International Journal of System Signal Control and Engineering Application 1 (1): 38-48, 2008
ISSN: 1997-5422

Robustness of Fuzzy Variable Structure Controllers

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Abstract: In this study, a fuzzy (FLC) based variable structure control (VSC) is presented. VSC is characterized by its ability to deal with non-linearities, uncertainties of control systems, invariance to parameters fluctuations and disturbances and its robustness. The main objective is to obtain an improved performance of highly non-linear unstable systems. Both the fuzzy controller and the controller system are represented by the Affine Takagi-Sugeno fuzzy model (T-S). The main aim of this research is to design a completely FLC-VSC, which means a one without an additional conventional VSC as usually done by several other researchers. This proposed, controller will also be able to solve the chattering problem which is considered the main drawback of the VSC control by introducing a boundary layer around the switching surface and applying the equivalent control method inside this layer. From implementation point of view, each fuzzy rule will have a unique fuzzy consequent part. The main feature of the proposed method is that the switching function s(x) is added as an additional fuzzy variable and will be introduced in the premise part of the fuzzy rules together with the state variables. The proposed method is explained by applying it on an inverted pendulum mounted on a cart. The results obtained show a robust and stable behavior without chattering.

Key words: Fuzzy control, T-S model, variable structure control, inverted pendulum, chattering

INTRODUCTION

The VSC approach is one of the robust control methods to handle systems with model uncertain-ties (Chen and Chang, 1998) or with imprecise knowledge of their parameters.

The most distinguished feature of VSC is its ability to result in very robust control systems; in many cases invariant control systems result.

The term invariant means that the system is completely insensitive to parametric uncertainty and external disturbances.

VSC is naturally attractive to control engineers because its basic concepts are rather easy to understand and has given satisfactory performance in many practical areas of industrial electronics. More importantly VSC is applicable to many control systems where there are no other design methods well developed. It has attracted interest recently because a fast calculation and switching action have been realized through the progress of micro and power electronics.

The main VSC feature is to drive the state trajectory towards a sliding plan previously determined by computing the feedback control structure. It is desirable to simultaneously: reach the sliding plane fastly; maintain the trajectory close to it and reduce the number of switching between structures (chattering). However, reaching the sliding plane fast implies also a fast departure, unless a frequent switching between structures is allowed. In a digital control scheme the trajectory will depart from the S.P. increasingly with the sampling time.

Motivated by observation on similarly between FLC control rules and the VSC, the robustness of the FLC's has been analyzed for nonlinear dynamic systems in this study. As results, the asymptotic behavior of the fuzzy control system can be clarified and the relationship between the design parameters of the FLC and tracking performances of the control system is addressed explicitly. The relationship is important since, it gives guidance on the design parameters of the FLC to achieve the specified control performances (Al-Hadithi *et al*., 2005; Shiuh-Jer and Lin, 2003; Hwang, 2004; Hwang and Lin, 2004; Hwang and Jan, 2003; Tong and Li, 2003; Yi and Chung, 1998). This analysis gives an account of the relationship between control performance and the design parameters of FLC, which has been obscure in the theory of fuzzy control.

Sliding modes are used to determined best values for parameters in fuzzy control rules in which the robustness in inherent in the VSC-SM with sliding modes thereby,

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robustness in fuzzy control can be improved. With the aid of sliding modes, it provides an effective design methods of fuzzy control to ensure robustness.

In this study, we will show how the VSC-SM, the structure of which is based on mathematical analysis, can be made more appropriate for actual implementation by introduction of fuzzy rules. In this study, we will develop a methodology for designing a fuzzy rule based controller to smooth the control input for a general class of VSC-SM's.

The motivation behind this scheme is to combine the best feature of fuzzy control and sliding mode control to achieve rapid and accurate tracking control of a class of nonlinear systems (Alberto, 2005; Feeny, 1994; Chanchun and Guan, 2004; Chen, n.d; Ha and Durrant, 2004; Ishigame *et al.*, 1993; Kawaji *et al.*, 1991; Kawaji and Matsunaga, 1991; Kim and Lee, 1995; Cung-Chun and Chen, 2005; Lam and Leung, 2005; Chi-Ying and Su, 2003; Lin and Kung, 1992; Lo and Kuo, 1998; Meda-Campana and Castillo-Toledo, 2005; Lu and Chun, 1994; Palm, 1994; Pedryez, 1989; Wu and Liu, 1996b; Yi and Cheng, 1995; Tao *et al.*, 2004).

In this study, a nonlinear plant is represented by the Affine T-S fuzzy model because it is more suitable approximately non linear systems than other fuzzy models with relation metrics. But the majority of research works (Tanaka *et al.*, 1996; Tanaka and Sano, 1994) analyzed T-S fuzzy model assuming that the non linear fuzzy system is linearized with respect to the origin in each IF-THEN rule, which means that the consequent part of each rule is a linear function with zero independent term. This will in turn reduce the accuracy of approximately non linear systems. Moreover, in linear control theory, the independent term dose not affect the dynamics of the system rather the input to it. In the case of fuzzy control, the fuzzy system is resulted from blending all the sub-systems. The blending of the independent term of each rule will no longer be a constant but a function of the variables of the system and thus affects the dynamics of the resultant system.

