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Synthesis of an LMI-Based Robust H∞ Fuzzy Filter for Uncertain Nonlinear Dynamic Systems

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Abstract: This study examines the problem of designing a robust H∞ fuzzy filter for a class of dynamic uncertain nonlinear systems described by a Takagi-Sugeno (TS) fuzzy model. Then, based on a linear matrix inequality (LMI) approach, LMI-based sufficient conditions for the uncertain systems to have an H∞ performance are derived. Finally, a numerical example is provided to illustrate the design developed in this study.

Key words: H∞ filter, fuzzy systems, Linear Matrix Inequality (LMI)

INTRODUCTION

Over the past few decades, the nonlinear H∞ filter theory has been extensively studied by many researchers (Nguang and Fu, 1996; Nguang and Shi, 2000a, b). The nonlinear H∞-filter problem can be stated as follows: given a dynamic system with the exogenous input noise and the measured output, find a filer such that the L_2-gain of the mapping from the exogenous input noise to the regulated output is less than or equal to a prescribed value. It has been shown that the existence of solution H∞ is in fact related to the solvability of an appropriate algebraic Riccati equation (ARE). This results is then extended into a class of linear system which are subjected to parametric uncertainty.

Recently, a great amount of effort has been made on the design of fuzzy H∞ control and filter for a class of nonlinear systems which can be represented by a Takagi-Sugeno (TS) fuzzy model (Takagi and Sugeno, 1985; Wang, 1995; Sugeno and Kang, 1988; Joh *et al.*, 1998; Tanaka *et al.*, 1996; Wang *et al.*, 1996). Fuzzy system theory enables us to utilize qualitative, linguistic information about a highly complex nonlinear system to construct a mathematical model for it. Recent studies (Wang *et al.*, 1996; Nauang and Assawinchaichote, 2003; Assawinchaichote and Nguang, 2004, 2005) show that a fuzzy linear model can be used to approximate global behaviors of a highly complex nonlinear system. In this fuzzy linear model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by "blending" of these linear models through nonlinear fuzzy membership functions. Unlike conventional modelling which uses a single model to describe the global behavior of a system, fuzzy modelling is essentially a multi-model approach in which simple sub-models (linear models) are combined to describe the global behavior of the system. Nevertheless, so far, to the best of our knowledge, the global robust H∞ fuzzy filter problem for a class of uncertain nonlinear dynamic systems via LMIs approach has not much yet been considered in the literature.

What we intend to do in this study is to develop the new technique of design a robust H∞ fuzzy filter for a class of uncertain nonlinear dynamic systems. First, we approximate this class of nonlinear systems by a Takagi-Sugeno fuzzy model. Then based on an LMI approach, we develop an H∞ filter such that the L_2-gain from an exogenous input to an estimate error is less or equal to a prescribed value.

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SYSTEM DESCRIPTIONS

The class of uncertain nonlinear dynamic systems under consideration is described by the following fuzzy system model:

Plant rule i:

IF $\nu_1(t)$ is M_{i1} and ... and $\nu_\vartheta(t)$ is $M_{i\vartheta}$ THEN

$$\begin{aligned}\dot{x}(t) &= [A_i + \Delta A_i]\, x(t) + [B_{1i} + \Delta B_{1i}]\, w(t)\\ z(t) &= [C_{1i} + \Delta C_{1i}]\, x(t) \qquad (1)\\ y(t) &= [C_{2i} + \Delta C_{2i}]\, x(t) + [D_{21i} + \Delta D_{21i}]\, w(t)\end{aligned}$$

where, x (0) = 0, i = 1, 2, ..., r, M_{ij} (j = 1, 2, ..., ϑ) are fuzzy sets, x (t) $\in \Re^n$ is the state vector, w(t) $\in \Re^p$ is the disturbance which belongs to L_2 (0, ∞), y (t) $\in \Re^\ell$ is the measurement, z (t) $\in \Re^s$ is the state to be es- timated, the matrices A_i, B_{1i}, C_{1i}, C_{2i} and D_{21i} are of appropriate dimensions, r is the number of IF-THEN rules. The matrices ΔA_i, ΔB_{1i}, ΔC_{1i}, ΔC_{2i} and ΔD_{21i} represent the uncertainties in the system and satisfy the following assumption:

Assumption 1:

$$\Delta A_i = F(x(t), t) H_{1i},\ \Delta B_{1i} = F(x(t), t) H_{2i},$$
$$\Delta C_{1i} = F(x(t), t) H_{3i},\ \Delta C_{2i} = F(x(t), t) H_{4i}$$
$$\text{and } \Delta D_{21i} = F(x(t), t) H_{5i}$$

where, H_{ji}, j = 1, 2, ..., 5 are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\|F(x(t), t)\| \le \rho \qquad (2)$$

for any known positive constant ρ.
Let

$$\varpi_i(\nu(t)) = \prod_{K=1}^{\vartheta} M_{ik}(\nu_k(t)) \text{ and}$$
$$\mu_i(x(t)) = \frac{\varpi_i(\nu(t))}{\sum_{i=1}^{r} \varpi_i(\nu(t))}$$

where, $M_{ik}(\nu_k(t))$ is the grade of membership of $\nu_k(t)$ in M_{ik}. It is assumed in this study that

$$\varpi_i(\nu(t)) \ge 0, i = 1, 2, \ldots, r;\ \sum_{i=1}^{r} \varpi_i(\nu(t)) > 0$$

for all t. Therefore,

$$\mu_i(\nu(t)) \ge 0, i = 1, 2, \ldots, r;\ \sum_{i=1}^{r} \mu_i(\nu(t)) = 1$$

for all t. For the convenience of notations, we let

$$\varpi_i = \varpi_i(\nu(t)) \text{ and } \mu_i = \mu_i(\nu(t))$$

The resulting fuzzy system model is inferred as the weighted average of the local models of the form:

$$\begin{aligned}\dot{x}(t) &= [A(\mu) + \Delta A(\mu)]x(t)\\ &\quad + [B_1(\mu) + \Delta B_1(\mu)]\omega(t),\\ z(t) &= [C_1(\mu) + \Delta C_1(\mu)]x(t) \qquad (3)\\ y(t) &= [C_2(\mu) + \Delta C_2(\mu)]x(t)\\ &\quad + [D_{21}(\mu) + \Delta D_{21}(\mu)]\omega(t)\end{aligned}$$

where, x (0) = 0,

$$A(\mu) = \sum_{i=1}^{r} \mu_i A_i,$$

$$B_1(\mu) \sum_{i=1}^{r} \mu_i B_{1i},\ C_1(\mu) = \sum_{i=1}^{r} \mu_i C_{1i},$$

$$C_2(\mu) \sum_{i=1}^{r} \mu_i C_{2i},\ D_{21}(\mu) = \sum_{i=1}^{r} \mu_i D_{21i},$$

$$\Delta A(\mu) = \sum_{i=1}^{r} \mu_i \Delta A_i, \triangleq \sum_{i=1}^{r} \mu_i F(x(t), t) H_{1i},$$

$$\Delta B_1(\mu) = \sum_{i=1}^{r} \mu_i \Delta B_{1i}, \triangleq \sum_{i=1}^{r} \mu_i F(x(t), t) H_{2i},$$

$$\Delta C_1(\mu) = \sum_{i=1}^{r} \mu_i \Delta C_{1i}, \triangleq \sum_{i=1}^{r} \mu_i F(x(t), t) H_{3i},$$

$$\Delta C_2(\mu) = \sum_{i=1}^{r} \mu_i \Delta C_{2i}, \triangleq \sum_{i=1}^{r} \mu_i F(x(t), t) H_{4i},$$

$$\Delta D_{21}(\mu) = \sum_{i=1}^{r} \mu_i \Delta D_{21i}, \triangleq \sum_{i=1}^{r} \mu_i F(x(t), t) H_{5i}.$$

SYNTHESIS OF AN LMI-BASED RO-BUST H∞ FUZZY FILTER

In this study, we present the new technique for designing the robust H∞ fuzzy filter described in study. The validity of this approach is demonstrated by an example from a literature described in this study.

