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The Modelling and Control of Interst and Coolling of Rolled Steell in Billet Mills

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Abstract: A generalized approach to the modelling of transient heat flow in rolled steel during air-cooling in billet mills is presented. The mathematical development starting with the relevant differential equations and their modelling using the Hills' integral profile method give polynomial functions which characterize the transient heat flow in the workpiece and establishes an optimal control law for static cooling conditions. An experimental validation test enabled the final model to be obtained as a function of the rolling speed and shows that a good functional correspondence exists between the model and the data reported in the literature.

Key words: Transient heat flow in rolled steel, modelling and control of interstand air-cooling, experimental validation tests, optimal control law

INTRODUCTION

Several models of heat flow in hot rolling processes have been reported in the existing literature. Most of these studies have been confined to laboratory models and relatively simple components, notably finite lengths of steel slabs, have been used to produce solutions to the heat conduction equations. These equations were often defined in one (Wartnann and Mertes, 1973) and more rarely two (Yu and Sang, 2007) dimensions in space.

In billet mills, accurate mathematical models of interstand cooling of rolled steel are required for the analysis and control of the shape distortions observed during air-cooling of the final product. In relation to the thermal distortions observed during hot rolling of steel in billet mills (Obinabo, 1991), these one-dimensional models cannot be used with confidence to describe the origins and orientations of these defects. Increasingly, the models reported in the existing literature are aimed at predicting the thermal conditions of the rolls in both hot and cold strip mills while a very scant treatment is, so far, given to the determination of the actual temperature distributions in the workpiece itself.

On the computational methods, various numerical methods have been developed for the computation of the heat flow problems. Finite difference techniques were mainly used in which direct approximations were made to the governing equations in terms of a finite number of the unknown temperature values chosen at strategic mesh points. These mesh points are a pattern of discrete points used to replace a continuous domain on the test sample, each point being taken to represent a region within the domain. Therefore, instead of obtaining a continuous solution throughout the domain for the temperature values, only an approximation of these values at these isolated points is obtained.

This study presents a model of interstand cooling of rolled steel for 2 transverse surfaces of the workpiece in a billet mill. The decision to model the two-dimensional problem was conceived from the results of experimental tests on the samples (Obinabo, 1991), which confirmed the desirability of this approach by exhibiting asymmetry of the temperature distributions in the edge dimensions of the test samples.

The overall objective of the study, therefore, was to develop modelling and control techniques that are of potential interest to steel industry. Because the deformation process in a billet mill is a complex operation, this analysis is seen as an important research tool, complimentary to and capable of extending knowledge obtained from laboratory and on-line plant observations and measurements (Obinabo and Chijioke, 1991).

MATHEMATICAL FORMULATION

The problem considered is one of time-and space dependent heat flow in flat bar products rolled in Delta Steel mill at Ovwian-Aladja, Warri, Nigeria. The geometrical model of the bar sample used in the analysis (Fig. 1) is of the dimensions 110×70×6 mm and produced from RST 37-2 steel grade. The approach in the development of the model is based on the assumption that the RST 37-2 steel is homogeneous in terms of its metallurgical constitutions. This enables the thermal conductivity and the specific heat capacity of the material to be assumed constant across any dimension of the

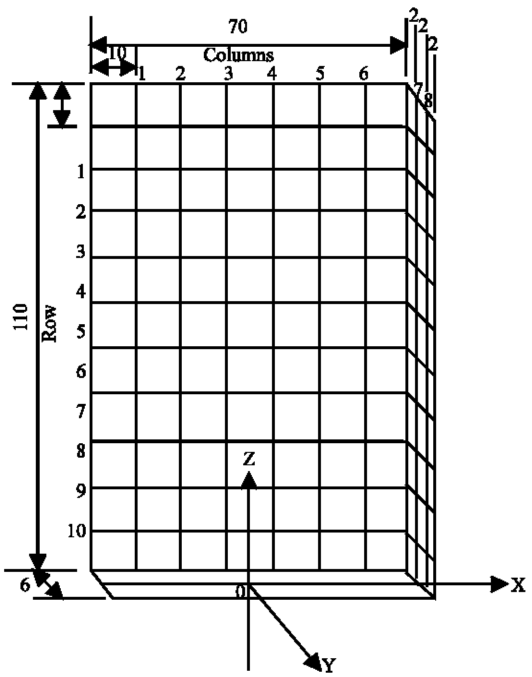

Fig. 1: Geometrical model of the RST 37-2 steel bar sample

workpiece. Another important aspect of the model is the provision for temperature independence of these functions. A series of empirical equations has been adopted in the literature (Yang and Lu, 1986) for the different phases in steel over the whole temperature range. It is indicated that above the temperature of 900°C, the thermal conductivity is temperature-dependent.

Rolling temperature models exist for plate and hot strip mills (Pedersen, 1999; Bryant and Heselton, 1982) and for billet mills (Obinabo, 1991) and are defined by a set of differential equations in terms of transfer function components in the frequency domain and analyzed using transform methods of solution. The problem considered in this investigation is one of time- and space-dependent heat flow in rectangular cross-sectional bars rolled from RST 37-2 steel. The model assumes the RST 37-2 steel to be homogeneous in its metallurgical constitution so that the thermal conductivity and the specific heat capacity of the material were assumed constant across the entire dimensions.

Two simultaneous ordinary differential equations were derived for these parameters which satisfied the following unsteady heat conduction equation,

$$\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{\partial^2\theta}{\partial z^2}\right)-\left(v_x\frac{\partial\theta}{\partial x}+v_y\frac{\partial\theta}{\partial y}+v_z\frac{\partial\theta}{\partial z}\right)+Q'=\frac{\partial\theta}{\partial\tau} \qquad (1)$$

Where,

$$\alpha=\frac{k}{\rho c},\ Q'=\frac{Q}{\rho c},\ v_x,\ v_y,\ v_z$$

are component of the velocity vector $\vec{v}$. $\partial\theta/\partial\tau$ denotes the rate of change of temperature in space and time along the workpiece. The length of the deforming bar was considered infinite since the rolling process in the mill was continuous. Consequently, conduction of heat in that dimension relative to the other dimensions was negligibly small, so that the temperature changes in that direction is a function of time only.

