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Parameter Estimation of Flexible Space Craftusing Least Square Estimation

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Abstract: A well modeled system response deviate from the desired response due to lack of complete knowledge about the system dynamics and other practical limitations. Also, all the critical systems exhibit stochastic nature. Hence, the parameters change dynamically. It is essentially required to estimate the parameters dynamically. The system identification (SI) helps to estimate the parameters, assuming the model of the system under operation is known. The parameters of the dynamics are estimated from the input and output data of the system. Here, an attempt is made to estimate the parameters of a Flexible Space Craft (FSC) using least square (LS) estimation method for both noise and noise free environment. Since, the digital computers operate on digital data easily, the transfer function of the FSC is transformed into discrete domain and computations are made. It is observed that the sampling time and sampling technique play critical role in estimating the parameter. The SI is a useful mathematical tool to estimate appropriate parameters by correlating the excitation and response of the system under consideration.

Key words: System Identification (SI), sampling (Ts) time, least square estimation

INTRODUCTION

The system Identification (SI) is defined, as the determination (Rodriguez, 1984) of parameters on the basis of input and output data of the system within the specified class of systems under consideration. Any model represents 3 types of knowledge base, viz, structure, flow and connections expressed in terms of mathematical identities the parameter values i.e., those quantities show no dependence on input and variables/states which are dependent (Fig. 1). The SI process is of 2 types. They are off-line or batch identification process and on-line or recursive or real-time process (Ljung and Soderstrom, 1983).

The SI, in general, as follows:

- Determine the appropriate linear model.

$$Y[k] = x[k] + n[k] \quad (1)$$

$$x[k] = h[k] * u[k] \quad (2)$$

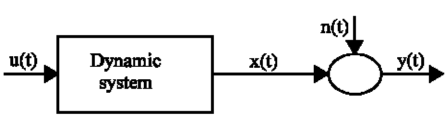

Fig. 1: Generic system block diagram

- Selection criteria for determining the accuracy of the model.
- Designing appropriate input signal which will optimize the system parameters.

The SI process requires the repeatedly selecting model structure and evaluating the model properties in order to obtain satisfactory results. There is no fixed procedure for identification and the following procedure may be followed satisfactorily:

i Collect the input and output data of the process by conducting a suitable experiment.
ii Examine the data physically so as to remove the trends, outliers and consistent data to select the useful portion from the original data by applying proper techniques such as filtering etc.
iii Select and define a model structure within which the model is to be identified.
iv Compute the best model in the structure according to the data and the selected criterion of fit.
v Examine the obtained model properties.
vi If the model obtained is satisfactory terminate the process otherwise repeat (iii) to (v).

Any SI ultimately lies in the method used to estimate the parameters of the model. Parameter estimator is the

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information and processing the elements of the entire process. The parameter estimation is done in either in Time domain or in Frequency domain. A large variety of methods have been applied to SI for both Off-line and On-line identification processes (Eykhoff, 1974). One such schema of classification is as follows:

Classical Methods:

- Frequency response identification.
- Identification from the step response.
- Impulse response identification by the disconsolation.

Model adjustment techniques:

- Least-Square estimation
- Generalized least-square
- Instrument variable method
- Boot-strap method.
- Maximum likely-hood method
- Correlation method.
- Stochastic approximation.

The literature suggests the following conclusions:

- The least-square algorithm is the most efficient approach for parameter estimation forthe low noise level data. This method converges rapidly.
- The stochastic approximation of quantity requires the least amount of computation and yields good estimations. But, the convergence rate is slow.
- For high noise contaminated data the correlation method is well suitable for parameter estimation but this approach requires sufficient amount of data to get a reasonable initial estimation of the correlation ordinates.
- The general least-square method gives quite better results but requires large amount of computation.
- The maximum likely-hood method gives very good estimation for Off-line process. In the present work the least-square estimation is presented as the data is contaminated by little noise.

THE FLEXIBLE SPACE-CRAFT MODEL

The altitude and orbit control system (AOCS) is responsible for maintaining the desired orbit and orientation of a satellite to accomplish the mission goals. As shown in Fig. 2, it consists of a set of actuators and sensors such as thrusters, wheels, earth sensors, gyros, sun sensors etc. and the control electronics implement the control strategy (Eleftheriou and Falconer, 1986). Any

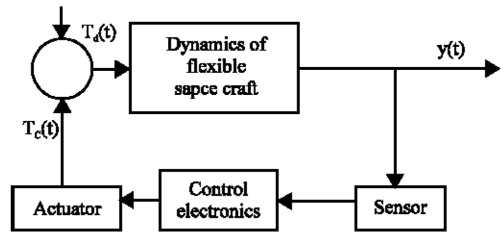

Fig. 2: Control block diagram of flexible spacecraft

hidden accuracy in the model is directly reflected in the degradation of the system performance. SI approach is therefore removes various model inaccuracies and finally results in high performance system.