In this study, a design of FLC-VSC is presented based on T-S fuzzy model (Takagi and Sugeno, 1985) taking into account the effect of the independent term in both the fuzzy system and controller. A necessary condition has been added to deal with the independent term. A solution is suggested to overcome the chattering problem by introducing a boundary layer around the switching surface and applying the equivalent control method inside this layer.

The proposed controller is applied to control an inverted pendulum mounted on a cart. The results obtained show a robust and stable behavior when the system is subjected to a initial condition, an impulse input and to disturbances.

VARIABLE STRUCTURE THEORY

In order to design the suggested fuzzy controller to be described in this study, a mathematical background of a VSC-SM is firstly presented here.

The VSC-SM is a combination of subsystems together with a suitable switching logic. The control actions are discontinuous function of system states. In VSC-SM, the design algorithm includes choosing the desired sliding functions which are formed by a choice of their parameters as will be explained. Then a discontinuous control is found which assures the existence of sliding modes at each point of the sliding plane s(x) = 0. In the final stage, the control should drive the system states to the sliding plane. The robustness of VSC stems from the property that the behavior of the controlled system in the sliding mode only depends on the parameters of the sliding mode, not on the system parameters or any disturbances and fluctuations.

Let us design a VSC for nth order system represented by:

$$\dot{x}(t) = Ax + Bu \quad (1)$$

The structure of the VCS is determined by the sign of the vector valued function s (x), which is defined to be the switching function. A switching function is generally assumed to be r dimensional and linear, i.e.,

$$s(x)= Cx \quad (2)$$

where,

$$s(x) = [s_1(x)\ s_2(x)\ldots\ s_r(X)]^T$$

and

$$C = \left[c_1^T\ c_2^T \ldots c_r^T \right]^T$$

i.e.,

$$s_i(x) = c_i^T x \quad (3)$$

Where,

c(x)	=	Is an arbitrary.
(r × n)	=	Matrix chosen such that.
s(x) = 0	=	Defines a stable dynamic system of reduced order.

The next step in the design of the VCS includes choosing the structure of the control of satisfy a reaching condition. There exist various structure of control algorithms which guarantee the existence of sliding modes. Sometimes, it is convenient to preassign the structure of the VSC and then determine the value of the controller gain. The design of VSC can be proceed with the structure of the control u (x) free or preassigned. In either case, the objective is to satisfy the reaching condition. In the free structure approach, the control structure can be solved by constraining the switching function to any one of various reaching conditions mentioned in Huang *et al.* (1993). Among them are Emelyanov (1967) and Utkin (1978):

$$\dot{s}_i(x) < 0 \text{ when } s_i(x) > 0 \tag{4}$$

$$\dot{s}_i(x) > 0 \text{ when } s_i(x) < 0 \tag{5}$$

Or equivalently

$$\dot{s}_i(x)\, \dot{s}_i(x) < 0\ \forall_i = \{1,...,r\} \tag{6}$$

In some cases, it is convenient to preassgin the structure of VSC and then determine the values of the controller gains such that the desired reaching law is satisfied. In this study, a feedback control with switched gains is used. Figure 1 shows the VCS.

$$u(x) = -\psi(x)\, x \tag{7}$$

Where $\psi(x) = [\psi_{ij}(x)]$ is an $r \times n$ matrix

$$\psi_{ij}(x) = \begin{cases} \alpha_{ij} & \text{when } s_i(x)x_i > 0 \\ \beta_{ij} & \text{when } s_i(x)x_i < 0 \end{cases} \tag{8}$$

with α_{ij} and β_{ij} as constants, $\forall I = \{1,..., r\}$, $\forall j = \{1,..., n\}$ to satisfy the desired reaching condition.

PROPOSED CONTROLLER (FLC-VSC)

There exist variousTechniques to design the FLC-VSC. In most of the researshs carried out in this field (Hwang and Tomizuka, 1994; Ishigame *et al.*, 1993; Jacob and Munighan, 1997; Lo and Kuo, 1998; Lu and Chen, 1994; Wu and Liu, 1996a, b), the fuzzy system is described as follows:

$$\dot{x} = A(x)\, x + B(x)u \tag{9}$$

This means that the non linear fuzzy system is linearized with respect to the origin in each IF-THEN rule, which means that the consequent part of each rule is a linear function with zero independent term. This will in turn reduce the accuracy of approximating non linear systems. In this study, a design of FLC-VSC is presented based on T-S fuzzy model (Takagi and Sugeno, 1985)

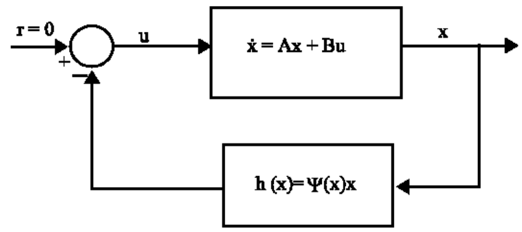

Fig. 1: Variable structure control scheme

taking into account the effect of the independent term in both the fuzzy system and controller.

The main feature of the proposed algorithm is that the both the fuzzy system and the fuzzy and controller are represented by the affine T-S fuzzy model (Al-Hadithi, 2002; Al-Hadithi *et al.*, 2002).

$$\dot{x} = a_0(x) + A(x)x + b(x)u \tag{10}$$

Fuzzy logic controller based on variable structure controller (FLC-VCS): Let us consider the problem of designing a fuzzy controller based on variable structure with VSC theory. The proposed algorithm is modified such that the reaching conditions depend upon the states of the controlled system. As regards to the independent term, an additional necessary condition has been added to treat this term. It should be mentioned that the resultant feedback controlled system is an approximation of the non-linear original system.