Applicability of Eq. (1) was based on the assumption that the rolling process in the mill was steady relative to the roll stand and that the motion of the workpiece was restricted to the direction of rolling only. The heat input, Q, due to the deformation in the roll gap was assumed uniformly distributed in the workpiece. Thus

$$\frac{\partial\theta}{\partial\tau}=0 \qquad (2)$$

$$V_x = V_y = 0 \qquad (3)$$

At any point *z* along the length of the workpiece from a chosen position, the time during which the workpiece was exposed to the cooling effects of the mill was defined (Obinabo, 1991) as from a chosen position, this time was defined as:

$$\tau=\frac{z}{v} \qquad (3)$$

The origin of z was at the instance the workpiece exits the roll gap and this is at time $\tau = 0$. Equation (3) applies to the workpiece at any point between two roll stands, that is, at exit from the roll gap of one stand to the point just before entry into the roll gap of the next stand. It also applies to the portion of the mill between the last finishing stand and the cooling bed. In each of these regions the speed of the workpiece is assumed constant. Therefore, to transform the foregoing results Eq. (1) in terms of this variable τ, the following operator was derived from Eq. (3) as:

$$\frac{\partial}{\partial z}=\frac{\partial}{\partial\tau}\frac{d\tau}{dz}=\frac{1}{V}\frac{\partial}{\partial\tau} \qquad (4)$$

and on substitution into (1), yields (Yu and Sang, 2007; Kwon and Bang, 2000):

$$\alpha\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}+\frac{1}{V^2}\frac{\partial^2\theta}{\partial\tau^2}\right)+Q'=\frac{\partial\theta}{\partial\tau} \qquad (5)$$

In the weighted form (Yu and Sang, 2007), Eq. (5) becomes

$$\int_{\Omega} W\left[\frac{\partial\theta}{\partial t}-\frac{\partial}{\partial x}\left(\alpha\frac{\partial\theta}{\partial x}\right)-\frac{\partial}{\partial y}\left(\alpha\frac{\partial\theta}{\partial y}\right)\right]d\Omega=0 \qquad (6)$$

and reduced using Green's theorem (Pepper and Heinrich, 1992) as follows:

$$\int_{\Omega}\left[W\frac{\partial\theta}{\partial t}+\alpha\left(\frac{\partial W}{\partial x}\frac{\partial\theta}{\partial x}+\frac{\partial W}{\partial y}\frac{\partial\theta}{\partial y}\right)\right]d\Omega+ \int_{\psi_B} W\left(-\alpha\frac{\partial\theta}{\partial n}\right)d\psi=0 \qquad (7)$$

where, $\alpha = k(\theta)/\rho c(\theta)$ and Ω denotes the two-dimensional domain. ψ_B and $d\psi$ represent respectively the boundary and the surface element of ψ_B over which the normal gradients were applied. Also, n is the outward normal unit vector at the boundary ψ. To facilitate computation of (5) the dimensionless variables due to Hills (Obinabo, 1991) were employed to transform the equation to the following form

$$\frac{\partial^2\theta^*}{\partial x^{*2}}+\frac{\partial^2\theta^*}{\partial y^{*2}}+\left(\frac{1}{V^*}\right)^2\frac{\partial^2\theta^*}{\partial\xi^2}=\frac{\partial\theta^*}{\partial\xi} \qquad (8)$$

which, on introduction of the integral sign, gave

$$\iint\left(\frac{\partial^2\theta^*}{\partial x^{*2}}+\frac{\partial^2\theta^*}{\partial y^{*2}}\right)dx^*dy^* = \iint\left(\frac{\partial\theta^*}{\partial\xi}-\left(\frac{1}{V^*}\right)^2\frac{\partial^2\theta^*}{\partial\xi^2}\right)dx^*dy^* \qquad (9)$$

Where:

$$V^*=\frac{V\rho c\theta_i}{[q_o]_o}$$

and which, on further simplification and ignoring the asterisk, gave

$$h\left(\theta_{t_x}+\theta_{t_y}\right)-q_{oy}t_x-q_{ox}t_y = \frac{d}{d\xi}\left(\int_0^{t_x}\int_0^{t_y}\theta\,dxdy\right)-\theta_{t_x}\left(\frac{dt_x}{d\xi}\right)-\theta_{t_y}\left(\frac{dt_y}{d\xi}\right) -\left(\frac{1}{V}\right)^2\left(\frac{d^2}{d\xi^2}\int_0^{t_x}\int_0^{t_y}\theta\,dxdy\right)-\theta_{t_x}\left(\frac{d^2t_x}{d\xi^2}\right)-\theta_{t_y}\left(\frac{d^2t_y}{d\xi^2}\right) \qquad (10)$$

The left hand side of Eq. (10) was evaluated first by integrating with respect to y*, then with respect to x* to yield the following:

$$LHS = t_y\cdot\left.\frac{\partial\theta^*}{\partial x^*}\right|_{t_x}-t_y\cdot\left.\frac{\partial\theta^*}{\partial x^*}\right|_{x=0}+t_x\cdot\left.\frac{\partial\theta^*}{\partial y^*}\right|_{t_y}-t_x\cdot\left.\frac{\partial\theta^*}{\partial y^*}\right|_{y=0} \qquad (11)$$

where suffix o represents the origin of the coordinate axes, x* and y* are dimensionless spatial extents in the x and y directions respectively. Writing Eq. (11) in terms of the surface heat flux, q gives

$$q^*_{t_y}=h^*\Delta\theta^*_{t_y} \qquad (12)$$

then Eq. (12) becomes:

$$h^*\left(\theta^*_{t_x}+\theta^*_{t_y}\right)-q^*_{ox}t^*_y-q^*_{oy}t^*_x \qquad (13)$$

