)$$

The proportional integral observer can be applied to onlooker gain L planning for the proportional-integral observer with this extended onlooker model for tracking systems^[4].

Fuzzy controller design: In closed-loop dominion systems, the classical controller has been replaced by the FLC. This quantity that the IF-THEN rules and fuzzy membership functions replace the mathematical rule to experiment the system^[5]. For the fuzzy logic mechanism, the signal variables are error (e) and derivative of error (de/dt) and the output Sus_def (suspension deflection) (u). Gaussian membership functions are utilized for inputs variables and the output. An error has 9 membership functions as shown in Fig. 2, the derivative of error has 9 membership functions as shown in Fig. 4 and 5. The Mamdani-based fuzzy logic controller is selected for this paper as shown in Fig. 6.

Fuzzy rule base: The fuzzy input variable e has nine membership functions and fuzzy input variable Δe has nine membership functions and the output variable has nine membership functions. There are 81 rules generated as shown in Table 1.

The proposed controller design: To dominion the tracking system, a fuzzy logic supervisor has been added





Fig. 3: Change of error membership function



Fig. 4: Output membership function



Fig. 5: Surface membership function



Fig. 6: Mamdani based fuzzy logic controller

to the system to regulator the process using the difference between system output y and reference input r. The block design of the fuzzy PIO schemes is shown in Fig. 7.



Fig. 7: Block diagram of the plant with FPIO

RESULTS AND DISCUSSION

The system with fuzzy PIO is designed in this section with nonlinear quarter vehicle active suspension with hydraulic actuator for state and output estimation by comparing the actual state with the estimated state with FPIO and PIO tracking systems.

Case study: The nonlinear model description of the quarter vehicle active suspension system and the hydraulic actuator can be described as:

$$\begin{split} \mathbf{M}_{1}\ddot{\mathbf{x}}_{1} &= \mathbf{k}_{1}^{1}\left(\mathbf{x}_{2} - \mathbf{x}_{1}\right) + \mathbf{k}_{1}^{nl}\left(\mathbf{x}_{2} - \mathbf{x}_{1}\right)^{3} + \mathbf{B}_{1}^{1}\left(\dot{\mathbf{x}}_{2} - \dot{\mathbf{x}}_{1}\right) \\ &- \mathbf{B}_{1}^{mix}\left|\dot{\mathbf{x}}_{2} - \dot{\mathbf{x}}_{1}\right| + \mathbf{B}_{1}^{nl}\sqrt{\left|\dot{\mathbf{x}}_{2} - \dot{\mathbf{x}}_{1}\right|} \mathrm{sgn}\left(\dot{\mathbf{x}}_{2} - \dot{\mathbf{x}}_{1}\right) - \mathbf{F} \end{split}$$

$$\begin{split} M_{2}\ddot{x}_{2} &= -k_{1}^{1}\left(x_{2} - x_{1}\right) - k_{1}^{nl}\left(x_{2} - x_{1}\right)^{3} + B_{1}^{l}\left(\dot{x}_{2} - \dot{x}_{1}\right) - B_{1}^{mix}\left|\dot{x}_{2} - \dot{x}_{1}\right| \\ &- B_{1}^{nl}\sqrt{\left|\dot{x}_{2} - \dot{x}_{1}\right|sgn\left(\dot{x}_{2} - \dot{x}_{1}\right)} + k_{2}^{nl}\left(x_{2} - \dot{z}\right) + B_{2}^{nl}\left(x_{2} - \dot{z}\right) + F \end{split}$$

Where:

- M_1 = Quarter body mass
- M_2 = Quarter suspension mass
- = Body travel displacement \mathbf{X}_1
- = Suspension deflection displacement \mathbf{X}_2
- $k_1^{\overline{l}}$ = Linear spring stiffness between mass 1 and mass 2
- k^{nl} = Nonlinear spring stiffness between mass 1 and mass 2
- k^{nl}2 = Nonlinear spring stiffness between mass 2 and road profile
- \mathbf{B}^{1} = Linear damping between mass 1 and mass 2
- B^{nl} = Nonlinear damping between mass 1 and mass 2
- B_{1}^{mix} = Mixed damping effect between mass 1 and mass 2
- \mathbf{B}^{nl} = Nonlinear damping between mass 2 and road profile
- Ζ = Road profile displacement

And:

$$Q = sgn\left[P_{s} - sgn(y_{d})P_{0}\right]R_{d}\sqrt{\frac{1}{\rho}}\left[P_{s} - sgn(y_{d})P_{0}\right]$$

Where:

- Q = The hydraulic load drift
- \mathbf{R}_{d} = The release coefficient, is the spool valve vicinity gradient
- P_0 = The pressure inside the chamber of the hydraulic piston
- y_d = The valve displacement from its closed position
- ρ = The hydraulic fluid density P_s = The supply pressure

The parameters of the nonlinear quarter car active suspension and the hydraulic actuator are shown in Table 2.