Both the fuzzy system and the fuzzy controller are represented by the affine T-S model. Let the $(i_1...i_n)^{th}$ rule of the T-S model be represented as:

$S^{(i1..., in)}$: If x is M_1^{i1} and $\dot{x}$ is M_2^{i2} and $x^{(n-1)}$ is M_n^{in} then

$$\dot{x} = a_0^{(i_1...,i_n)} + A^{(i_1...,i_n)}x + b^{(i_1...,i_n)}u \tag{11}$$

where

M_1^{i1} (i_1= 1,2,..., r_1) are fuzzy sets for x

M_2^{i2} (i_2= 1,2,..., r_2) are fuzzy sets for $\dot{x}$,

M_n^{in} (i_n= 1,2,..., r_n) are fuzzy sets for $x^{(1-n)}$.

Therefore, the complete fuzzy system has $r_1 \times r_2 \times ... r_n$ rules. The membership of the fuzzy system is shown in Fig. 2. The state vector is:

$$X^T = [x\ \dot{x} ...\ x^{n-1}]$$

n: Dimension.

u: Is a scalar input.

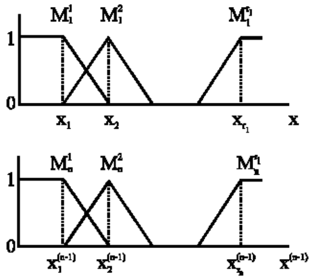

Fig. 2: Membership function of the fuzzy system

The vectors and matrices in these rules are described in a canonical controllable form:

$$a_0^{(i_1\ldots i_n)} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a_0^{(i_1\ldots i_n)} \end{bmatrix}$$

$$A^{(i_1\ldots i_n)} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ a_0^{(i_1\ldots i_n)} & a_2^{(i_1\ldots i_n)} & \cdots & a_{n-1}^{(i_1\ldots i_n)} & a_n^{(i_1\ldots i_n)} \end{bmatrix}$$

and

$$b^{(i_1\ldots i_n)} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_n^{(i_1\ldots i_n)} \end{bmatrix}$$

The weight of each rule is:

$$\begin{aligned} w^{i_1\ldots i_n}(x) &= \mu_{M_1^{i1}}(x)\ldots\mu_{M_n^{in}}(x^{(n-1)}) \\ &= \prod_{l=1}^{n} \mu_{M_l^{il}}(x^{(l-1)}) \end{aligned} \tag{12}$$

and therefore, $w^{i1\ldots in}(x) \geq 0$, $\forall x$, $\forall_i,\ldots,i_n$

In Matia and Jimeneze (1996) it has been proved that, if the membership functions are similar to those shown in Fig. 2, then:

$$\sum_{i_1=1}^{r_1}\cdots\sum_{i_n=1}^{r_n} w^{i_1\ldots i_n}(x) = 1, \forall x$$

and the resultant fuzzy system becomes:

$$\dot{x} = a_0(x) + A(x)x + b(x)u \tag{13}$$

where,

$$a_0(x) = \sum_{i_1=1}^{r_1}\cdots\sum_{i_n=1}^{r_n} w^{i_1\ldots i_n}(x)\, a_0^{i_1\ldots i_n}$$

$$A(x) = \sum_{r_n}^{r_1}\cdots\sum_{i_n=1}^{r_n} w^{i_1\ldots i_n}(x)\, A^{i_1\ldots i_n}$$

and

$$b(x) = \sum_{i_1=1}^{r_1}\cdots\sum_{i_n=1}^{r_n} w^{i_1\ldots i_n}(x)\, b^{i_1\ldots i_n}$$

The same membership function described above for the system are used for the FLC-VSC controller and therefore, the same number of rules. The $(j_1\ldots j_n)^{th}$ fuzzy controller rule can be written as follows assuming that it is a feedback control:

$CV^{(i_1\ldots,i_n)}$: if x is $M_1^{i_1}$ and $\dot{x}$ is $M_2^{i_2}$ and $x^{(n-1)}$ is $M_n^{i_n}$ then

$$u = r - [k_0^{(i_1\ldots i_n)} + K_u^{(i_1\ldots i_n)}x] \tag{14}$$

where, the references input r and $k_0^{(i_1\ldots i_n)}$ are scalar and $K_u^{(i_1\ldots i_n)}$ is a $1\times n$ matrix:

Substituting Eq. (14) in the fuzzy system model, a fuzzy model of the feedback system is obtained:

$SV^{(i_1\ldots,i_n)}$: if x is $M_1^{i_1}$ and $\dot{x}$ is $M_2^{i_2}$ and $x^{(n-1)}$ is $M_n^{i_n}$

then

$$b^{(i_1\ldots i_n)}\left(r - [k_0^{(i_1\ldots i_n)} + K_u^{(i_1\ldots i_n)}x]\right) \tag{15}$$

Let us suppose that we choose $k_0^{(i_1\ldots i_n)}$ such that:

$$a_0^{(i_1\ldots i_n)} - b^{(i_1\ldots i_n)}\, k_0^{(i_1\ldots i_n)} = 0 \tag{16}$$

then the feedback system (15) becomes:

$SC^{(i_1\ldots,i_n)}$: If x is $M_1^{i_1}$, $\dot{x}$ is $M_2^{i_2}$ and $x^{(n-1)}$ is $M_n^{i_n}$

then

$$\dot{x} = (A^{(i_1\ldots i_n)} - b^{(i_1\ldots i_n)}\, K_u^{(i_1\ldots i_n)}\, x + b^{(i_1\ldots i_n)} r \tag{17}$$

Let us suppose that r = 0 to calculate the controller matrix $K_v^{(i_1\ldots i_n)}$.