We first consider the following full order H∞ fuzzy filter which is inferred as the weighted average of the local models of the form:

$$\dot{\hat{x}}(t) = \hat{A}(\hat{\mu})\,\hat{x}(t) + \hat{B}(\hat{\mu})y(t)$$
$$\hat{z}(t) = \hat{C}(\hat{\mu})\,\hat{x}(t) \tag{4}$$

Where,

$$\lambda \geq \left\| I + \rho^2\,[H_2^T(\mu)H_2(\mu) + H_5^T(\mu)H_5(\mu)] \right\|^{\frac{1}{2}}$$

Note that

$$\hat{A}(\hat{\mu}) \in \Re^{n\times n}$$

Problem formulation: Given a prescribed H∞ performance $\gamma > 0$, design a robust fuzzy H∞ filter of the form (4) such that the following inequality holds

$$\int_0^{Tf} \left(z(t) - \hat{z}(t)\right)^T \left(z(t) - \hat{z}(t)\right) dt \leq \gamma^2 \left[\int_0^{Tf} \omega^T(t)\omega(t)\,dt\right] \tag{5}$$

Theorem 1: Consider the system (3). Given a prescribed H∞ performance γ and a positive constant δ, if there exists a matrix P satisfying the following matrix inequalities:

$$\begin{pmatrix} \begin{pmatrix} A_{cl}(\mu,\hat{\mu})P \\ +PA_{cl}(\mu,\hat{\mu})^T \end{pmatrix} & B_{cl}(\mu,\hat{\mu}) & (*)^T \\ B_{cl}(\mu,\hat{\mu})^T & -\gamma^2 I & 0 \\ C_{cl}(\mu,\hat{\mu})P & 0 & -1 \end{pmatrix} < 0 \tag{6}$$

$$P > 0 \tag{7}$$

Where,

$$A_{cl}(\mu,\hat{\mu}) = \begin{bmatrix} A(\mu) & 0 \\ \hat{B}(\hat{\mu})C_2(\mu) & \hat{A}(\hat{\mu},\varepsilon) \end{bmatrix},$$

$$B_{cl}(\mu,\hat{\mu}) = \begin{bmatrix} \tilde{B}_1(\mu) \\ \hat{B}(\hat{\mu})\,D_{21}(\mu) \end{bmatrix}$$

and $C_{cl}(\mu,\hat{\mu}) = [\tilde{C}_1(\mu) \quad \tilde{D}_{12}\hat{C}(\hat{\mu})]$

With

$$\tilde{B}_1(\mu) = [\delta I \;\; I \;\; 0 \;\; B_1(\mu) \;\; 0],$$

$$\tilde{C}_1(\mu) = \begin{bmatrix} \frac{\gamma\rho}{\delta}H_1^T(\mu) & \frac{\gamma\rho}{\delta}H_4^T(\mu) \\ \sqrt{2}\lambda\rho H_3^T(\mu) & \sqrt{2}\lambda\rho C_1^T(\mu) \end{bmatrix}^T,$$

$$\tilde{D}_{12} = \left[0 \;\; 0 \;\; 0 \;\; -\sqrt{2}\lambda I\right]^T,$$

$$\tilde{D}_{21}(\mu) = [0 \;\; 0 \;\; \delta I \;\; D_{21}(\mu) \;\; I]$$

$\lambda \geq \left\| I + \rho^2[H_2^T(\mu)H_2(\mu) + H_5^T(\mu)H_5(\mu)] \right\|^{\frac{1}{2}}$, $H_1(\mu) = \sum_{i=1}^r \mu_i H_{1i}$, $H_2(\mu) = \sum_{i=1}^r \mu_i H_{2i}$, $H_3(\mu) = \sum_{i=1}^r \mu_i H_i$, $H_4(\mu) = \sum_{i=1}^r \mu_i H_{4i}$ and $H_5(\mu) = \sum_{i=1}^r \mu_i H_{5i}$, then the inequality (5) is guaranteed.

Proof: The state space form of the fuzzy system model (3) with the filter (4) is given by

$$\dot{\breve{x}}(t) = A_{cl}(\mu,\hat{\mu})\,\breve{x}(t) + B_{cl}(\mu,\hat{\mu})\,\tilde{\omega}(t) \tag{8}$$

Where,

$$\breve{x}(t) = [\tilde{x}^T(t) \quad \hat{x}^T(t)]$$

and the matrix functions $A_{cl}(\mu,\hat{\mu})$ and $B_{cl}(\mu,\hat{\mu})$ are defined in Theorem 1 and the disturbance $\tilde{\omega}(t)$ is

$$\tilde{\omega}(t) = \begin{bmatrix} \frac{1}{\delta}F(x(t),t)H_1(\mu)x(t) \\ F(x(t),t)H_2(\mu)x(t) \\ \frac{1}{\delta}F(x(t),t)H_4(\mu)x(t)\,\omega(t) \\ F(x(t),t)H_5(\mu)\omega(t) \end{bmatrix} \tag{9}$$

Let choose a Lyapunov function

$$V(\breve{x}(t)) = \breve{x}^T(t)\,Q\breve{x}(t) \tag{10}$$

where, $Q = P^{-1}$. Differentiate $V(\breve{x}(t))$ along the system (8) yields

$$\begin{aligned} \dot{V}(\breve{x}(t)) &= \dot{\breve{x}}^T(t)\,Q\breve{x}(t) + \breve{x}^T(t)\,Q\dot{\breve{x}}(t) \\ &= \{\breve{x}^T(t)A_{cl}(\mu,\hat{\mu}))^T\;Q\breve{x}(t) \\ &\quad + \breve{x}^T(t)QA_{cl}(\mu,\hat{\mu})\,\breve{x}(t) \\ &\quad + \tilde{\omega}^T(t)B_{cl}(\mu,\hat{\mu}))^T\;Q\breve{x}(t) \\ &\quad + \breve{x}^T(t)QB_{cl}(\mu,\hat{\mu})\tilde{\omega}(t)\} \end{aligned} \tag{11}$$