THE FSC TRANSFER FUNCTION

In contrast to a rigid body, a flexible body has some internal potential energy and also the kinetic energy of motion. The resulting motions are called vibrations of flexible structures. In a FSC, these vibrations interact with the rigid body motion, thereby altering the dynamic behavior. In the present work the following model of FSC has been adopted (Sinha and Kutza, 1983).

$$G(s)=\frac{1}{Is^2}\left[1-s^2\sum\frac{r_i}{s^2+2\xi_i\omega_i s+\omega_i^2}\right]^{-1} \qquad (3)$$

Where,

I : Moment of inertia.
r : Model coefficient.
ξ : Model damping.
ω : Model frequency.
n : No. of modes included in the model. The range of 'I' is taken from 1- 4 and is known as ith mode of vibration.

The parameters considered as:

r_i = (0.4 0.2 0.1 0.08).
ω_1 = (0.8 1.5 4.2 10.5).
ξ_1 = (0.01 0.05 0.08 0.09).
I = 1000 Kg m^2

The transfer function polynomials of both numerator (NUM) and denominator (DEN) are tabulated in Table 1, for the parameter r_i, ω_i, ξ_i and I as mentioned. The mapping 's' domain to 'z' domain is done by choosing appropriate sampling period and the method of sampling like 'ZOH', 'FOH', 'TUSTIN'.

Table 1: The list of coefficients of ‘s’

Coefficients		s^0	s^1	s^2	s^3	s^4	s^5	s^6	s^7	s^8	s^9	s^{10}
G_1 (s)	NUM	1.067e-3	2.667e-5	0.0017	0	0						
	DEN	0	0	1.0667	0.0267	1						
G_2 (s)	NUM	0.0036	3.3e-4	7.0231e-3	4.015e-3	2.5e-3	0	0				
	DEN	0	0	3.6	0.33	4.661	0.257	1				
G_3 (s)	NUM	0.847	0.0109	0.175	0.0167	0.0688	2.793e-3	3.33e-3	0			
	DEN	0	0	84.67	10.99	114.2	10.62	29	1.183	1		
G_4 (s)	NUM	12.73	1.87	26.48	2.947	10.63	0.62	0.6022	0.124	4.545e-3	0	0
	DEN	0	0	1.273e4	1870	1.731e4	1904	4524	265.3	185.4	3.886	1

Off-line identification of noise frees system: The parameter estimation of a linear of a linear model from the input-output data of a SISO system is considered and assumes that the model order is known priori. The off-line parameter estimation algorithm for a noise free data is as:

$$\dot{X}(t) = AX(t) + BU(t) \tag{4}$$

$$Y(t) = CX(t) + DU(t) \tag{5}$$

- Select the DTM of a SISO system. (The sampling must be uniform).
- Transform into ‘z’ domain from ‘s’ domain and choose appropriate sampling technique:

$$H(z) = \frac{X(z)}{U(z)} = \frac{\sum_{i=0}^{m} a_i z^{-1}}{1 + \sum_{j=0}^{n} b_j z^{-1}} \tag{6}$$

- Convert the ‘z’-transfer function into difference equation:

$$X(k) = \sum_{i=0}^{m} a_i u(k-i) - \sum_{j=0}^{n} b_j x(k-j) \tag{7}$$

- Collect various sets of data by conduction suitable experiments:

$$X = A^T T \tag{8}$$

- Verify for the non-singularity and positive definiteness for A(k).
- Compute the parameters from:

$$T = [A^T]^{-1} X \tag{9}$$

In the SI, the coefficients a_i, b_j, are estimated for various sampling times and various sampling techniques. These results are tabulated. It is observed that the ZOH sampling technique gives a closer true value of the approximation of the parameters than the TUSTIN sampling technique. However, the TUSTIN in well suitable for order determination.