The first term on the right hand side of Eq. 10 was evaluated (Obinabo, 1991) to yield the following result:

$$\int\int\left(\frac{\partial\theta^*}{\partial\xi}\right)dx^*dy^*=\frac{d}{d\xi}\left(\int\int\theta^*dx^*dy^*\right) -\theta^*_{t_x}\left(\frac{dt^*_x}{d\xi}\right)-\theta^*_{t_y}\left(\frac{dt^*_y}{d\xi}\right) \qquad (14)$$

Similarly, evaluating the second term on the same right hand side of the equation yields:

$$-\left(\frac{1}{V^*}\right)^2\int\int\left(\frac{\partial^2\theta^*}{\partial\xi^2}\right)dx^*dy^*=-\left(\frac{1}{V^*}\right)^2\frac{d^2}{d\xi^2}\left(\int\int\theta^*dx^*dy^*\right)-\theta^*_{t_x}\left(\frac{d^2t_y}{d\xi^2}-\theta^*_{t_y}\left(\frac{d^2t_y}{d\xi^2}\right)\right) \qquad (15)$$

Combining these results and ignoring the *asterisks*, the following was obtained:

$$h\left(\theta_{t_x}+\theta_{t_y}\right)-q_{oy}t_x-q_{ox}t_y =\frac{d}{d\xi}\left(\int_0^{t_x}\int_0^{t_y}\theta\,dxdy\right)-\theta_{t_x}\left(\frac{dt_x}{d\xi}\right)-\theta_{t_y}\left(\frac{dt_y}{d\xi}\right) -\left(\frac{1}{V}\right)^2\left(\frac{d^2}{d\xi^2}\left(\int_0^{t_x}\int_0^{t_y}\theta\,dxdy\right)-\theta_{t_x}\left(\frac{d^2t_x}{d\xi^2}\right)-\theta_{t_y}\left(\frac{d^2t_y}{d\xi^2}\right)\right) \qquad (16)$$

Auxiliary function for θ (x, y): The result shown in Eq. (16) could not as yet be solved because the temperature distributions appearing in the integrals were not known. Some functions were required to represent these temperature distributions in the workpiece during cooling. Consequently it was imperative to design an accurate temperature profile q which, in itself, satisfies the boundary conditions that prevail in the cooling of the bars during rolling and was a function of x, y and ξ. In considering the 2-dimensional steady state heat conduction problem, the integral approaches proposed by Yang and Lu (1986) report auxiliary functions with at least one unspecified parameter. Ritz method assumes a quadratic function in the dimension that runs across the width of the workpiece and an exponential function in the dimension that runs along the length of the workpiece. The result of the 2-dimensional profile was defined mathematically as:

$$\theta(x, y) = A\left(\ell^2 - y^2\right) e^{-BX} \tag{17}$$

where, A and B were determined from the boundary conditions and ℓ represents the width of the workpiece of infinite length. Kantorovich's method was almost similar to Ritz's. The difference was that the form of the profile assumed in the dimension that runs along the length of the flat bar was an unknown function. This reduced the Ritz function to the form.

$$\theta(x, y) = (\ell^2 - y^2)X(x) \tag{18}$$

where X(x) was required to be determined from the boundary conditions.

$$\theta = a_o + a_1\left(\frac{x}{t}\right) + a_2\left(\frac{x}{t}\right)^2 \tag{19}$$

In current investigation, a 2-D auxiliary function based on the spatial cooling profiles reported by Obinabo (1991) was proposed for the surface and width dimensions of the workpiece as follows

$$\theta(x, y) = \left(a_o + a_2x^2 + a_4x^4\right)\left(b_o + b_1y + b_2y^2\right) \tag{20}$$

The spatial distribution was symmetrical in the surface dimension and asymmetrical in the width dimension. During air cooling the workpiece rested surface-wise on the cooling bed. In this condition, the top surface was exposed to the free air stream surrounding it while the bottom surface exchanged heat by conduction with the cooling bed. This condition gave rise to Eq. (20). The a's and b's were determined from the boundary conditions. Expanding Eq. (20) and ignoring terms containing powers of x's and y's higher than 2, the following was obtained.

$$\theta(x, y) = a_o b_o + a_o b_1 y + a_o b_2 y^2 + a_2 b_o x^2 \tag{21}$$

The justification for truncating Eq. (21) is embodied in the reasoning that the variables x and y became non-dimensionalised by defining the following:

$$x^* = \frac{t_x}{x} \tag{22}$$

$$y^* = \frac{t_y}{y} \tag{23}$$

where t_x and t_y are instantaneous spatial extents along the directions of x and y, respectively. It then follows that the maximum value either t_x or t_y can take in Eq. (23) and (24) is x or y. Consequently, in analyzing the heat distributions within the workpiece, the values of x and y in the auxiliary function will always be fractional and higher powers of fractions reduced them to negligibly small quantities and the terms containing them tend to zero. For all values of x and y, the expansion to power 2 obtained in Eq. (22) seemed quite reasonable and therefore, represents the approximate auxiliary function required to compute the temperature distributions in the workpiece.