Add and subtract $-\breve{z}^T(t)\breve{z}(t) + \gamma^2\,\tilde{\omega}(t)^T\,\tilde{\omega}(t)$ to and from (11) yields

$$\dot{V}(\breve{x}(t)) = [\breve{x}^T(t) \quad \tilde{\omega}^T(t)] \times \begin{pmatrix} \begin{Bmatrix} A_{cl}^T(\mu,\hat{\mu})Q \\ +QA_{cl}(\mu,\hat{\mu}) \\ +C_{cl}^T(\mu,\hat{\mu})C_{cl}(\mu,\hat{\mu}) \end{Bmatrix} & QB_{cl}(\mu,\hat{\mu}) \\ & -\gamma^2 I \\ B_{cl}^T(\mu,\hat{\mu})Q & \end{pmatrix} \times \begin{bmatrix} \breve{x}(t) \\ \tilde{\omega}(t) \end{bmatrix} - \breve{z}^T(t)\,\breve{z}(t) + \gamma^2\,\tilde{\omega}^T(t)\,\tilde{\omega}(t) \tag{12}$$

Where,

$$\breve{z}^T(t) = \breve{z}(t) - \breve{z}(t)$$

Now suppose there exits a matrix p > 0 such that (6) holds, i.e.,

$$\begin{pmatrix} \begin{pmatrix} A_{cl}(\mu,\hat{\mu})P \\ +PA_{cl}^T(\mu,\hat{\mu}) \end{pmatrix} & (*)^T & (*)^T \\ B_{cl}^T(\mu,\hat{\mu}) & -\gamma^2 I & (*)^T \\ C_{cl}(\mu,\hat{\mu})P & 0 & -1 \end{pmatrix} < 0 \tag{13}$$

Pre and post multiply (13) by

$$\begin{pmatrix} Q & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{pmatrix}$$

yields

$$\begin{pmatrix} \begin{pmatrix} A_{cl}^T(\mu,\hat{\mu})Q \\ +QA_{cl}(\mu,\hat{\mu}) \end{pmatrix} & (*)^T & (*)^T \\ B_{cl}^T(\mu,\hat{\mu})Q & -\gamma^2 I & (*)^T \\ C_{cl}(\mu,\hat{\mu}) & 0 & -1 \end{pmatrix} < 0 \tag{14}$$

The Schur complement of (14) is

$$\begin{pmatrix} A_{cl}^T(\mu,\hat{\mu})Q + QA_{cl}^T(\mu,\hat{\mu}) & QB_{cl}(\mu,\hat{\mu}) \\ B_{cl}^T(\mu,\hat{\mu})Q & -\gamma^2 I \end{pmatrix} + \begin{pmatrix} C_{cl}^T(\mu,\hat{\mu}) \\ 0 \end{pmatrix} \begin{pmatrix} C_{cl}(\mu,\hat{\mu}) & 0 \end{pmatrix} < 0 \tag{15}$$

or

$$\begin{pmatrix} \begin{pmatrix} A_{cl}^T(\mu,\hat{\mu})Q \\ +QA_{cl}(\mu,\hat{\mu}) \\ +C_{cl}^T(\mu,\hat{\mu})C_{cl}(\mu,\hat{\mu}) \end{pmatrix} & QB_{cl}(\mu,\hat{\mu}) \\ B_{cl}^T(\mu,\hat{\mu})Q & -\gamma^2 I \end{pmatrix} < 0 \tag{16}$$

Using (16) in (12), we have

$$\dot{V}(\breve{x}(t)) \le -\breve{z}^T(t)\,\breve{z}(t) + \gamma^2\,\tilde{\omega}^T(t)\,\tilde{\omega}(t) \tag{17}$$

Integrate both sides of (17) yields

$$\int_0^{T_f} \dot{V}(\breve{x}(t))dt \le \int_0^{T_f} (-\breve{z}^T(t)\breve{z}(t) + \gamma^2\tilde{\omega}^T(t)\,\tilde{\omega}(t))\,dt$$

$$V(\breve{x}(t)) + V(\breve{x}(0)) \le \int_0^{T_f} (-\breve{z}^T(t)\breve{z}(t) + \gamma^2\tilde{\omega}^T(t)\,\tilde{\omega}(t))\,dt$$

Using the fact that $\breve{x}(0) = 0$ and $V(\breve{x}(t)) > 0$ for all $T_f \neq 0$, we have

$$0 \le \int_0^{Tf} (-\breve{z}^T(t)\breve{z}(t) + \gamma^2\tilde{\omega}^T(t)\,\tilde{\omega}(t))\,dt$$

or

$$\int_0^{T_f} \breve{z}^T(t)\breve{z}(t)dt \ge \gamma^2 \left[\int_0^{T_f} \tilde{\omega}^T(t)\,\tilde{\omega}(t)\right] dt \tag{18}$$

Putting $\breve{z}(t)$ and $\tilde{\omega}(t)$ respectively given in (8) and (9) into (18) and using the fact that $\|F(x(t),t)\| \le \rho$ and

$$\lambda^2 \ge \left\| I + \rho^2 \left[H_2^T(\mu)H_2(\mu) + H_5^T(\mu)H_5(\mu)\right]\right\|$$

we have

$$\int_0^{T_f} \left\{2\lambda^2\,\breve{x}^T(t)[C_1(\mu) - \hat{C}(\hat{\mu})]^T\,[C_1(\mu) - \hat{C}(\hat{\mu})]\,\breve{x}(t) + 2\lambda^2\rho^2 x^T(t)H_3^T(\mu)H_3(\mu)x(t)\right\} dt \le \gamma^2\lambda^2\left[\int_0^{T_f} \omega^T(t)\,\omega(t)\,dt\right] \tag{19}$$

Adding and subtracting

$$\lambda^2\breve{z}^T(t)\breve{z}(t) = \lambda^2\breve{x}^T(t) \times \left[C_1(\mu) + F(x(t),t)H_3(\mu) \quad -\hat{C}(\hat{\mu})\right]^T \times \left[C_1(\mu) + F(x(t),t)H_3(\mu) \quad -\hat{C}(\hat{\mu})\right]\breve{x}(t)$$

to and from (19), one obtains

$$\begin{aligned} &\int_0^{T_f} \Big\{\lambda^2\,\breve{z}^T(t)\breve{z}(t) \\ &+2\lambda^2\breve{x}^T(t)[C_1(\mu)\text{-}\hat{C}(\hat{\mu})]^T\,[C_1(\mu)\text{ - }\hat{C}(\hat{\mu})]\,\breve{x}(t) \\ &+2\lambda^2\,\rho^2 x^T(t)\,H_3^T(\mu)H_3(\mu)x(t) \\ &-\lambda^2\breve{x}^T(t)[C_1(\mu)+F(x(t),t)\,H_3(\mu)\text{- }\hat{C}(\hat{\mu})]^T \\ &[C_1(\mu)+F(x(t),t)H_3(\mu)\text{ -}\hat{C}(\hat{\mu})]\breve{x}(t)\Big\}\,dt \\ &£\,\gamma^2\lambda^2\left[\int_0^{T_f} \omega^T(t)\,\omega(t)\,dt\right] \end{aligned} \tag{20}$$