In accordance with the VSC methodology, we firstly define a sliding plane as follows:

$$s(x) = C(x) = 0 \qquad (18)$$

Where,

C = An arbitrary.

$1 \times n$ = Matrix chosen such that.

$s(x) = 0$ = Defines a stable dynamic system of reduces order.

The coefficients of the controller are chosen to satisfy the reaching condition (Barbashin, n.d; Emelyanov, 1967; Utkin, 1978) under which the state will move towards and reach a sliding plane.

$$\dot{s}(x) = \begin{cases} < 0 \text{ when } s(x) > 0 \\ > 0 \text{ when } s(x) < 0 \end{cases} \qquad (19)$$

Differentiating the equation of the switching function (18), we get:

$$\dot{s}(x) = C\dot{x}$$

which can be modelled by the fuzzy system:

$DS^{(i_1..., i_n)}$: If x is $M_1^{i_1}$, $\dot{x}$ is $M_2^{i_2}$ and $x^{(n-1)}$ is $M_n^{i_n}$
then

$$\dot{s} = \dot{s}^{(i_1 \dots i_n)} \qquad (20)$$

where,

$$s^{(i_1 \dots i_n)} = (CA^{(i_1 \dots i_n)} - Cb^{(i_1 \dots i_n)} K_u^{(i_1 \dots i_n)})x$$

In order to verify the reaching conditions, the coefficients of the feedback matrix $K_u^{(i_1 \dots i_n)}$ are chosen as follows :

$$k_{u1j}^{(i_1 \dots i_n)} = \begin{cases} \alpha_{1j}^{(i_1 \dots i_n)} \text{ when } s(x)x_j > 0 \\ \beta_{1j}^{(i_1 \dots i_n)} \text{ when } s(x)x_j < 0 \end{cases} \qquad (21)$$

where, $\alpha_{1j}^{(i_1 \dots i_n)}$ and $\beta_{1j}^{(i_1 \dots i_n)}$ are constants.

such that:

$$\dot{s}^{(i_1 \dots i_n)} = \begin{cases} < 0 \text{ when } s(x) > 0 \\ > 0 \text{ when } s(x) < 0 \end{cases}$$

Chattering reduction: As mentioned before, the main drawback of sliding mode control is the chattering problem resulted from the switching from one value to another. In this study, we present the idea of the equivalent control which yields the ideal sliding mode define as the one found by recognizing that $\dot{s}$ (x) = 0 is a necessary and sufficient condition for the state trajectory to stay on the switching surface s (x) = 0.

In the order to overcome the chattering problem, a boundary layer is introduced around the switching surface. The resultant controller is a combination of the discontinuous one used to create the sliding mode (analyzed in the previous section) outside the layer and applying the equivalent control method inside this layer.

Therefor, it is suggested in this research to switch from the sliding model control explained before to the equivalent control according to the position of the state trajectory. The necessary and sufficient condition $\dot{s}$ (x) = 0 required for the equivalent control to exist can be modelled as follow:

$DC^{(i1..., in)}$: if x is $M_1^{i_1}$ and $\dot{x}$ is $M_2^{i_2}$ and $x^{(n-1)}$ is $M_n^{i_n}$

then

$$C\left(a_0^{(i_1 \dots i_n)} + A^{(i_1 \dots i_n)}x + b^{(i_1 \dots i_n)} u\right) = 0 \qquad (22)$$

In order to satisfy this condition, it is suggested to use a fuzzy controller assuming that it is a feedback one similar to the discontinuous controller develop in the previous section:

$CC^{(i1..., in)}$: if x is $M_1^{i_1}$ and $\dot{x}$ is $M_2^{i_2}$ and $x^{(n-1)}$ is $M_n^{i_n}$

$$u = r - [k_0^{(i_1 \dots i_n)} + K_c^{(j_1 \dots j_n)} x] \qquad (23)$$

where, the constant $k_0^{(i1...in)}$ is chosen to be the same as the value calculated in the previous section, which means that it satisfies the following condition:

$$a_0^{(i1...in)} - b^{(i1...in)} k_0^{(i1...in)} = 0 \qquad (24)$$

assuming that r = 0 and substituting (23) in (24) we obtain:

$DC^{(i_1..., i_n)}$: If x is $M_1^{i_1}$ and $\dot{x}$ is $M_2^{i_2}$ and $x^{(n-1)}$ is $M_n^{i_n}$

$$C = (A^{(i_1 \dots i_n)} x - b^{(i_1 \dots i_n)} K_c^{(i_1 \dots i_n)} x) = 0 \qquad (25)$$

which gives:

$$K_c^{(i_1 \dots i_n)} (Cb^{(i_1 \dots i_n)})^{-1} CA^{(i_1 \dots i_n)}$$

In this study, fuzzy inference is applied to switch from the discontinuous controller to the equivalent one by regarding the distance from the switching hyperplane as a variable of the premise of the control laws. For this reason fuzzy sets S_0 y S_Φ are defined which are shown in

