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A New Approach to Non-Fragile H_∞ Fuzzy Controller for Uncertain Fuzzy Dynamical Systems with Multiple Time-Scales

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Abstract: This study examines the problem of designing a new approach to non-fragile H_∞ controllers for a class of nonlinear uncertain dynamical systems with multiple time-scales described by a Takagi-Sugeno (TS) fuzzy model. Based on a Linear Matrix Inequality (LMI) approach, we develop a non-fragile H_∞ controller which guarantees the L_2-gain of the mapping from the exogenous input noise to the regulated output to be less than some prescribed value for this class of uncertain fuzzy dynamical systems with multiple time-scales. A numerical example is provided to illustrate the design developed in this study.

Key words: Fuzzy control, Linear Matrix Inequality (LMI), non-fragile H_∞ control, multiple time-scale systems, exogenous, dynamical system

INTRODUCTION

The problem of control design for dynamical systems with multiple time-scale has been intensively studied by a number of researchers for the past three decades (Suzuki and Miura, 1976; Kokotovic *et al*., 1976, 1986; O'Reilly, 1979; Pan and Basar, 1993; 1994; Fridman, 2001; Shi and Dragan, 1999; Shao and Sawan, 1993; Su *et al*., 1992; Su, 1999; Grodt and Gajic, 1998). This is due not only to theoretical interest but also to the relevance of this topic in control engineering applications. Singularly perturbed systems are dynamical systems with multiple time-scales. Singularly, perturbed systems often occur naturally due to the presence of small parasitic parameter, typically small time constants, masses, etc. Indeed multiple time-scales phenomena are almost unavoidable in real-life systems. Examples of such systems abound and include convection-diffusion systems, diffusion-drift motion systems, power systems, scheduling systems, economic models, telecommunication systems and bifurcations.

The main purpose of the singular perturbation approach to analysis and design is the alleviation of high dimensionality and ill-conditioning resulting from the interaction of slow and fast dynamics modes. The separation of states into slow and fast ones is a nontrivial modelling task demanding insight and ingenuity on the part of the analyst. In state space, such systems are commonly modelled using the mathematical framework of singular perturbations with a small parameter say determining the degree of separation between the slow and fast modes of the system.

In the last few years, many researchers have studied the H_∞ control design for a general class of linear singularly perturbed systems due to a great practical impor-tance. For examples, Pan and Basar have investigated the H_∞-optimal control of singularly perturbed systems under either perfect or imperfect state measurements (Pan and Basar, 1993, 1994). In the mean time, Shi and Dragan (1999) have considered the asymptotic H_∞ control of singularly perturbed systems with parametric uncertainties.

Although, many researchers have studied linear singularly perturbed systems for many years, the H_∞ control design of nonlinear singularly perturbed systems remains as an open research area. This is because in general, nonlinear singularly perturbed systems can not be separated into slow and fast subsystems.

Over the past two decades, there has been rapidly growing interest in application of fuzzy logic to control problem. Researches have been focused on its application to industrial processes and a number of successful results have been reported in the literature. In spite of these successes, there are many basic issues remain to be addressed. One of them is how to achieve a systematic design that guarantees closed loop stability and performance. Recently, a great amount of effort has been devoted to describing a nonlinear system using a Takagi-Sugeno fuzzy model (Zadeh, 1965, 1973; Mamdani and Assilian, 1975; Wang, 1997; Yoneyama *et al*., 2000; Tanaka and Sugeno, 1992; Tanaka, 1995; Tanaka *et al*., 1996; Wang *et al*., 1996; Cao *et al*., 1996; Assawinchaichote and Nguang, 2002; Takagi and

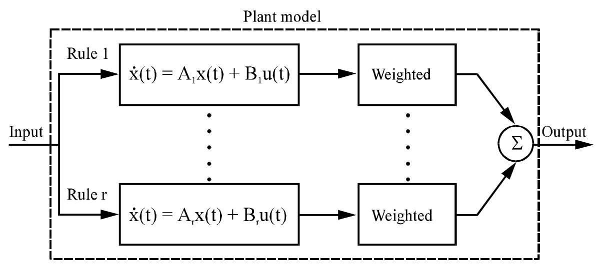

Fig. 1: The TS type fuzzy system

Sugeno, 1985; Chen *et al*., 1995, 2000; Ma *et al*., 1998; Nguang and Shi, 2000, 2001a, b, 2003; Teixeira and Zak, 1999; Zak, 1999; Wang, 1997, 1995; Nguang and Assawinchaichote, 2003; Assawinchaichote and Nguang, 2004a, b; Han and Feng, 1998; Han *et al*., 2000). The Takagi-Sugeno (TS) fuzzy model represents a nonlinear system by a family of local linear models which smoothly blended together through fuzzy membership functions. Figure 1 shows the structure diagram of TS fuzzy model. Unlike conventional modelling techniques which uses a single model to describe the global behavior of a nonlinear system, fuzzy modelling is essentially a multi-model approach in which simple sub-models (typically linear models) are fuzzily combined to described the global behavior of a nonlinear system. Based on this fuzzy model, a number of systematic model-based fuzzy control design methodologies have been developed.