Using the triangular inequality and the fact that $\|F(x(t), t)\| \le \rho$ we have

$$\lambda^2 \breve{x}^T(t)\left[C_1(\mu)+F(x(t),t)H_3(\mu)-\hat{C}(\hat{\mu})\right]^T \times \left[C_1(\mu)+F(x(t),t)H_3(\mu)-\hat{C}(\hat{\mu})\right]\breve{x}(t)$$
$$\le 2\lambda^2 \breve{x}^T(t)\left[C_1(\mu)-\hat{C}(\hat{\mu})\right]^T\left[C_1(\mu)-\hat{C}(\hat{\mu})\right]\breve{x}(t) + 2\lambda^2 x^T(t)[F(x(t),t)H_3(\mu)]^T[F(x(t),t)H_3(\mu)]x(t)$$
$$\le 2\lambda^2 \breve{x}^T(t)\left[C_1(\mu)-\hat{C}(\hat{\mu})\right]^T\left[C_1(\mu)-\hat{C}(\hat{\mu})\right]\breve{x}(t) + 2\lambda^2\rho^2 x^T(t)H_3^T(\mu)H_3(\mu)x(t) \quad (21)$$

Using (21) on (20), we obtain

$$\int_0^{T_f} \breve{z}^T(t)\breve{z}(t) \le \gamma^2 \int_0^{T_f} \omega^T(t)\omega(t)dt \quad (22)$$

Where,

$$\breve{z}^T(t) = z(t) - z^T(t)$$

Hence, the inequality (5) is guaranteed.

Case I-ν(t) is available for feedback: The premise variable of the fuzzy model ν is available for feedback which implies that μ is available for feedback. Thus we can select our filter that depends on μ as follows:

$$\begin{aligned}\dot{\hat{x}}(t) &= \hat{A}(\mu)\hat{x}(t) + \hat{B}(\mu)y(t)\\ \hat{z}(t) &= \hat{C}(\mu)\hat{x}(t)\end{aligned} \quad (23)$$

Knowing that the filter's premise variable is the same as the plant's premise variable, the left hand of (6) can be re-expressed as follows:

$$\begin{aligned}&A_{cl}(\mu,\mu)P + PA_{cl}^T(\mu,\mu)\\ &+\gamma^{-2}B_{cl}(\mu,\mu)B_{cl}^T(\mu,\mu) +\\ &PC_{cl}^T(\mu,\mu)\,C_{cl}(\mu,\mu)P\end{aligned} \quad (24)$$

Before providing LMI-based sufficient conditions for the system (3) to have an H∞ performance, let us partition the matrix P as follows:

$$P = \begin{bmatrix} X & Y^{-1} - X \\ Y^{-1} - X & X - Y^{-1}\end{bmatrix} \quad (25)$$

where, $X \in \Re^{n\times n}$ and $Y \in \Re^{n\times n}$. Utilizing the partition above, we define the new filter's input and output matrices as

$$\begin{aligned}\mathbf{B}(\mu) &\triangleq [Y^{-1} - X]\,\hat{B}(\mu)\\ \mathbf{C}(\mu) &\triangleq \hat{C}(\mu)\,Y\end{aligned} \quad (26)$$

Using these changes of variable, we have the following theorem.

Theorem 2: Consider the system (3). Given a prescribed H∞ performance γ > 0, a positive constant γ. If there exist matrices X, Y, $\mathbf{B}(\mu)$ and $\mathbf{C}(\mu)$ satisfying the following nonlinear matrix inequalities:

$$\begin{bmatrix} X & I \\ I & Y\end{bmatrix} > 0 \quad (27)$$

$$X > 0 \quad (28)$$

$$Y > 0 \quad (29)$$

$$\psi 11(\mu) < 0 \quad (30)$$

$$\psi 22(\mu) < 0 \quad (31)$$

Where,

$$\begin{aligned}\Psi_{11}(\mu) &= A(\mu)Y + YA^T(\mu) + \gamma^{-2}\tilde{B}_1(\mu)\tilde{B}_1^T(\mu)\\ &+\left[Y\tilde{C}_1^T(\mu) + \mathbf{C}^T(\mu)\tilde{D}_{12}^T\right]\times\\ &\left[Y\tilde{C}_1^T(\mu) + \mathbf{C}^T(\mu)\tilde{D}_{12}^T\right]^T\end{aligned} \quad (32)$$

$$\begin{aligned}\Psi_{22}(\mu) &= A^T(\mu)X + XA(\mu)\\ &+\mathbf{B}(\mu)C_2(\mu) + C_2^T(\mu)\mathbf{B}^T(\mu) + \tilde{C}_1^T(\mu)\tilde{C}_1(\mu)\\ &+\left(\gamma^{-2}[X\tilde{B}_1(\mu) + \mathbf{B}(\mu)\tilde{D}_{21}(\mu)]\times\right.\\ &\left.[X\tilde{B}_1(\mu) + \mathbf{B}(\mu)\tilde{D}_{21}(\mu)]^T\right)\end{aligned} \quad (33)$$

then the prescribed H∞ performance γ > 0 is guaranteed. Furthermore, a suitable filter

$$(\hat{A}(\mu),\ \hat{B}(\mu) \text{ and } \hat{C}(\mu))$$

is given as follows:

$$\begin{aligned}\hat{B}(\mu) &= [Y^{-1} - X]^{-1}\,\mathbf{B}(\mu)\\ \hat{C}(\mu) &= \mathbf{C}(\mu)Y^{-1}\\ \hat{A}(\mu) &= [Y^{-1} - X]^{-1}\,\mathbf{M}(\mu)\,Y^{-1}\end{aligned} \quad (34)$$

Where,

$$\begin{aligned} M(\mu) &= -A^T(\mu) - XA(\mu)Y \\ &-[Y^{-1}-X]\,\hat{B}(\mu)C_2(\mu)\,Y \\ &-\tilde{C}_1^T(\mu)[\tilde{C}_1(\mu)\,Y + \tilde{D}_{12}\,\tilde{C}(\mu)Y] \\ &-\gamma^{-2}\left\{X\hat{B}_1(\mu) + [Y^{-1}-X]\,\hat{B}(\mu)\tilde{D}_{21}(\mu)\right\}\tilde{B}_1^T(\mu) \end{aligned} \tag{35}$$

Proof: Suppose there exist X and Y such that there inequalities (27) and (30)-(31) holds. The inequalities (27) imply that the matrix P defined in (25) is a positive definite matrix. Using the partition (25), the filter (34) and multiplying (24) to the left by

$$\begin{bmatrix} Y & I \\ Y & 0 \end{bmatrix}$$

and to the right by

$$\begin{bmatrix} Y & Y \\ I & 0 \end{bmatrix}$$

we have

$$\begin{bmatrix} \Psi_{11}(\mu) & 0 \\ 0 & \Psi_{22}(\mu) \end{bmatrix}$$

According to (30)-(31) and Theorem 1, we learn that the prescribed H∞ performance $\gamma > 0$ is guaranteed.