The final form of the model: Apart from the roll gap where heat was generated within the workpiece due to deformation, no heat sources were known to exist in the mill train. Consequently, a zero heat flow condition across the centre line of the workpiece was assumed so that the following result was obtained from (16).

$$h\left(\theta_{t_x} + \theta_{t_y}\right) = \frac{d}{d\xi}\left(\int_0^{t_x}\int_0^{t_y} \theta\, dx\, dy\right) - \theta_{t_x}\left(\frac{dt_x}{d\xi}\right) - \theta_{t_y}\left(\frac{dt_y}{d\xi}\right) - \left(\frac{1}{V}\right)^2\left(\frac{d^2}{d\xi^2}\left(\int_0^{t_x}\int_0^{t_y} \theta\, dx\, dy\right) - \theta_{t_x}\left(\frac{d^2t_x}{d\xi^2}\right) - \theta_{t_y}\left(\frac{d^2t_y}{d\xi^2}\right)\right) \tag{24}$$

Difficulties associated with measurement of the surface temperature of the workpiece during the rolling process made direct measurement of the heat transfer coefficients at these locations almost impossible. Indirect method of measurement which involves use of radiation pyrometers has been adopted generally (Kim and Huh, 2000; Polukhin, 1975). The disadvantage of this technique of temperature measurement was that the other modes of cooling were not monitored. Consequently, the accuracy of the results so obtained depends largely on the effectiveness of the radiation mechanism and the surface heat flux of the material becomes a direct function of the radiation mechanism. Harding (1976) argued that this is misleading since convection was a more important heat transfer mechanism than was generally thought. Polukhin (1975), Hills (1963) and Obinabo (1991) also considered a combined effect of convection and radiation mechanisms and related it to the surface heat flux of the workpiece. Meanwhile, in their classical experiments on heat flow in continuous casting of steel ingots, Savage and Pritchard (Hills, 1963) obtained a relationship that expresses the surface flux as a function of time. This was done by measuring the rise in the temperature of the cooling water. The data so generated was used to estimate the total quantity of heat removed from the surface of the cooling steel ingot. The expression obtained from the heat flux was of the form

$$q^{*}_{o} = [q_{o\cdot o} - b\sqrt{\tau} \tag{25}$$

for which the values of 2628 and 221.9 were obtained for q_0 and b respectively; b is constant of linear relationship between the heat flux and dwell time. In terms of the dimensionless variables used in the development of this work, this expression reduces to

$$q* = 1 - \beta\sqrt{\xi} \tag{26}$$

Where:

$$\beta = \frac{221.9}{2628}\ (= 0.08)$$

and is a constant of linear relationship obtained by transforming Eq. (25) to its dimensionless form. This result was reduced in Obinabo (1991) to the following:

$$\frac{\partial q}{\partial \xi} = f_{\xi} = \frac{-\beta}{2\sqrt{\xi}} \tag{27}$$

From Eq. (27) the following result was obtained:

$$\frac{\partial^2 q}{\partial \xi^2} = f^{'}_{\xi} = \frac{\beta}{4\sqrt{\xi^3}} \tag{28}$$

Hills (1963) shows that the heat transfer coefficient at the surface of the workpiece bears a linear relationship with time and gives the surface heat flux as:

$$q^{*}_{o} = -h_{o}(1-\gamma\xi)\theta_{o} \tag{29}$$

where the subscript o represents the values on the surface of the workpiece and γ represents a constant of linear relationship. In terms of the dimensionless variables this result becomes:

$$q^{*} = (1-\gamma\xi)\theta \tag{30}$$

which yields:

$$\frac{\partial q}{\partial \xi} = f^{'}_{\xi} = -\gamma\theta \tag{31}$$

The following result was deduced from Eq. (31)

$$\frac{\partial^2 q}{\partial \xi^2} = f^{*}_{\xi} = 0 \tag{32}$$

For the modes of cooling the workpiece considered in this work, the surface heat flux was given by an equation of the form (Eckert *et al.*, 1993)

$$q = -\left(\sigma F\left(\theta^4 - \theta_A^4\right) + h\left(\theta - \theta_A\right)\right) \tag{33}$$

Where,

θ_A	=	Ambient temperature
θ	=	Measurement surface temperature
σ	=	Stefan-Boltzman constant = 56.7×10^{-12} $kWm^{-2}\,k^{-4}$.
F	=	Shape factor accounting for the geometry of The surface of the workpiece radiating heat.

In terms of the dimensionless variables Eq. (33) becomes:

$$q^{*} = \frac{\theta^4 - \phi^4 + h^{*}(\theta - \phi)}{1 - \phi^4 + h^{*}(1-\phi)} \tag{34}$$

where, $h^* = h/\phi F\theta^3$, ϕ = dimensionless absolute ambient temperature.

On the surface heat transfer coefficient of steel products cooling in air, a number of results has been deduced by in the existing literature. On the run-out table of a strip mill, Labiesh (1982) reported a wide range of total heat transfer coefficient in the range 60-120 $Wm^{-2}\,k^{-1}$ for a strip piece being transported from the roll stand to the cooling bed. Several other publications have been made on the predication of this value and some of them have been discussed extensively in Obinabo (1991).

A complete analysis of Eq. (24) was possible only when an auxiliary function was defined and the surface heat flux adequately accounted for. When the three forms of the surface heat flux variation were considered, the following results were obtained.

From Eq. (26)

$$\theta(x,y) = \theta_o + (5y/2)\,(\theta_{17} - \theta_{18}) + (y/2)(1-\beta\sqrt{\zeta}) + (x/2)(1-\beta\sqrt{\zeta}) \tag{35}$$

From Eq. (30), the auxiliary function becomes:

$$\theta(x,y) = \theta_O + (5y/2)\,(\theta_{17} - \theta_{18}) + (y/2)(1-\beta\sqrt{\zeta}) + (x/2)(1-\beta\sqrt{\zeta}) \tag{36}$$

From Eq. (34), the auxiliary function becomes:

$$\theta(x,y) = \theta_o + (5y/2)(\theta_{17} - \theta_{18}) + (y/2)(\theta) + (x/2)\theta \tag{37}$$

Now taking the integrals:

$$\int_0^{t_x}\int_0^{tt_y} \theta dxdy = \int_0^{t_y}\left(\int_0^{t_x} \theta dx\right)dy$$