Tuning method: PIO is an algorithm that iteratively runs until it manages to get the minimum of a function. The PIO is used to moderate the membership function tuning for FLC. The objective function was from the predicted output compared to the input given. It is a simple mathematical method that is based on a differentiation equation where the initial point output was the move towards the targeted output by calculating the errors. The 2 important parameters need to be considered which are the direction of movement and the size of the step that needs to be used. The direction of movement defines by the tangential of the initial point. The sharpness of the tangent line also shows how near the point to the minimum point and how to decide the learning rate that should be selected. Fig. 8 shows the flow diagram of the Table 2: System parameters

Parameters	Symbols	Values
Quarter body mass	M ₁	600 kg
Quarter suspension mass	M ₂	130 kg
Linear spring stiffness between	k_1^l	1700 N/m
mass 1 and 2		
Nonlinear spring stiffness	k ^{nl} 1	1850 N/m
between mass 1 and 2		
Nonlinear spring stiffness between	k_2^{nl}	19700 N/m
mass 2 and road profile		
Linear damping between mass 1 and 2	\mathbf{B}_{1}^{1}	900 Ns/m
Nonlinear damping between mass 1 and 2	\mathbf{B}_{1}^{nl}	980 Ns/m
Mixed damping effect between	$\mathbf{B}_{1}^{\text{mix}}$	1280 Ns/m
mass 1 and 2		
Nonlinear damping between mass 2	\mathbf{B}_{2}^{nl}	680 Ns/m
and road profile		
Release coefficient of the spool valve	R _d	0.4
vicinity gradient		
Hydraulic fluid density	ρ	2900 kg m ⁻³
Supply pressure	Ps	6.8 MPa



Fig. 8: Flow diagram of FLC tuning algorithm

FLC tuning algorithm. From Fig. 8, the PIO will keep on running until the optimum condition is generated or the iteration reach. The initial state of the system is $x_0 = (0.0)$ $(0 0)^{T}$. The PIO state-space representation is:

$$A_{\text{PIO}} = \begin{pmatrix} -22.43 & 8 & 19.43 \\ -31.4 & -18 & 105.4 \\ -44.63 & 40 & 14.83 \end{pmatrix},$$
$$B_{\text{PIO}} = \begin{pmatrix} 27.13 \\ 101.7 \\ 38.43 \end{pmatrix}, C_{\text{PIO}} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, D_{\text{PIO}} = 0$$

The MATLAB/Simulink model of the quarter vehicle active suspension system with fuzzy PIO and PIO tracking systems is shown in Fig. 9.

The output feedback gains K1, K2 and K3 for tracking the system without observer are obtained by using pole placement and it becomes as:

$$K_1 = 1.25, K_2 = 0.25, K_3 = 0.75$$

The input for the system is a bump road disturbance of 10 cm as shown in Fig. 10. The simulation

Int. J. Soft Comput., 16 (6): 127-133, 2021



Fig. 9: Simulink model of the proposed system



Fig. 10: Road disturbance input



Fig. 11: Estimated and actual state variable x1



Fig. 12: Estimated and actual state variable x2



Fig. 13: Estimated and actual state variable x3



Fig. 14: Estimated and actual system output y

of the actual and estimated state variables x1, x2, x3 and the output variable y are shown in Fig. 11-14, respectively.

From the simulation result in Fig. 11-14, the estimated state variables x1 and x2 and the system output y exactly match the actual ones perfectly using FPIO tracking systems. The estimation of the state variable x3 (body travel) shows a small variation which is dependent on the nonlinear design of the quarter car active suspension system.

CONCLUSION

In this study, a fuzzy proportional integral observer for tracing a nonlinear gadget has been designed to eliminate the sensitivity of estimating the state and output variables of the proposed system. The proposed controller has been tested with a nonlinear quarter vehicle active suspension system with a hydraulic actuator. Comparison of this system with FPIO and PIO controllers have been made for tracing a bump road disturbance. The estimated state and tracking of the design capability comparison results show that the system with FPIO estimated and actual state and design authority have exactly matched.

RECOMMENDATION

In future work, this intrigue can be improved by modifying the fuzzy membership functions.

REFERENCES

01. Dong, X., 2016. Proportional Integral Observer (PIO) design for linear control systems. Master, Thesis, University of Rhode Island, South Kingstown, Rhode Island.

- 02. Namazov, M., 2018. Fuzzy logic control design for 2-link robot manipulator in MATLAB/simulink via robotics toolbox. Proceedings of the 2018 Global Smart Industry Conference (GloSIC), November 13-15, 2018, IEEE, Chelyabinsk, Russia, pp: 1-5.
- 03. Bakhshande, F. and D. Soffker, 2015. Proportional-integral-observer: A brief survey with special attention to the actual methods using ACC benchmark. IFAC. Papers Online, 48: 532-537.
- 04. Liu, B., X. Luo and J. Li, 2014. Conventional and added-order proportional nonlinear integral observers. Int. J. Mod. Nonlinear Theory Appl., 3: 210-220.
- Abad, A.C., D.D. Ligutan, E.P. Dadios, L.J.S. Cruz, M.C.D.P. Del Rosario and J.N.S. Kudhal, 2018. Fuzzy logic-controlled 6-DOF robotic arm color-based sorter with machine vision feedback. Int. J. Adv. Comput. Sci. Appl., 9: 21-31.