Theorem 3: Consider the system (3). Given a prescribed H∞ performance $\gamma > 0$ and a positive constant δ. If there exist matrices X, Y, B_i and C_i satisfying the following nonlinear matrix inequalities:

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \tag{36}$$

$$X > 0 \tag{37}$$

$$Y > 0 \tag{38}$$

$$\psi_{11ii} < 0,\ i = 1, 2, \ldots, r \tag{39}$$

$$\psi_{22ii} < 0,\ i = 1, 2, \ldots, r \tag{40}$$

$$\psi_{11ij} + \psi_{11ji} < 0,\ i < j \le r \tag{41}$$

$$\psi_{22ij} + \psi_{22ji} < 0,\ i < j \le r \tag{42}$$

Where,

$$\Psi_{11ij} = \begin{bmatrix} \begin{pmatrix} A_i\,Y + Y A_{1i}^T \\ +\gamma^{-2}\,\tilde{B}_{1i}\,\tilde{B}_{1i}^T \end{pmatrix} & (*)^T \\ [Y\tilde{C}_{1i}^T + C_i^T\,\tilde{D}_{12}^T] & -1 \end{bmatrix} \tag{43}$$

$$\Psi_{22ij} = \begin{bmatrix} \begin{pmatrix} A_i^T X + XA_i \\ +B_i\,C_{2j} + C_{2i}^T\,B_j^T \\ +\tilde{C}_{1i}^T\,\tilde{C}_{1i} \end{pmatrix} & (*)^T \\ [X\tilde{B}_{1i}^T + B_i\,\tilde{D}_{21j}] & -\gamma^2 I \end{bmatrix} \tag{44}$$

with

$$\begin{aligned} \tilde{B}_{1i} &= [\delta I \quad I \quad 0 \quad B_{1i} \quad 0 \\ \tilde{C}_{1i} &= \left[\tfrac{\gamma\rho}{\delta}H_{1i}^T \quad \tfrac{\gamma\rho}{\delta}H_{4i}^T \quad \sqrt{2}\lambda\rho\,H_{3i}^T \quad \sqrt{2}\lambda C_{1i}^T\right]^T \\ \tilde{D}_{12} &= \left[0 \quad 0 \quad 0 \quad \sqrt{2}\lambda I\right]^T \\ \tilde{D}_{21i} &= [0 \quad 0 \quad \delta I \quad D_{21i} \quad I \end{aligned}$$

and

$$\lambda \ge \left\| I + \rho^2\ [H_2^T(\mu)H_2(\mu) + H_5^T(\mu)H_5(\mu)] \right\|^{\frac{1}{2}}$$

then the prescribed H∞ performance $\gamma > 0$ is guaranteed. Furthermore, a suitable filter

$$\hat{A}_{ij}(\varepsilon),\ \hat{B}_i\ \text{and}\ \hat{C}_i$$

is given as follows:

$$\begin{aligned} \hat{B}_i &= [Y^{-1} - X]^{-1}\,B_i \\ \hat{C}_i &= C_i\ Y^{-1} \\ \hat{A}_{ij} &= [Y^{-1} - X]^{-1}\,M_{ij}\ Y^{-1} \end{aligned} \tag{45}$$

Where,

$$\begin{aligned} M_{ij} &= -A_i^T - XA_i\,Y - [Y^{-1} - X]\,\hat{B}_i\,C_{2j}\,Y \\ &-\hat{C}_{1i}^T[\hat{C}_{1j}\,Y + \tilde{D}_{12}\,\hat{C}_j\,Y] \\ &-\gamma^{-2}\left\{X\tilde{B}_{1i} + [Y^{-1} - X]\,\hat{B}_i\,D_{21i}\right\}\hat{B}_{1j}^T \end{aligned} \tag{46}$$

Proof: From Theorem 2, using the fact that

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\ \mu_j\ \mu_m\ \mu_n\ M_{ij}^T\ N_{mn}$$
$$\le \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\ \mu_j\ [M_{ij}^T\ M_{ij} + N_{ij}\,N_{ij}^T]$$

we obtain

$$\begin{bmatrix} \Psi_{11}(\mu) & 0 \\ 0 & \Psi_{22}(\mu) \end{bmatrix} \le \sum_{i=1}^{r} \mu_i\ \mu_i \begin{bmatrix} \Psi_{11ii} & 0 \\ 0 & \Psi_{22ji} \end{bmatrix} + \sum_{i=1}^{r}\sum_{i<j}^{r} \mu_i\ \mu_j \left\{ \begin{bmatrix} \Psi_{11ij} & 0 \\ 0 & \Psi_{22ij} \end{bmatrix} \begin{bmatrix} \Psi_{11ji} & 0 \\ 0 & \Psi_{22ji} \end{bmatrix} \right\} \quad (47)$$

Using (39)-(42), we have (47) less than zero. Hence by Theorem 2, we learn that the inequality (5) holds.

Case II-ν(t) is unavailable for feedback: Now the premise variable of the fuzzy model ν is unavailable for feedback which implies μ is unavailable for feedback. Hence, we can not select our filter which depends on μ. Thus we select our filter as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}(\hat{\mu})\hat{x}(t) + \hat{B}(\hat{\mu})\ y(t) \\ z(t) &= \hat{C}(\hat{\mu})\ \hat{x}(t) \end{aligned} \quad (48)$$

where, $\hat{\mu}$ depends on the premise variable of the filter which is different from μ.

Let us re-express the system (3) in the term of $\hat{\mu}$, thus that the plant's premise variable becomes thesame as the filter's premise variable. By doing so, the result given in the previous case can then be applied here. After some manipulation, we get

$$\begin{aligned} \dot{x}(t) &= [A(\hat{\mu}) + \Delta\bar{A}(\hat{\mu})]x(t) \\ &\quad + [B_1(\hat{\mu}) + \Delta\bar{B}_1(\hat{\mu})]\ \omega(t), \\ z(t) &= [C_1(\hat{\mu}) + \Delta\bar{C}_1(\hat{\mu})]x(t) \\ y(t) &= [C_2(\hat{\mu}) + \Delta\bar{C}_2(\hat{\mu})]x(t) \\ &\quad + [D_{21}(\hat{\mu}) + \Delta\bar{D}_{21}(\hat{\mu})]\omega(t) \end{aligned} \quad (49)$$

where x (0) = 0,

$$A(\hat{\mu}) = \sum_{i=1}^{r} \hat{\mu}_i A_i, \quad B_1(\hat{\mu}) = \sum_{i=1}^{r} \hat{\mu}_i B_{1i},$$

$$C_1(\hat{\mu}) = \sum_{i=1}^{r} \hat{\mu}_i C_{1i}, \quad C_2(\hat{\mu}) = \sum_{i=1}^{r} \hat{\mu}_i C_{2i},$$