From Eq. (35) the following was obtained

$$\int_0^{t_x} \theta\, dx = t_x\theta_o + \frac{5}{2}t_x y\left(\theta_{17} - \theta_{18}\right) + t_x\frac{y}{2}\left(1-\beta\sqrt{\xi}\right) + \frac{t_x^2}{4}\left(1-\beta\sqrt{\xi}\right)$$

and

$$\int_0^{t_y}\left(\int_0^{t_x} \theta dx\right)dy = t_x t_y\theta_o + \frac{5}{4}t_x t_y^2\left(\theta_{17} - \theta_{18}\right) + \frac{1}{4}t_x t_y^2\left(1-\beta\sqrt{\xi}\right) + \frac{1}{4}t_x t_y\left(1-\beta\sqrt{\xi}\right) \tag{38}$$

Similarly from Eq. (36) and (37) respectively the following were obtained:

$$\int_0^{t_x}\int_0^{t_y} \theta\, dxdy = t_x t_y\theta_o + \frac{5}{4}t_x t_y^2\left(\theta_{17} - \theta_{18}\right) + \frac{1}{4}t_x t_y^2\left(1-\gamma\xi\right)\theta_{t_y} + \frac{1}{4}t_x\left(1-\gamma\xi\right)\theta_{t_x} \tag{39}$$

$$\int_0^{t_x}\int_0^{t_y} \theta\, dxdy = t_x t_y\theta_o + \frac{5}{4}t_x t_y^2\left(\theta_{17} - \theta_{18}\right) + \frac{1}{4}t_x t_y^2\theta t_y + \frac{1}{4}t_x^2 t_y\theta t_x \tag{40}$$

From these results, therefore, (16) is written for each of the cases considered above in the x-and y- dimensions as follows: From Eq. (38), the following were obtained:

- In the x-dimension:

$$h\theta_{t_x} = \frac{d}{d\xi}\left(t_x t_y\theta_o + \frac{1}{4}t_x^2 t_y\left(1-\beta\sqrt{\xi}\right)\right) - \theta_{t_x}\frac{dt_x}{d\xi} - \left(\frac{1}{V}\right)^2\left(\frac{d^2}{d\xi^2}\left(t_x t_y\theta_o + \frac{1}{4}t_x^2 t_y\left(1-\beta\sqrt{\xi}\right)\right)\right) - \theta_{t_x}\frac{d^2 t_x}{d\xi^2} \tag{41}$$

In the y-dimension:

$$h\theta_{t_x} = \frac{d}{d\xi}(t_x t_y\theta_o + \frac{5}{4}t_x t_y^2(\theta_{17} - \theta_{18}) + \frac{1}{4}t_x t_y\left(1-\beta\sqrt{\xi}\right)) - \theta_{t_y}\frac{dt_y}{d\xi} - (\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_x t_y\theta_o + \frac{5}{4}t_x t_y^2(\theta_{17} - \theta_{18}) + \frac{1}{4}t_x t_y(1-\beta\sqrt{\xi})) - \theta_{t_y}(\frac{d^2 t_t}{d\xi^2})) \tag{42}$$

From Eq. (39), the following were obtained:

- In the x-dimension:

$$h\theta_{t_x} = \frac{d}{d\xi}(t_x t_y\theta_o + \frac{1}{4}t_x^2 t_y(1-\gamma\xi)\theta_{t_x}) - \theta_{t_x}\frac{dt_x}{d\xi}) - (\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_x t_y\theta_o + \frac{1}{4}t_x^2 t_y(1-\gamma\xi)\theta_{t_x}) - \theta_{t_x}\frac{d^2 t_x}{d\xi^2}) \tag{43}$$

- In the y-dimension:

$$h\theta_{t_y} = \frac{d}{d\xi}(t_x t_y\theta_o + \frac{5}{4}t_x t_y^2(\theta_{17} - \theta_{18}) + \frac{1}{4}t_x t_y^2\left(1-\gamma\xi\right)\theta_{t_y}) - \theta_{t_y}\frac{dt_y}{d\xi} - (\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_x t_y\theta_o + \frac{5}{4}t_x t_y^2(\theta_{17} - \theta_{18}) + \frac{1}{4}t_x t_y^2(1-\gamma\xi)\theta_{t_y}) - \theta_{t_y}\frac{d^2 t_y}{d\xi^2} \tag{44}$$

From Eq. (40), the following were obtained:

- In the x-dimension:

$$h\theta_{t_x} = \frac{d}{d\xi}(t_x t_y \theta_o + \frac{1}{4}t_x^2 t_y (q_{t_x})) - \theta_{t_x}(\frac{dt_x}{d\xi}) - (\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_x t_y \theta_o + \frac{1}{4}t_x^2 t_y (q_{t_x})) - \theta_{t_x}(\frac{d^2 t_x}{d\xi^2})) \quad (45)$$

- In the y-dimension:

$$h\theta_{t_y} = \frac{d}{d\xi}(t_x t_y \theta_o + \frac{1}{4}t_x t_y^2 q_{t_x}) - \theta_{t_x}\frac{dt_y}{d\xi} - (\frac{1}{V})^2(\frac{d^2}{d\xi^2}(t_x t_y \theta_o + \frac{1}{4}t_x t_y^2 q_{t_y}) - \theta_{t_y}\frac{d^2 t_y}{d\xi^2} \quad (46)$$

From Eq. (41) and (42), the following were obtained:

$$(\frac{1}{V})^2\frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = -(\frac{1}{t_x t_y}(1-\beta\zeta^{\frac{1}{2}}) + \frac{1}{8}\beta t_x \xi^{-\frac{1}{2}}(1+2\left(\frac{1}{v}\right)^2\zeta^3)) - \frac{\theta_{t_x}}{t_x t_y}(\frac{dt_x}{d\xi}) - \frac{\theta_{t_x}}{t_x t_y}(\frac{d^2 t_x}{d\xi^2}) \quad (47)$$

and

$$(\frac{1}{V})^2\frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = -\frac{1}{t_x t_y}(1-\beta\zeta^{\frac{1}{2}}) + \frac{1}{8}\beta\xi^{\frac{1}{2}}(1+\frac{1}{2}(\frac{1}{V})^2\xi^3)) - \frac{\theta_{t_y}}{t_x t_y}(\frac{dt_y}{d\xi}) - \frac{\theta_{t_y}}{t_x t_y}(\frac{d^2 t_y}{d\xi^2}) \quad (48)$$