Off-line identification of noisy system: Consider the dynamics of a noise corrupted system is as follows:

$$x(k+1) = Ax(k) + Bu(k) + KE(k) \tag{10}$$

$$y(k+1) = Cx(k) + Du(k) + E(k) \tag{11}$$

The algorithm for parametric identification of Noisy System (Grenander and Rosenblatt, 1997) is same as the algorithm discussed earlier except that the noise is also considered in the system dynamics and appropriate transformation are to be made to achieve the goal i.e., required parameters of the dynamic system. Substitute Eq. (3) in Eq. (1), upon simplification, we get that Y - E = AT and consider that the splitting the parameter vector to be estimated T into actual system parameter then:

$$Y - E = A^T \left\{ \tilde{T} - \hat{T}_e \right\} \tag{12}$$

i.e., $Y = A^T \tilde{T}$ and the error $E = A^T \hat{T}$

LEAST SQUARE ESTIMATION

Consider that obtaining is a measure of vector error equation and E (k) is to be minimized. The simplest case occur when the minimization of the norm-squared of the E (k) i.e., the performance index:

$$J = Y^T Y - Y^T A\hat{T} - \hat{T}^T A^T Y + \hat{T} A^T AT \tag{13}$$

For minimizing the J differentiating with respect to $\hat{T}$ and equating to zero we get:

$$\hat{T} = \left(A^T A\right)^{-1} A^T Y \tag{14}$$

Estimating for Eq. (14) is called least square estimation, as it minimizes the sum of the squares of the components of the error vector. A more general error criterion may be obtained by introducing the weighted sum of the squares of the components of Y is minimized i.e.,

$$J = Y^T W Y - Y^T W A \hat{T} - \hat{T}^T A^T W Y + \hat{T} A^T W A T \quad (15)$$

Where,

$$\hat{T} = \left(A^T W A\right)^{-1} A^T W Y$$

The Solution exist if A is of full Rank and the W must be positive definite and symmetric.

SIMULATION RESULTS

Both the noise free and noise contaminated data are considered. The experimental results are tabulated for various sampling techniques. Fertig and McClellan (1990) Transfer function is considered by the Eq. (3). Table 2 illustrates the coefficients of the polynomial of the system under examination in discrete domain with a sampling

Table 2: The list of coefficients of 'z' (for 'ZOH' Sampling)

Coefficients		z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}
G1 (z)	NUM	8.311e-6	-8.268e-6	-8.264e-6	8.327e-6	0						
	DEN	0.9973	-3.981	5.971	-3.987	1						
G_2((z)	NUM	1.22e-5	-3.462e-5	2.431e-5	2.427e-5	-3.681e-5	1.244e-5	0				
	DEN	0.974	-5.287	14.56	-19.47	14.69	-5.929	1				
G_3 (z)	NUM	1.488e-5	-7.23e-5	1.287e-4	-7.198e-5	-7.168e-5	1.336e-4	-7.74e-5	1.637e-5	0		
	DEN	0.884	-6.955	24.07	-48.13	60.78	-49.64	25.6	-7615	1		
G_4 (z)	NUM	1.534e-5	-9.103e-5	2.298e-4	-2.998e-4	1.453e-4	1.587e-4	-3.458e-4	2.852e-4	-1.19e-4	2.107e-5	
	DEN	0.678	-5.825	23.31	-57.7	98.49	-121.4	109.3	-70.4	30.81	-80201	1

Table 3: The list of coefficients of 'z' (using 'TUSTIN' Sampling)

Coefficients		z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}
G_1 (z)	NUM	4.153e-6	1.992e-8	-8.26e-6	3.32e-8	4.16e-6						
	DEN	0.9973	-3.9891	5.971	-3.987	1						
G_2 (z)	NUM	6.904e-6	-1.211e-5	-5.992e-6	2.423e-5	-6.295e-6	-1.211e-5	6.195e-6				
	DEN	0.9748	-5.829	14.57	-19.48	14.69	-5.929	1				
G_3 (z)	NUM	7.42e-6	-2.879e-5	2.852e-5	2.757e-5	-7.125e-5	3.121e-5	2.727e-5	3.0e-5	8.045e-6		
	DEN	0.893	-6.997	24022	-48.42	61.11	-49.86	25.68	-7.628	1		
G_4 (z)	NUM	7.876e-6	-4.022e-5	7.689e-5	-4.331e-5	-7.642e-5	1.599e-4	-9.943e-5	-2.891e-5	8.118e-5	-4.741e-5	9.906e-6
	DEN	0.734	-6.482	26.51	-66.52	113.9	-139.1	122.8	-77.05	32.72	-8.444	1