$$D_{21}(\hat{\mu}) = \sum_{i=1}^{r} \hat{\mu}_i D_{21i}$$

with

$$\begin{aligned} \Delta\bar{A}(\hat{\mu}) &= \bar{F}(x(t),t)\ \bar{H}_1\ (\hat{\mu}), \\ \Delta\bar{B}_1(\hat{\mu}) &= \bar{F}(x(t),t)\ \bar{H}_2\ (\hat{\mu}), \\ \Delta\bar{C}_1(\hat{\mu}) &= \bar{F}(x(t),t)\ \bar{H}_3\ (\hat{\mu}), \\ \Delta\bar{C}_2(\hat{\mu}) &= \bar{F}(x(t),t)\ \bar{H}_4\ (\hat{\mu}), \\ \text{and } \Delta\bar{D}_{21}(\hat{\mu}) &= \bar{F}(x(t),t)\ \bar{H}_5\ (\hat{\mu}), \end{aligned}$$

with

$$\bar{F}(x(t),t) = \begin{bmatrix} F^T(x(t),t) \\ (\mu_1 - \hat{\mu}_1) \\ \vdots \\ (\mu_1 - \hat{\mu}_1) \\ F^T(x(t),t)\ (\mu_1 - \hat{\mu}_1) \\ \vdots \\ F^T(x(t),t)\ (\mu_r - \hat{\mu}_r) \end{bmatrix}$$

$$\bar{H}_1(\hat{\mu}) = \left[\sum_{i=1}^{r} \hat{\mu}_i H_{1i}^T\ A_1^T\ \dots A_r^T\ H_{1i}^T\ \dots H_{1r}^T \right]^T$$

$$\bar{H}_2(\hat{\mu}) = \left[\sum_{i=1}^{r} \hat{\mu}_i H_{2i}^T\ B_{1_1}^T\ \dots B_{1r}^T\ H_{2i}^T\ \dots H_{2r}^T \right]^T$$

$$\bar{H}_3(\hat{\mu}) = \left[\sum_{i=1}^{r} \hat{\mu}_i H_{3i}^T\ B_{2_1}^T\ \dots B_{2r}^T\ H_{3i}^T\ \dots H_{3r}^T \right]^T$$

$$\bar{H}_4(\hat{\mu}) = \left[\sum_{i=1}^{r} \hat{\mu}_i H_{4i}^T\ C_{1_1}^T\ \dots C_{1r}^T\ H_{4i}^T\ \dots H_{4r}^T \right]^T$$

$$\bar{H}_5(\hat{\mu}) = \left[\sum_{i=1}^{r} \hat{\mu}_i H_{5i}^T\ C_{2_1}^T\ \dots C_{2r}^T\ H_{5i}^T\ \dots H_{5r}^T \right]^T$$

Note that

$$\left\| \bar{F}(x\ (t),\ t) \right\| \le \bar{\rho}$$

where, $\bar{\rho} > 0$. Now the premise variable of the system is the same as the premise variable of the filter, thus we can apply the results given in Case I.

Theorem 4: Consider the system (3). Given a prescribed H∞ performance γ > 0 and a positive constant δ. If there exist matrices X, Y, B_i and C_i satisfying the following matrix inequalities:

$$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \quad (50)$$

$$X > 0 \quad (51)$$

$$Y > 0 \quad (52)$$

$$\psi_{11ii} < 0,\ i = 1, 2, \ldots, r \tag{53}$$

$$\psi_{22ii} < 0,\ i = 1, 2, \ldots, r \tag{54}$$

$$\psi_{11ij} + \psi_{11ji} < 0,\ i < j \le r \tag{55}$$

$$\psi_{22ij} + \psi_{22ji} < 0,\ i < j \le r \tag{56}$$

Where,

$$\Psi_{11ij} = \begin{bmatrix} \begin{pmatrix} A_i Y + Y A_i^T \\ + \gamma^{-2} \bar{\bar{B}}_{1i} \bar{\bar{B}}_{1i}^T \end{pmatrix} & (*)^T \\ [Y \bar{\bar{C}}_{1_i}^T + C_2^T \bar{\bar{D}}_{12}^T]^T & -I \end{bmatrix} \tag{57}$$

$$\Psi_{22ij} = \begin{bmatrix} \begin{pmatrix} A_i^T X + X A_i \\ + \mathcal{B}_i C_{2_j} + C_{2_i}^T \mathcal{B}_j^T \end{pmatrix} & (*)^T \\ [X \bar{\bar{B}}_{1_i} + \mathcal{B}_i \bar{\bar{D}}_{21_j}]^T & -\gamma^2 I \end{bmatrix} \tag{58}$$

with

$$\begin{aligned} \bar{\bar{B}}_{1_i} &= [\delta I \quad I \quad 0 \quad B_{1i} \quad 0] \\ \bar{\bar{C}}_{1_i} &= \left[\tfrac{\gamma\bar{\rho}}{\delta} \bar{H}_{1i}^T \quad \tfrac{\gamma\bar{\rho}}{\delta} \bar{H}_{4i}^T \quad \sqrt{2}\lambda\bar{\rho} \bar{H}_{3i}^T \quad \sqrt{2}\lambda C_{1i}^T\right]^T \\ \bar{\bar{D}}_{12} &= \left[0 \quad 0 \quad 0 \quad \sqrt{2}\lambda I\right]^T \\ \bar{\bar{D}}_{21_i} &= [0 \quad 0 \quad \delta I \quad D_{21i} \quad I] \end{aligned}$$

and

$$\bar{\lambda} \ge \left\| I + \bar{\rho}^2 [\bar{H}_2^T(\mu)\, \bar{H}_2(\mu) + \bar{H}_5^T(\mu)\, \bar{H}_5(\mu)] \right\|^{\frac{1}{2}}$$

then the prescribed H∞ performance $\gamma > 0$ is guaranteed. Furthermore, a suitable filter

$$(\hat{A}_{ij}, \hat{B}_i \text{ and } \hat{C}_i)$$

is given as follows:

$$\begin{aligned} \hat{B}_i &= [Y^{-1} - X]^{-1} \mathcal{B}_i \\ \hat{C}_i &= \mathcal{C}_i\ Y^{-1} \\ \hat{A}_{ij} &= [Y^{-1} - X]^{-1} \mathcal{M}_{ij}\ Y^{-1} \end{aligned} \tag{59}$$

Where,

$$\begin{aligned} \mathcal{M}_{ij} = & -A_i^T - X A_i Y - [Y^{-1} - X]\, \hat{B}_i C_{2j} Y \\ & - \bar{\bar{C}}_{1i}^T [\bar{\bar{C}}_{1j} Y + \bar{\bar{D}}_{12} \hat{C}_j Y] \\ & - \gamma^{-2} \left\{ X \bar{\bar{B}}_{1i} + [Y^{-1} - X]\, \hat{B}_i \bar{\bar{D}}_{21i} \right\} \bar{\bar{B}}_{1j}^T \end{aligned} \tag{60}$$

Proof: It can be shown by employing Theorem 3.