Similarly from Eq. (43) and (44), the following were obtained:

$$-(\frac{1}{V})^2\frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = \frac{1}{t_x t_y}(-\frac{1}{4}t_x^2 t_y \gamma\theta_{t_x} - ((1-\gamma\xi)\theta_{t_x}) - \theta_{t_x}(\frac{dt_x}{d\xi}) - \theta_{t_x}(\frac{d^2 t_x}{d\xi^2}) \quad (49)$$

and

$$(\frac{1}{V})^2\frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = \frac{1}{t_x t_y}(-\frac{1}{4}t_x t_y^2\gamma\theta_{t_y} - ((1-\gamma\xi)\theta_{t_x}) - \theta_{t_y}(\frac{dt_y}{d\xi}) - \theta_{t_y}(\frac{d^2 t_y}{d\xi^2})) \quad (50)$$

From Eq. (45 and (46), the following were obtained:

$$(\frac{1}{V})^2\frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = \frac{1}{4}t_x\frac{d}{d\xi}(q_{t_x}) - (\frac{1}{4V^2})t_x\frac{d^2}{d\xi^2}(q_{t_x}) - \frac{k}{t_x t_y}(q_{t_x}) - \theta_{t_x}(\frac{dt_x}{d\xi}) - \theta_{t_x}(\frac{d^2 t_x}{d\xi^2}) \quad (51)$$

$$(\frac{1}{V})^2\frac{d^2\theta_o}{d\xi^2} - \frac{d\theta_o}{d\xi} = \frac{1}{4}t_y\frac{d}{d\xi}(q_{t_y}) - (\frac{1}{4V^2})t_y\frac{d^2}{d\xi^2}(q_{t_x}) - \frac{1}{t_x t_y}(q_{t_y}) - \theta_{t_y}(\frac{dt_y}{d\xi}) - \theta_{t_y}(\frac{d^2 t_y}{d\xi^2}) \quad (52)$$

The problem was finally represented globally using matrix notation as follows:

$$\begin{bmatrix} (\frac{1}{v})^2 & 0 \\ 0 & (\frac{1}{v})^2 \end{bmatrix}\begin{bmatrix} D_x{}^2\theta_o \\ D_x{}^2\theta_o \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} D_x\ \theta_o \\ D_y\ \theta_o \end{bmatrix} = \begin{bmatrix} \Gamma \\ \Lambda \end{bmatrix} \quad (53)$$

where, the D's represent derivatives with respect to time. Γ and Λ were deduced directly from the preceding equations as:

$$\Gamma = -(\frac{1}{t_x t_y}((1-\beta\xi^{\frac{1}{2}}) + \frac{1}{8}\beta t_x \xi^{-\frac{1}{2}}(1+2(\frac{1}{v})^2\xi^3)) - \frac{\theta_{t_x}}{t_x t_y}(\frac{dt_x}{d\xi}) - \frac{\theta_{t_x}}{t_x t_y}(\frac{d^2 t_x}{d\xi^2})) \quad (54)$$

$$\Lambda = -(\frac{1}{t_x t_y}((1-\beta\xi^{\frac{1}{2}}) + \frac{1}{8}\beta\xi^{-\frac{1}{2}}(1+\frac{1}{2}(\frac{1}{v})^2\xi^3)) - \frac{\theta_{t_y}}{t_x t_y}(\frac{dt_y}{d\xi}) - \frac{\theta_{t_y}}{t_x t_y}(\frac{d^2 t_y}{d\xi^2}) \quad (55)$$

We now let the state variable $x_1 = 0_o(t)$ in Eq. (39) and (40) so that

$$\frac{d}{dt}\ x_1 = x_2$$

$$\frac{d}{dt}\ V^2(x_2 + \Gamma)$$

or, using the matrix notation, the above result becomes

$$\frac{d}{d\xi}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0&1\\0&V^2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0\\\Phi\end{bmatrix}$$

Where, Φ (= $\Gamma - V^2 x_2$) and

$$\Gamma=V^2\left(\frac{t_x}{4}\frac{dq_n}{d\xi}-\frac{t_x}{4V^2}\frac{d^2q_n}{d\xi^2}-\frac{q_n}{t_xt_y}-\theta_\alpha\frac{d^2t_x}{d\xi^2}\right)$$

in the x-dimension

$$\Gamma=V^2\begin{pmatrix}\frac{t_y}{4}\frac{dq_y}{d\xi}-\frac{t_y}{4V^2}\frac{d^2q_y}{d\xi^2}\\-\frac{q_y}{t_xt_y}-\theta_\alpha\frac{d^2t_y}{d\xi^2}-\theta_y\frac{d^2t_y}{d\xi^2}\end{pmatrix}$$

in the y-dimension

$$\frac{d}{d\xi}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0&1\\0&V^2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0\\\Phi\end{bmatrix}u$$

or generally

$$\frac{d}{dt}\{x\}=A(u_\circ(\mu),\mu)x$$

OPTIMAL CONTROL OF THE STATE MODEL

In general, stability is a very important characteristic of the transient performance of dynamic systems. Almost every functional system is designed to be stable and within the boundaries of parameter variations the system performance can be improved. The system represented by (52) can be investigated for asymptotic stability by studying the eigenvalues of the system when A (u_0 (μ, μ)) is constant (Mayne, 1973; Obinabo, 2008), or by studying the solution system ϕ (μ) of Eq. (52) with the initial condition ϕ (μ) = I where I is the unit diagonal matrix if A (u_0 (μ, μ)) happens to be an arbitrary function of μ. Here, an optimal feedback control law u (μ) was obtained for the linear inhomogeneous system based on the quadratic performance index and was expressed as a function of x given by u (μ) = f (x), which assures asymptotic stability (x (μ) → 0) as $\mu \to \infty$. The system was represented by

$$\frac{d}{dt}\{x\}=A(\mu)x+B(\mu)u,\ x(x)=x_\circ$$

Minimizing the quadratic performance index

$$J=\int_0^\infty\left[x^r(\mu)Q(\mu)x(\mu)+u^T(\mu)R(\mu)u(\mu)\right]d\mu$$

where, Q (μ) and R (μ) are positive and semi-definite and positive respectively, leads to an optimal control which us a linear function of the state and is given by:

$$u(\mu)=-R^{-1}(\mu)B^T(\mu)S(\mu)x(\mu)$$

where, S (μ) is the solution of the matrix Riccati equation given by

$$S=-S(\mu)A(\mu)-A^T(\mu)S(\mu)+S(\mu)B(\mu)$$
$$R^{-T}(\mu)S(\mu)-Q(\mu),\ S(\mu\to\infty)=0$$