Table 4: The list of estimated coefficients of 'z' from the experiment Data

Coefficients		z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}
G_1 (z)	NUM	8.311e-6	-8.268e-6	-8.264e-6	8.237e-6	-2.512e-17						
	DEN	0.9973	-3.981	5.971	-3.987	1						
G_2((z)	NUM	1.22e-5	-3.642e-5	2.431e-5	2.427e-5	-3.681e-5	1.244e-5	7.147e-17				
	DEN	0.9746	-5.287	14.56	-19.47	14.69	-5.929	1				
G_3 (z)	NUM	1.488e-5	-7.328e-5	1.287e-4	-7.198e-5	-7.168e-5	1.336e-4	-7.748e-5	1.637e-5	-2.154e-16		
	DEN	0.884	-6.955	24.07	-48.13	60.78	-49.64	25.6	-7.615	1		
G_4 (z)	NUM	1.534e-5	-9.103e-5	2.298e-4	-2.2898e-4	1.453e-4	1.587e-4	-3.458e-4	2.852e-4	-1.197e-4	2.707e-5	-6289e-16
	DEN	0.678	-5.825	23.31	-57.7	98.49	121.4	109.3	-70.4	30.81	-8.201	1

Table 5(a): The estimated coefficients of's' (using 'ZOH')

Coefficients		z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}
G_1(z)	NUM	1.067e-3	2.667e-5	0.001667	9.423e-15	0.00						
	DEN	9.022e-11	-2.813e-11	1.0667	0.02667 1							
G_2(z)	NUM	0.0036	0.00033	0.007231	0.000415	0.0025	-9.659e-15	1.11e-16				
	DEN	1.145e-7	-1.688e-8	3.6	0.33	4.661	0.257	1				
G_3(z)	NUM	0.088467	0.01099	0.1752	0.1668	0.06881	0.002793	0.00333	-3.419e-15	-2.22e-16		
	DEN	-1.99e-5	2.017e-5	84.667	10.9867	114.233	10.62	29	1.183	1		
G_4(z)	NUM	17.53	1.757	36.22	2.388	14.26	0.1633	0.7404	-0.0215	0.004133	2.452e-4	2.849e-7
	DEN	1.234e-3	4.23e-5	1.753e-4	2635	2.373e-4	2689	6122	375.5	2236.5	5.299	1

Table 5(b): The Estimated coefficients of's' (using TUSTIN')

Coefficients		z_0	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9	z_{10}
G_1(z)	NUM	7.423e-10	-8.33e-5	0.001667	-2.669e-5	0.001069						
	DEN	6.245e-11	-1.81e-11	1.064	0.02674	1						
G_2(z)	NUM	0.003628	0.0001519	0.007258	0.0000552	0.002486	0.0001251	-9.55e-9				
	DEN	2.4299	3.3848	3.628	0.33333	4.685	0.2592	1				
G_3(z)	NUM	0.882	0.00717	0.1827	0.008465	0.0707	0.0006215	0.003232	0.000168	-4.995e-8		
	DEN	2.789e-7	5.022e-17	88.81	11.61	119.4	11.21	30.03	1.243	1		
G_4(z)	NUM	12.73	1.87	26.48	2.974	10.63	0.62	0.6022	0.0124	0.004545	-3149e-14	-6.601e-16
	DEN	0.03405	-4.852e-3	1.273e-4	1870	1.731e-4	1904	4524	265.4	185.4091	3.8864	1

period of 0.1 S. The Table 3 illustrates the coefficients of the polynomial of the system under examination in discrete domain with a sampling period of 0.1 S for TUSTIN. The estimated coefficients from the experimental data of the FSC model using various sampling approaches in the Least Square sense are tabulated in Table 4 and 5 (a and b). The coefficients of the discrete transfer function of G_1 (z), G_2 (z), G_3 (z), G_4 (z) are listed detailed in the tables shown.

CONCLUSION

Before applying the parameter estimation algorithm, the order of the transfer function was determined by converting the continuous time transfer function to the pulse transfer function. The order of DTM remains same irrespective of the method used the transform the CTM. However, the coefficients of the pulse transfer function, particularly the numerator coefficients do vary with the methods of transformation for a given sampling period. The order of the pulse transfer function also caries with the number of flexible modes, included in the FSC model. The results of parameter estimation of the FSC for noise free data are presented in the Table 4 and 5 (a and b). It is observed that the ZOH sampling method is well suitable for parameter estimation. For a dynamic system the parameters are also estimated and the performance of the controller with the dynamically varying parameters (estimated) gave satisfactory results. A fast convergence has been observed for low order noise free models. The convergence is remarkable even with a coarse sampling period in some cases.

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