AN EXAMPLE

Consider a tunnel diode circuit shown in Fig. 1 where the tunnel diode is characterized by

$$i_D(t) = 0.002 \upsilon_D(t) + 0.01 \upsilon_D^3(t)$$

Let $x_1(t) = v_C(t)$ and $x_2(t) = i_L(t)$ as the state variables, then the circuit is governed by the following state equations:

$$\begin{aligned} C\dot{x}_1(t) &= -0.002 x_1(t) - 0.01 x_1^3(t) + x_2(t) \\ L\dot{x}_2(t) &= -x_1(t) - R x_2(t) + 0.1 \omega_2(t) \\ y(t) &= Sx(t) + 0.1 \omega_1(t) \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{61}$$

where, $\omega(t)$ is the disturbance noise input, $y(t)$ is the measurement output, $z(t)$ is the state to be estimated and S is the sensor matrix. The parameters in the circuit are given as follows: C = 20 mF, L = 1000 mH and R = 10±10% Ω With these parameters, (61) can be rewritten as:

$$\begin{aligned} \dot{x}_1(t) &= -0.1 x_1(t) - 0.5 x_1^2(t) \cdot x_1(t) + 50 x_2(t) \\ \dot{x}_2(t) &= -x_1(t) - (10 + \Delta R) x_2(t) + 0.1 \omega_2(t) \\ y(t) &= Sx(t) + 0.1 \omega_1(t) \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{62}$$

For the sake of simplicity, we will use as few rules as possible. Assuming that $|x_1(t)| \le 3$ the nonlinear network system (62) can be approximated by the following 2 rules TS model:

Plant rule 1: IF $x_1(t)$ is $M_1(x_1(t))$ THEN

$$\begin{aligned} \dot{x}(t) &= [A_1 + \Delta A_1] x(t) + B_{1_1} \omega(t), \\ z(t) &= C_{1_1 x(t)}, \\ y(t) &= C_{2_1} x(t) + D_{21_1} \omega(t) \end{aligned}$$

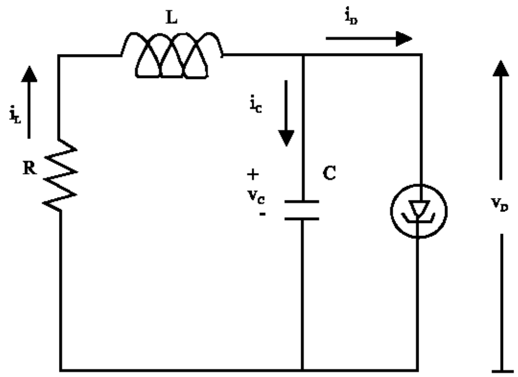

Fig. 1: Tunnel diode circuit

Plant rule 2: IF $x_1(t)$ is $M_2(x_1(t))$ THEN

$$\begin{aligned} \dot{x}(t) &= [A_2 + \Delta A_2]x(t) + B_{1_2}\omega(t), \\ z(t) &= C_{1_2}x(t), \\ y(t) &= C_{2_2}x(t) + D_{21_2}\omega(t) \end{aligned}$$

Where,

$$A_1 = \begin{bmatrix} -0.1 & 50 \\ -1 & -10 \end{bmatrix}, B_{1_1} \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C_{1_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D_{21_1} = [0.1 \quad 0],$$

$$A_2 = \begin{bmatrix} -4.6 & 50 \\ -1 & -10 \end{bmatrix}, B_{1_2} \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C_{1_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, D_{21_2} [0.1 \quad 0],$$

$$C_{2_1} = C_{2_2} = S,$$

$$\Delta A_1 = F(x(t),t)H_{1_1} \text{ and } \Delta A_2 = F(x(t),t)H_{1_2}$$

Now, by assuming that in (2), $\|F(x(t), t)\| \le \rho = 1$ and since, the values of R are uncertain but bounded within 10% of their nominal values given in (61), then we have

$$H_{1_1} = H_{1_2} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Figure 2 shows the plots of the membership functions for Rules 1 and 2.

Case I-ν(t) is available for feedback: In this case, $x_1(t) = \nu(t)$ is assumed to be available for feedback, i.e., $S = [1 \quad 0]$. This implies that μ is available for feedback. Using the LMI optimization algorithm and Theorem 3 with $\gamma = 1$ and $\delta = 1$, we obtain

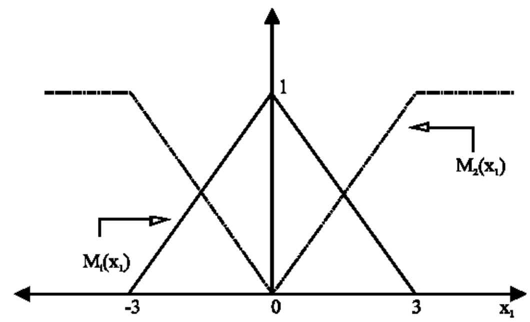

Fig. 2: Membership functions for the two fuzzy set

$$X = \begin{bmatrix} 34.5536 & -2.4910 \\ -2.4910 & 0.8883 \end{bmatrix},$$

$$Y = \begin{bmatrix} 0.8986 & 1.7528 \\ 1.7528 & 27.4284 \end{bmatrix},$$

$$\mathsf{B}_1 = \begin{bmatrix} -3.3132 \\ -19.2791 \end{bmatrix}, \mathsf{B}_2 = \begin{bmatrix} -0.8017 \\ -13.3587 \end{bmatrix}$$

$$\mathsf{C}_1 = [35.3349 \quad -2.5095],$$

$$\mathsf{C}_2 = [34.7013 \quad -2.5098],$$

Substituting X, Y , B_i and C_j into (45), we obtain

$$\hat{A}_{11} = \begin{bmatrix} -9.4003 & -1.1377 \\ 63.2915 & -3.7526 \end{bmatrix},$$

$$\hat{A}_{12} = \begin{bmatrix} -14.7653 & 1.3877 \\ 79.5268 & -2.9644 \end{bmatrix},$$

$$\hat{A}_{21} = \begin{bmatrix} -5.8973 & -0.9794 \\ 49.9964 & -4.1557 \end{bmatrix},$$

$$\hat{A}_{22} = \begin{bmatrix} -11.3243 & -1.2277 \\ 61.4584 & -3.3736 \end{bmatrix},$$

$$\hat{B}_1 = \begin{bmatrix} -0.1292 \\ 1.0828 \end{bmatrix}, \hat{B}_2 = \begin{bmatrix} -0.0312 \\ 0.7246 \end{bmatrix}$$

$$\hat{C}_1 = [-35.3890 \quad -1.5720],$$

$$\hat{C}_2 = [-34.7556 \quad -1.5891]$$

Hence, the resulting fuzzy filter is

$$\begin{aligned} \dot{\hat{x}}(t) &= \hat{A}(\mu)\hat{x}(t) + \hat{B}(\mu)y(t) \\ \hat{z}(t) &= \hat{C}(\mu)\hat{x}(t) \end{aligned}$$