Q (μ) and R (μ) May be chosen as unit diagonal matrices for convenience. Such a choice also implies that all the control and state variables are equally weighted in the cost function.

Now from (50) we derive equations for static cooling of the work piece on the cooling bed of the mill (where V = 0 and assumed unit) as follows:

$$\frac{d}{d\xi}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}u$$

The performance criteria is rewritten as

$$J(u)\begin{bmatrix}1\\0\end{bmatrix}^T x(1)+\int_0^1\left\{\begin{bmatrix}1\\0\end{bmatrix}^T x(t)+\lambda u^2(t)\right\}dt$$

or

$$J(u)\begin{bmatrix}1\\0\end{bmatrix}^T\begin{bmatrix}x_1&(1)\\x_2&(1)\end{bmatrix}+\int_0^1\left\{\begin{bmatrix}1\\0\end{bmatrix}^T\begin{bmatrix}x_1&(t)\\x_2&(t)\end{bmatrix}+\lambda u^2(t)\right\}dt$$

which, on comparison with the general form

$$J(u)=\alpha^Tx(T)+\int_0^T\left\{\beta^T(t)x(t)+g(u(t),t\right\}dt$$

gives

$$\alpha=\begin{bmatrix}1\\0\end{bmatrix}$$

Therefore,

$$\alpha^T = [1 \quad 0], \quad \beta = \begin{bmatrix}1\\0\end{bmatrix}$$

giving $\beta^T [1 \quad 0]$ and $g = \lambda u^2$
From (57)

$$A = \begin{bmatrix}0 & 1\\0 & 0\end{bmatrix} \text{ and } B = \begin{bmatrix}0\\1\end{bmatrix}.$$

Now the costate variable is

$$\frac{d}{dt}\{p(t)\} = -A^T(t)p(t) - \beta(t),\ p(T) = \alpha \tag{61}$$

Now substitute for A^T (t) and B (t) so that the following may be obtained

$$\frac{d}{dt}\{p(t)\} = -\begin{bmatrix}0 & 0\\1 & 0\end{bmatrix}\begin{bmatrix}p_1(t)\\p_2(t)\end{bmatrix} - \begin{bmatrix}1\\0\end{bmatrix}$$
$$= \begin{bmatrix}0 & 0\\-1 & 0\end{bmatrix}\begin{bmatrix}p_1(t)\\p_2(t)\end{bmatrix} - \begin{bmatrix}1\\0\end{bmatrix}$$

From which

$$\frac{d}{dt}\{p_1(t)\} = -1 \tag{62}$$

And

$$\frac{d}{dt}\{p_2(t)\} = -p_1(t) \tag{63}$$

From Eq. (62)

$$p_1(t) = -t + C \tag{64}$$

$$p(T) = \alpha,\ p(1) = \begin{bmatrix}0\\1\end{bmatrix}, p_1(1) = 1 \Rightarrow$$
$$p_1(t) = 1 \text{ and } p_2(1) = 1 \Rightarrow p_2(t) = 0$$

Now substitute these for p_1 (t) in Eq. (64) to yield 1 + t = C, that is, 1+1=C :.C = 2 giving p_1 (t) = 2-t
From Eq. (63)

$$\frac{d}{dr}\{p_2(t)\} = -p_1(t) = -(2-t)$$
$$= t - 2 \therefore p_2(t) = \frac{t^2}{2} - 2t + \frac{3}{2}$$

Now we define the Hamiltonian

$$H(x,p,u,t) = \beta^T(t)x + g(u,t) + p^T(Ax + Bu)$$
$$[1 \quad 0]\begin{bmatrix}x_1\\x_2\end{bmatrix} \lambda u^2 + p^T(Ax + Bu)$$
$$= x_1 + \lambda u^2 + p^T(Ax + Bu) \tag{65}$$
$$= x_1 + \lambda u^2 + p^T\left(\begin{bmatrix}0 & 1\\0 & 0\end{bmatrix}x + \begin{bmatrix}0\\1\end{bmatrix}u\right)$$
$$= x_1 + \lambda u^2 + p_1 x_2 + p_2 u$$

u is unconstrained hence we find $\partial H/\partial u = 0$
From Eq. (65) $\partial H/\partial u = 2\lambda u + p_2$ Equating to zero gives:

$$2\lambda u = -p_2 - \left(\frac{t^2}{2} - 2t + \frac{3}{2}\right) \therefore -\frac{1}{2\lambda}\left(\frac{t^2}{2} - 2t + \frac{3}{2}\right)$$