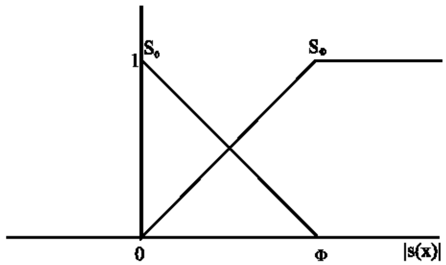

Fig. 3: Fuzzy sets of S_0 and S_Φ

Fig. 3, where ϕ represents the thickness of the layer. Finally the proposed controller becomes:

$CP^{(i_1,\ldots,i_n,\Phi)}$: if x is $M_1^{i_1}$, $\dot{x}$ is $M_2^{i_2}$ and $x^{(n-1)}$ is $M_n^{i_n}$ and $|s(x)|$ is S_Φ

then

$$u = r - [k_0^{(i_1 \ldots i_n)} + K_u^{(j_1 \ldots j_n)} x] \tag{26}$$

$CP^{(i_1,\ldots,i_n,0)}$: if x is $M_1^{i_1}$, $\dot{x}$ is $M_2^{i_2}$ and $x^{(n-1)}$ is $M_n^{i_n}$ and $|s(x)|$ is S_0

then,

$$u = r - [k_0^{(i_1 \ldots i_n)} + K_c^{(j_1 \ldots j_n)} x] \tag{27}$$

where, the first set of rules represents the VSC outside the boundary layer, while the second one corresponds to the equivalent control inside the layer.

ILLUSTRATIVE EXAMPLES

In this study, 2 algorithms for the design of VSC-FLC are illustrated by 2 examples. In the first one, a combination of the fuzzy control and the conventional variable structure control are used to control an inverted pendulum, without solving the chattering problem. The second one implements the proposed algorithm that solves the chattering problem with a completely fuzzy controller.

Example 1: inverted pendulum (case a): To examine the robustness of the proposed FLC-VSC approach, consider the problem of stabilizing and balancing of swing up of the an inverted pendulum (Fig. 4). The control of this system is a widely used performance measure of a controller, since this system is unstable and highly nonlinear. The objective is to maintain the inverted pendulum upright with θ despite small disturbances due to wind or system noises. The inverted pendulum can be represented as follows:

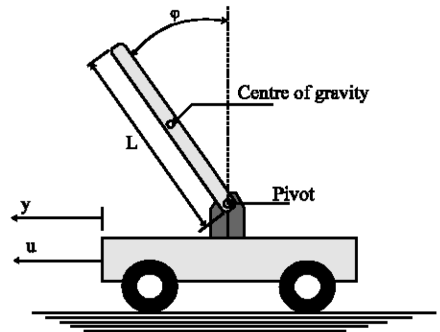

Fig. 4: inverted pendulum system

$$\ddot{\theta} = \frac{g\,\mathrm{sen}\theta - \cos\theta\left(\dfrac{u + ml\dot{\theta}^2 \mathrm{sen}\theta}{M+m}\right)}{1\left(\dfrac{4}{3} - \dfrac{m\cos^2\theta}{M+m}\right)} \tag{28}$$

Where,

θ = Denotes the angular position (in radians) deviated from the equilibrium position (vertical axis) of the pendulum.
$\dot{x}$ = Is angular velocity.
g = (gravity acceleration) = 9.8 m/sec^2
m = (mass) of the cart = 1 kg.
m = (mass) of the pole = 0.1 kg.
1 = Is the distances from the center of the mass m) of the pole to the cart = 0.5 m.

Assuming that $x_1 = x = \theta$ and

$$x_2 = \dot{x} = \dot{\theta}$$

then (28) can be rewritten in state space from as follow:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{g\,\mathrm{sen}(x_1) - \cos(x_1)\left(\dfrac{u + ml(x_2^2 \mathrm{sen}(x_1))}{M+m}\right)}{1\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{M+m}\right)} \tag{29}$$

The pendulum is linearized with respect to its angular position θ, its derivative and the proposed FLC-VSC is applied to control both of them. The advantage is to have information of all sysetm states and control them as well.

Firstly, The model of the inverted pendulum is linearized in 3 operation points for both the angle and its derivative. Them universe of discourse of the angle is $(-\pi/4, \pi/4)$ rad and the one of the angular velocity is (-5, 5) rad/seg. Both membership function for the angle x and its derivative $\dot{x}$ are shown in Fig. 5 and 6, respectively. Supposing that the desired switching function is:

$$s(x) = c_1 x + c_2 \dot{x} = 0.5x + \dot{x}$$
$$\dot{s}(x) = 0.5\dot{x} + \ddot{x} \tag{30}$$

The inverted pendulum model can be represented by T-S modal as follow:

R^{11}: If (x is M_1^1) and ($\dot{x}$ is M_2^1) then
$\ddot{x} = -11.92 + 10.06x - 0.35\dot{x} - u$

R^{12}: If (x is M_1^1) and ($\dot{x}$ is M_2^2) then
$\ddot{x} = -10.76 + 10x - u$

R^{13}: If (x is M_1^1) and ($\dot{x}$ is M_2^3) then
$\ddot{x} = -11.36 + 10.06x + 0.35\dot{x} - \mu$