Where,

$$\hat{A}(\mu) = \sum_{i=1}^{2}\sum_{j=1}^{2}\mu_i\ \mu_j\ \hat{A}_{ij},$$

$$\hat{B}(\mu) = \sum_{i=1}^{2}\mu_i\ \hat{B}_i\ \text{and}\ \hat{C}(\mu) = \sum_{i=1}^{2}\mu_i\ \hat{C}_i$$

with

$$\mu_1 = M_1(x_1(t)) \text{ and } \mu_2 = M_2(x_1(t))$$

Case II-ν(t) is unavailable for feedback: In this case, $x_1(t) = \nu(t)$ is assumed to be unavailable for feedback, i.e., S = [0 1]. This implies that μ is unavailable for feedback. Using the LMI optimization algorithm and Theorem 4 with $\gamma = 1$ and $\delta = 1$, we obtain

$$X = \begin{bmatrix} 77.3789 & -15.9191 \\ -15.9191 & 5.3505 \end{bmatrix},$$

$$Y = \begin{bmatrix} 1.6782 & -1.6006 \\ -1.6006 & 27.7695 \end{bmatrix},$$

$$\mathsf{B}_1 = \begin{bmatrix} -2.8679 \\ -14.4564 \end{bmatrix}, \mathsf{B}_2 = \begin{bmatrix} -4.1318 \\ -1.0994 \end{bmatrix}$$

$$\mathsf{C}_1 = [109.4744 \quad -22.5321],$$

$$\mathsf{C}_2 = [109.4773 \quad -22.5336],$$

Substituting X, Y, B_i and C_j into (59), we obtain

$$\hat{A}_{11} = \begin{bmatrix} -11.2662 & -1.0266 \\ 92.4376 & -1.1388 \end{bmatrix},$$

$$\hat{A}_{12} = \begin{bmatrix} -15.0307 & -0.9550 \\ 132.5941 & -2.1872 \end{bmatrix},$$

$$\hat{A}_{21} = \begin{bmatrix} -11.8923 & -1.0235 \\ 77.7075 & -0.9892 \end{bmatrix},$$

$$\hat{A}_{22} = \begin{bmatrix} -15.6465 & -0.9299 \\ 115.8735 & -1.7290 \end{bmatrix},$$

$$\hat{B}_1 = \begin{bmatrix} -0.0181 \\ 0.5905 \end{bmatrix}, \hat{B}_2 = \begin{bmatrix} -0.0261 \\ 0.3875 \end{bmatrix}$$

$$\hat{C}_1 = [-111.7427 \quad 2.4315],$$

$$\hat{C}_2 = [-111.7360 \quad 2.4290]$$

Hence, the resulting fuzzy filter is

$$\dot{\hat{x}}(t) = \hat{A}(\mu)\hat{x}(t) + \hat{B}(\mu)y(t)$$
$$\hat{z}(t) = \hat{C}(\mu)\hat{x}(t)$$

Where,

$$\hat{A}(\hat{\mu}) = \sum_{i=1}^{2}\sum_{j=1}^{2}\hat{\mu}_i\ \hat{\mu}_j\ \hat{A}_{ij},$$

$$\hat{B}(\hat{\mu}) = \sum_{i=1}^{2}\hat{\mu}_i\ \hat{B}_i\ \text{and}\ \hat{C}(\hat{\mu}) = \sum_{i=1}^{2}\hat{\mu}_i\hat{C}_i$$

with

$$\hat{\mu}_1 = M_1(\hat{x}_1(t)) \text{ and } \hat{\mu}_2 = M_2(\hat{x}_1(t))$$

Remark 1: Figure 3 and 4 show the responses of z(t). The fuzzy estimated z(t) is yielded by the fuzzy H∞ filter and the linear estimated z(t) is generated by a linear H∞ filter linearized at the origin. The disturbance input signal, w(t), which was used during simulation is given in Fig. 5. Simulation results for the ratio of the filter error energy to the disturbance input noise energy obtained by using the

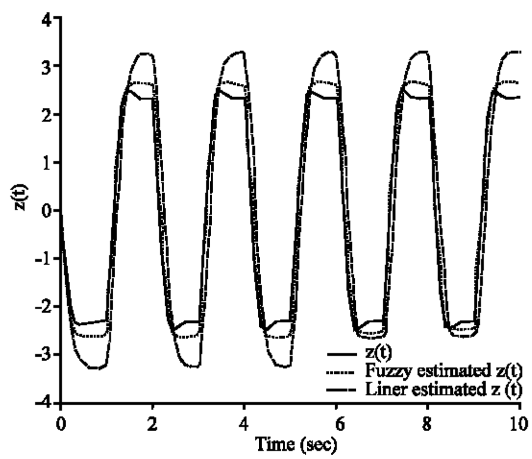

Fig. 3: Case I: The histories of z(t) and $\hat{z}(t)$

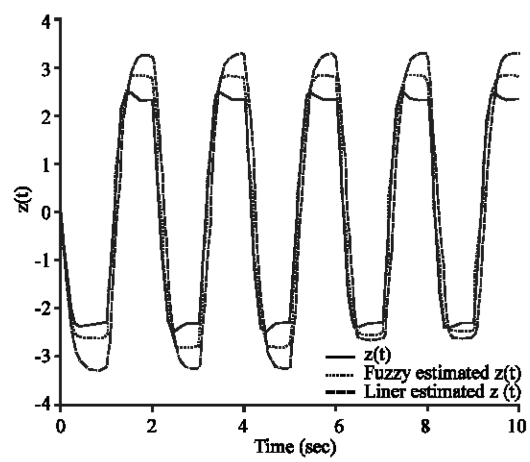

Fig. 4: Case II: The histories of z(t) and $\hat{z}(t)$

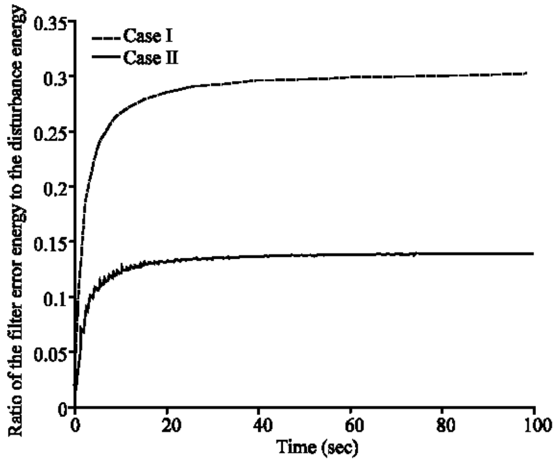

Fig. 5: The ratio of the filter error energy to the disturbance noise energy

$$\frac{\int_0^{Tf}(z(t)-\hat{z}(t))^T\,(z(t)-\hat{z}(t))dt}{\int_0^{Tf}\omega^T(t)\,\omega(t)\,dt}$$

H∞ fuzzy filter are depicted. After 100 sec, the ratio of the filter error energy to the disturbance input noise energy tends to a constant value which is about 0.14 for Case I and 0.30 for Case II. Thus, for Case I, $\gamma = \sqrt{0.14} = 0.37$ and $\gamma = \sqrt{0.30} = 0.55$ for Case II which are less than the prescribed value 1.

CONCLUSION

This study has investigated the problem of designing a robust H^∞ fuzzy filter for a class of uncertain nonlinear dynamic systems that given a dynamic system with the exogenous input noise and the measured output, the L_2-gain of the mapping from the exogenous input noise to the regulated output is less than or equal to a prescribed value. Based on an LMI approach, solutions to the problem of the robust H∞ fuzzy filter have been derived in terms of a family of linear matrix inequalities. A numerical simulation example is presented to illustrate the theory development.

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