Hence the control law for the static cooling of the rolled steel on the cooling bed of the mill is:

$$u^{\cdot} = -\frac{1}{2\lambda}\left(\frac{t^2}{2} - 2t + \frac{3}{2}\right)$$

which is optimal

RESULTS AND DISCUSSION

The results of the analysis of the model are presented for the conditions when (i) V = 0 and (ii) V has some finite values. In the first case, Fig. 2 and 3 represent typical temperature profiles obtained for the two planes of the sample considered. The average cooling rates of the various points on the sample were found to lie between 1.034 and 1.041°C per second over the 900-550°C temperature range. The expectation that portions of the samples nearest to the edge surface cool faster than those close to the middle, is supported by experimental data (Obinabo and Chijioke, 1991). The plots of the temperature distributions obtained for the X-Z plane of the sample show no significant difference in the cooling of the points on this plane. This confirms the homogeneity of the material and characteristics of the sample. It was also noted that there was little or no asymmetry in the spatial distribution of the heat flow across the sample as was expected because the bounding surfaces of this plane was equally free of constraints. In addition, the thickness of the oxide scales formed on the surfaces was approximately the same for the points monitored-a further confirmation of thermal symmetry. The importance of symmetry of temperature distribution is that it simplifies considerably the derivation of the modelling equations, allows computation carried out on a quarter section of the sample

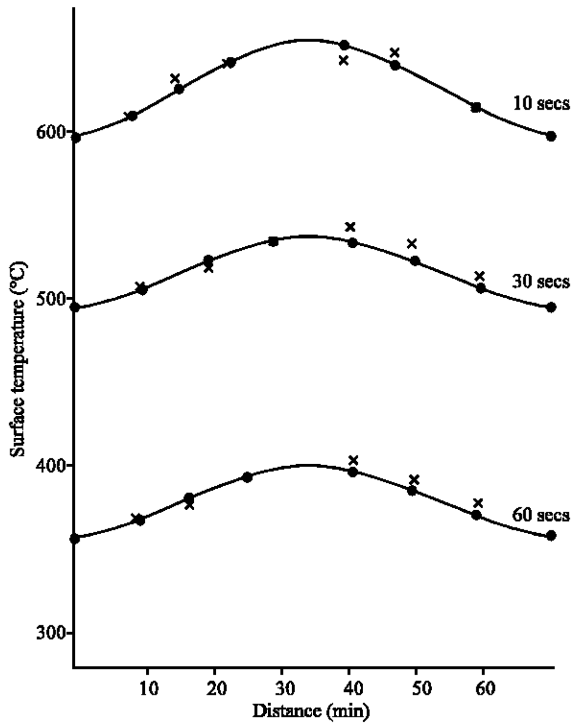

Fig. 2: Average cooling gradient for discrete points on the Y-Z plane

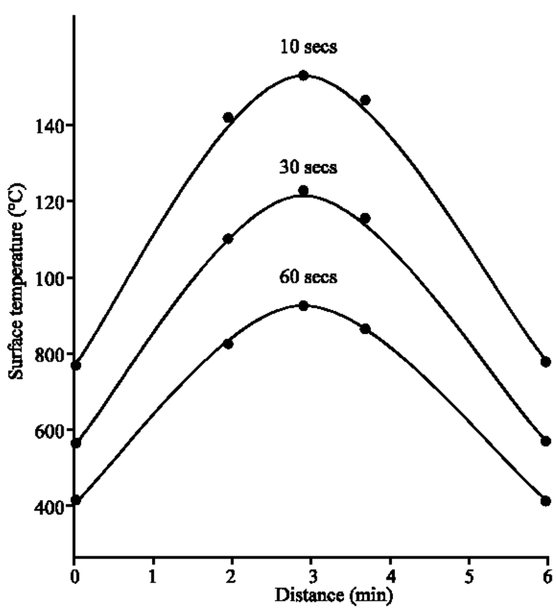

Fig. 3: Temperature distribution across X-Z plane on row 3 at the times 10, 30 and 60 sec

to represent the whole and confirm absence of unevenness in the temperature profile which could otherwise give rise to defects like buckling.

The plots obtained for the time-dependent profiles of the heat flow in the Y-Z plane of the sample do not portray basic departure from the general features observed in the case of the X-Z plane. However, by virtue of the manner in which the samples were positioned (surface-wise) on the cooling platform, the rate of cooling of the X-Z surface in contact with the platform was expected to be different from that of the opposite surface. Asymmetry in temperature was thus introduced between the top and bottom boundaries of the Y-Z faces, giving rise to asymmetrical Y-Z distributions, unlike the X-Z distributions.

The temperature profiles indicate that the cooling rates on the Y-Z plane were slightly higher than those recorded for the X-Z plane. An average value of 1.04°C per second was obtained for the Y-Z plane. The resulting spatial distributions of the heat flux on the sample were determined directly from these temperature data at the instantaneous times of 10, 30 and 60 sec after cooling had commenced, using techniques reported earlier by Obinabo (1991) and which were based on a rigorous mathematical formulation and digital computer solutions.

CONCLUSION

This study has established an optimal control law for static cooling conditions and an experimental validation test which enabled a two-dimensional heat flow model to be obtained as a function of the rolling speed for rectangular cross-sectional bars rolled from plain carbon steel. The model, which was based on the Hills' generalized integral profile method is of the form

$$\frac{d^2\theta}{d\zeta^2} - \frac{d\theta}{d\zeta} = f(q_{t,y}\varepsilon_{x,y}, v)$$

and applies to both interstand cooling and cooling of the final products on the cooling bed of the mill. The terms $q_{t_{x,y}}$, $\dot{\varepsilon}_{x,\,y}$ and V characterize the surface heat flux, rate of change of the dimensions of the workpiece during cooling and the rolling speed respectively. The validity of the model was confirmed in Obinabo (1991) by comparing the profiles of the heat flow determined by experiment for static models with the theoretical results. The study shows that a good functional correspondence exists between the model and the data reported in the literature.

Notation:

C	:	Specific heat capacity (J/kg°C).
h	:	Heat transfer coefficient (W/m2°C).
k	:	Thermal Conductivity (W/m°C).

Q, q : Heat flux (W/m^2).
t : Dimension of workpiece (m).
V : Speed of workpiece (m/s).
x, y, z : Spatial extent in space (m).
α : Thermal diffusivity (m^2/s).
ε : Strain.
θ : Temperature (^{O}C).
ρ : Density (kg/m^3).
τ : Time (s).
ξ : Dimensionless time.
Δ : Small increment.

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