R^{21}: If (x is M_1^2) and ($\dot{x}$ is M_2^1) then
$\ddot{x} = 13.95x - 1.46u$

R^{22}: If (x is M_1^2) and ($\dot{x}$ is M_2^2) then
$\ddot{x} = 15.78x - 1.46u$

R^{23}: If (x is M_1^2) and ($\dot{x}$ is M_2^3) then
$\ddot{x} = 13.95x - 1.46u$

R^{31}: If (x is M_1^3) and ($\dot{x}$ is M_2^1) then
$\ddot{x} = 11.37 + 10.06x - 0.35\dot{x} - u$

R^{32}: If (x is M_1^3) and ($\dot{x}$ is M_2^2) then
$\ddot{x} = 10.76 + 10x - u$

R^{33}: If (x is M_1^3) and ($\dot{x}$ is M_2^3) then
$\ddot{x} = +11.92 + 10.06x + 0.35\dot{x} - u$

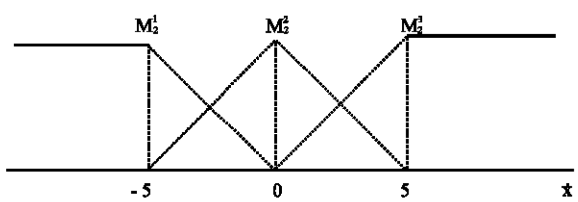

Fig. 5: Membership function for the angle x of the inverted pendulum for the example 4.1

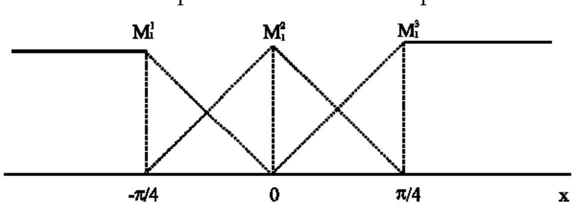

Fig. 6: Membership function for the angular velocity of the inverted pendulum for the example 4.1

Applying the combination of the fuzzy control and the conventional variable structure control, used by several authors, to control an inverted pendulum, the switching parameters:

$$\alpha_{ij}^{j_1 \dots j_n}$$

and

$$\beta_{ij}^{j_1 \dots j_n}$$

for the subsystem R^{22}, for instance, are:

$CV^{(22)}$: If (x is M_1^2) and ($\dot{x}$ is M_2^2) than

$$\begin{aligned}
&\text{If } x < 0\ \dot{x} > 0 \Rightarrow s(x) < 0 \Rightarrow \dot{s}(x) > 0 \\
&\alpha_{11}^{22} \geq 10.81, \alpha_{11}^{22} \geq 1.37 \\
&u = 24.51x - 15.07\dot{x} \\
&\text{If } x < 0\ \dot{x} > 0 \Rightarrow s(x) < 0 \Rightarrow \dot{s}(x) > 0 \\
&\alpha_{11}^{22} \geq 10.81, \alpha_{12}^{22} \geq 1.37 \\
&u = 24.51x - 12.33\dot{x} \\
&\text{If } x < 0\ \dot{x} > 0 \Rightarrow s(x) > 0 \Rightarrow \dot{s}(x) < 0 \\
&\beta_{11}^{22} \leq 10.81, \beta_{12}^{22} \geq 1.37 \\
&u = 2.89x - 15.07\dot{x} \\
&\text{If } x > 0\ \dot{x} < 0 \Rightarrow s(x) < 0 \Rightarrow \dot{s}(x) > 0 \\
&\beta_{11}^{22} \leq 10.81, \beta_{12}^{22} \geq 1.37 \\
&u = -2.89x - 15.07\dot{x} \\
&\text{If } x > 0\ \dot{x} < 0 \Rightarrow s(x) > 0 \Rightarrow \dot{s}(x) < 0 \\
&\alpha_{11}^{22} \geq 10.81, \alpha_{12}^{22} \leq 1.37 \\
&u = 24.51x - 12.33\dot{x} \\
&\text{If } x > 0\ \dot{x} > 0 \Rightarrow s(x) > 0 \Rightarrow \dot{s}(x) < 0 \\
&\alpha_{11}^{22} \geq 10.81, \alpha_{12}^{22} \geq 1.37 \\
&u = 24.51x - 15.07\dot{x}
\end{aligned} \tag{31}$$

Figure 7 shown the transient response of the inverted pendulum controlled by the proposed FLC-VSC controller.

The results obtained shows that the system is stabilized by applying the proposed FLC-VSC.

Although, the VSC is characterized by its ability to deal with non-linearities, uncertainties of control systems, invariance to prameters fluctuations and disturbances and its robustness, its main drawback is the switching between structures (chattering). Figure 8 shown this fenomenon.

Example 2: Inverted pendulum (Case b): In this example, a new algorithm is applied to deal with the previous example. This algorithm has 2 main features. Firstly, it is aimed at eliminating the chattering effect by applying the equivalent control method. Secondly, the switching function is added as an additional fuzzy variable in the premise part of fuzzy rules to implement a completely fuzzy controller. The membership functions of the switching function is shown in Fig. 9. The inverted pendulum model together with the VSC can be represented by T-S model as follows:

R^{11}: If (x is M_1^1) and ($\dot{x}$ is M_2^1) then
$\ddot{x} = 11.92+10.06x-0.35\dot{x} - u$

R^{12}: If (x is M_1^1) and ($\dot{x}$ is M_2^2) then
$\ddot{x} = 10.76+10x - u$

R^{13}: If (x is M_1^1) and ($\dot{x}$ is M_2^3) then
$\ddot{x} = 10.76+10x - u$

R^{14}: If (x is M_1^1) and ($\dot{x}$ is M_2^4) then
$\ddot{x} = 11.36+10.06x-0.35\dot{x} - u$

R^{21}: If (x is M_1^2) and ($\dot{x}$ is M_2^1) then
$\ddot{x} = 13.95-1.46u$

R^{22}: If (x is M_1^2) and ($\dot{x}$ is M_2^2) then
$\ddot{x} = 15.78-1.46u$

R^{23}: If (x is M_1^2) and ($\dot{x}$ is M_2^3) then
$\ddot{x} = 15.78-1.46u$

R^{24}: If (x is M_1^2) and ($\dot{x}$ is M_2^4) then
$\ddot{x} = 13.95-1.46u$

R^{31}: If (x is M_1^3) and ($\dot{x}$ is M_2^1) then
$\ddot{x} = 13.95-1.46u$

R^{32}: If (x is M_1^3) and ($\dot{x}$ is M_2^2) then
$\ddot{x} = 15.78-1.46u$

R^{33}: If (x is M_1^3) and ($\dot{x}$ is M_2^3) then
$\ddot{x} = 15.78-1.46u$

R^{34}: If (x is M_1^3) and ($\dot{x}$ is M_2^4) then
$\ddot{x} = 13.95-1.46u$

R^{41}: If (x is M_1^4) and ($\dot{x}$ is M_2^1) then
$\ddot{x} = 11.37+10.06x-0.35\dot{x} - u$

R^{42}: If (x is M_1^4) and ($\dot{x}$ is M_2^2) then
$\ddot{x} = 10.76+10x - u$

R^{43}: If (x is M_1^4) and ($\dot{x}$ is M_2^3) then
$\ddot{x} = 10.76+10x - u$

R^{44}: If (x is M_1^4) and ($\dot{x}$ is M_2^4) then
$\ddot{x} = 11.92+10.06x+0.35\dot{x} - u$

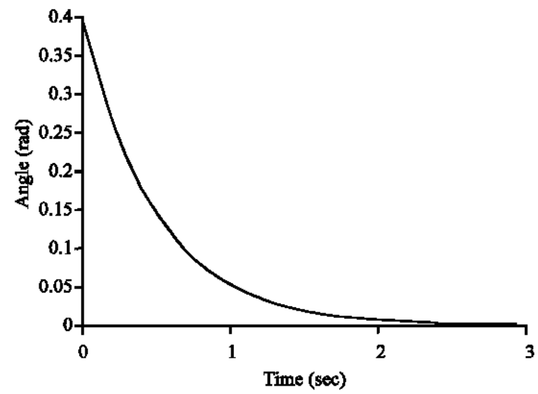

Fig. 7: Transient response of the inverted pendulum

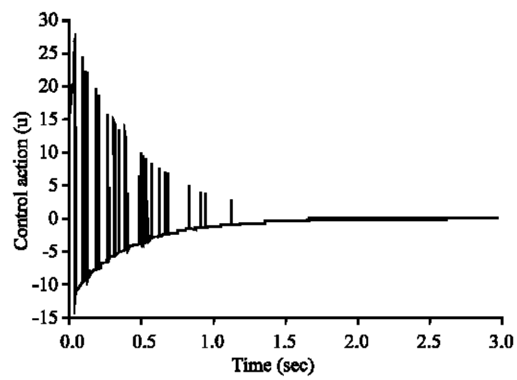

Fig. 8: Chattering effect on the control action s (x)

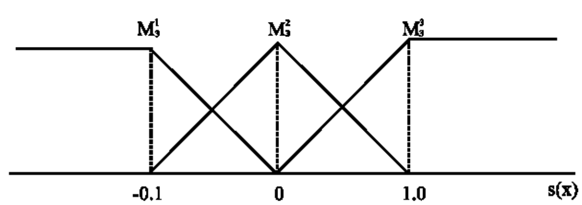

Fig. 9: Membership function of the switching function

Applying the designed FLC-VSC explained in this study, the control rules, equivalent to one explained in example 1 are:

$$\begin{gathered} CV^{(221)}: \text{If } (x \text{ is } M_1^2) \text{ and } (\dot{x} \text{ is } M_2^2) \\ \text{and } (s(x) \text{ is } M_3^1) \text{ then} \\ x < 0\ \dot{x} < 0 \Rightarrow s(x) < 0 \Rightarrow \dot{s}(x) > 0 \\ \alpha_{11}^{221} \geq 10.81,\ \alpha_{12}^{221} \geq 1.37 \\ u = 24.51x + 15.07\dot{x} \end{gathered} \tag{32}$$

$CC^{(222)}$: If (x is M_1^2) and ($\dot{x}$ is M_2^2)
and (s(x) is M_3^2) then
$\dot{s}(x) = 0$
$u = 10.81x + 1.37\dot{x}$

$CV^{(231)}$: If(x is M_1^2) and ($\dot{x}$ is M_2^3)
and (s(x) is M_3^1) then
$x < 0\ \dot{x} > 0 \Rightarrow s(x) < 0 \Rightarrow \dot{s}(x) > 0$
$\alpha_{11}^{231} \geq 10.81,\ \alpha_{12}^{231} \geq 1.37$
$u = 24.51x + 12.33\dot{x}$

$CC^{(232)}$: If (x is M_1^2) and ($\dot{x}$ is M_2^3)
and (s(x) is M_3^2) then
$\dot{s}(x) = 0$
$u = 10.81x + 1.37\dot{x}$

$CV^{(233)}$: If (x is M_1^2) and ($\dot{x}$ is M_2^3)
and (s(x) is M_3^3) then
$x < 0\ \dot{x} > 0 \Rightarrow s(x) > 0 \Rightarrow \dot{s}(x) < 0$
$\beta_{11}^{233} \leq 10.81,\ \beta_{12}^{233} \geq 1.37$
$u = -2.89x + 15.07\dot{x}$

$CV^{(321)}$: If(x is M_1^3) and ($\dot{x}$ is M_2^2)
and (s(x) is M_3^1) then
$x > 0\ \dot{x} < 0 \Rightarrow s(x) < 0 \Rightarrow \dot{s}(x) > 0$
$\beta_{11}^{321} \leq 10.81,\ \beta_{12}^{321} \geq 1.37$
$u = -2.89x + 15.07\dot{x}$

$CC^{(322)}$: If (x is M_1^3) and ($\dot{x}$ is M_2^2)
and (s(x) is M_3^2) then
$\dot{s}(x) = 0$
$u = 10.81x + 1.37\dot{x}$

$CV^{(323)}$: If(x is M_1^3) and ($\dot{x}$ is M_2^2)
and (s(x) is M_3^3) then
$x > 0\ \dot{x} < 0 \Rightarrow s(x) > 0 \Rightarrow \dot{s}(x) < 0$
$\alpha_{11}^{323} \geq 10.81,\ \alpha_{12}^{323} \leq 1.37$
$u = 24.51x + 12.33\dot{x}$

$CC^{(332)}$: If (x is M_1^3) and ($\dot{x}$ is M_2^3)
and (s(x) is M_3^2) then
$\dot{s}(x) = 0$
$u = 10.81x + 1.37\dot{x}$

$CV^{(333)}$: If(x is M_1^3) and ($\dot{x}$ is M_2^3)
and (s(x) is M_3^3) then
$x > 0\ \dot{x} > 0 \Rightarrow s(x) > 0 \Rightarrow \dot{s}(x) < 0$
$\alpha_{11}^{333} \geq 10.81,\ \alpha_{12}^{333} \geq 1.37$
$u = 24.51x + 15.07\dot{x}$

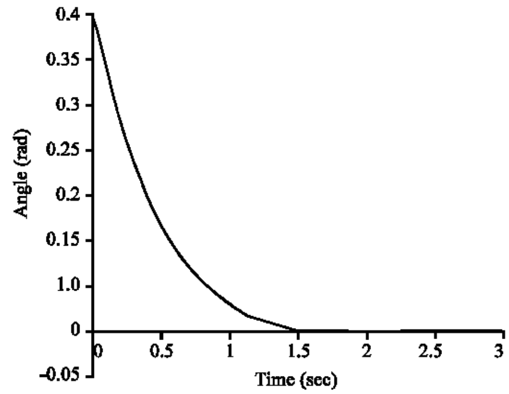

Fig. 10: Transient response of the inverted pendulum

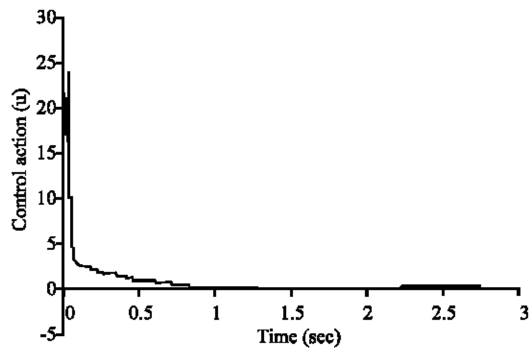

Fig. 11: Control action without chattering

Figure 10 shows the transient response of the inverted pendulum controlled by the proposed FLC-VSC controller subjected to an initial condition.

The results obtained show that the system is stabilized by applying the proposed FLC-VSC.

As it was explained that the equivalent method was applied to eliminate the effect of chattering. Figure 11 shows control action without chattering.

CONCLUSION

Due to the similarity between FLC and VSC-SM, a design of a non-linear controller based on both techniques have been presented. The fuzzy system and controller have been represented by Affine Takagi-Sugeno (T-S) model, taking into consideration the contribution of the independent term. The design methodology is illustrated by 2 example of balancing an inverted pendulum problem.

The aim is to control a highly unstable nonlinear system consisted of an inverted pendulum mounted on a cart. The inverted pendulum system has been linearized in 9 operating points in the first example. In the second example, the premise part of each of the rules is a function of the system state and its derivative, which implies the design of the switching parameters for both states. The results obtained in both examples show that the system is stabilized by applying the proposed FLC-VSC. Moreover, the results show, a smooth response without oscillation with a reduced chattering. The controlled system has been stabilized applying a new algorithm of an FLC-VSC controller. The results obtained showed a robust smooth response. The problem of chattering has been reduced applying a procedure of adding a boundary layer around the switching surface where inside this layer, the equivalent control method has been applied which reduces significantly the chattering effect.

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