The aim of this study is to design a non-fragile H_∞ fuzzy controller for a class of uncertain nonlinear dynamical systems with multuple time-scales. First, we approximate this class of uncertain nonlinear dynamical systems with multuple time-scales by a Takagi-Sugeno fuzzy model. Then based on an LMI approach, we develop the non-fragile H_∞ controller that guarantees the L_2-gain of the mapping from the exogenous input noise to the regulated output to be less than or equal to a prescribed value for this class of fuzzy dynamical systems with multuple time-scales. In order to alleviate the ill-conditioned linear matrix inequalities resulting from the interaction of slow and fast dynamic modes, the ill-conditioned LMIs are decomposed into ε-independent and ε-dependent LMIs.

The ε-independent LMIs are not ill conditioned and the ε-dependent LMIs tend to zero when ε approaches to zero. It can be shown that when ε is sufficiently small, the original ill-conditioned LMIs are solvable if and only if the ε-independent LMIs are solvable.

The proposed approach does not involve the separation of states into slow and fast ones and it can be applied not only to standard but also to nonstandard singularly perturbed systems.

SYSTEM DESCRIPTIONS AND DEFINITIONS

In this study, we consider the TS fuzzy system with multiple time-scales to represent a TS fuzzy multiple time-scale system with parametric uncertainties as follows:

$$\begin{aligned}
E_\varepsilon \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\nu(t))\big[[A_i + \Delta A_i]x(t) + \\
&\quad [B_{1_i} + \Delta B_{1_i}]w(t) + \\
&\quad [B_{2_i} + \Delta B_{2_i}]u(t)],\ x(0)=0\big] \\
z(t) &= \sum_{i=1}^{r} \mu_i(\nu(t))\big[[C_{1_i} + \Delta C_{1_i}]x(t) + \\
&\quad [D_{12_i} + \Delta D_{12_i}]u(t)\big] \\
y(t) &= \sum_{i=1}^{r} \mu_i(\nu(t))\big[[C_{2_i} + \Delta C_{2_i}]x(t) + \\
&\quad [D_{21_i} + \Delta D_{21_i}]w(t)\big]
\end{aligned} \tag{1}$$

Where:

$$E_\varepsilon = \begin{bmatrix} I & 0 \\ 0 & \varepsilon I \end{bmatrix}$$

$\nu(t) = [\nu_1(t) \ldots \nu_\vartheta(t)]$ is the premise variable vector that may depend on states in many cases, $\varepsilon>0$ is the singular perturbation parameter, $\mu_i(\nu(t))$ denotes the normalized time-varying fuzzy weighting functions for each rule (i.e.,

$$\mu_i(\nu(t)) \geq 0$$

and;

$$\sum_{i=1}^{r} \mu_i(\nu(t)) = 1$$

where, ϑ is the number of fuzzy sets, $x(t)\in\Re^n$ is the state vector, $u(t)\in\Re^m$ is the input, $w(t)\in\Re^p$ is the disturbance which belongs to $L_2[0, \infty)$, $y(t)\in\Re^l$ is the measurement, $z(t)\in\Re^s$ is the controlled output, the matrices $A_i, B_{1_i}, B_{2_i}, C_{1_i}$, C_{2_i}, D_{12_i} and D_{21_i} are of appropriate dimensions and r is the number of IF-THEN rules. The matrices $\Delta A_i, \Delta B_{1_i}, \Delta B_{2_i}, \Delta C_{1_i}$, $\Delta C_{2_i}, \Delta D_{12_i}$ and ΔD_{21_i} .

Assumption 1:

$$\begin{aligned}
&\Delta A_i = F(x(t),t)H_{1_i} \\
&\Delta B_{1_i} = F(x(t),t)H_{2_i},\ \Delta B_{2_i} = F(x(t),t)H_{3_i} \\
&\Delta C_{1_i} = F(x(t),t)H_{4_i},\ \Delta C_{2_i} = F(x(t),t)H_{5_i} \\
&\Delta D_{12_i} = F(x(t),t)H_{6_i},\ \Delta D_{21_i} = F(x(t),t)H_{7_i}
\end{aligned}$$

where, H_j, j = 1, 2, . . ., 7 are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\|F(x(t), t)\| \leq \rho \tag{2}$$

for any known positive constant ρ. Next, let us recall the following definition.

Definition 1: Suppose γ is a given positive number. A system (Eq. 1) is said to have an L_2-gain $\leq\gamma$ if:

$$\int_0^{T_f} z^T(t)z(t)dt \leq \gamma^2\left[\int_0^{T_f} w^T(t)w(t)dt\right] \quad (3)$$

For all $T_f\geq 0$, $x(0) = 0$ and $w(t)\in L_2[0, T_f]$. For the symmetric block matrices, we use (*) as an ellipsis for terms that are induced by symmetry.

NON-FRAGILE H_∞ STATE-FEEDBACK CONTROL DESIGN

The aim of this study is to design a non-fragile H_∞ fuzzy state-feedback controller of the form:

$$u(t)=\sum_{j=1}^{r}\mu_j K_j x(t) \quad (4)$$

Where, K_j is the controller gain such that the inequality (Eq. 3) holds. The state space form of the fuzzy system model (Eq. 1) with the controller (Eq. 4) is given by:

$$\begin{aligned}E_e\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big[&[(A_i + B_{2_i}K_j)+\\ &(\Delta A_i + \Delta B_{2_i}K_j)]x(t)+\\ &[B_{1_i} + \Delta B_{1_i}]w(t)\Big],\ x(0) = 0\end{aligned} \quad (5)$$

Sufficient conditions for the existence of a non-fragile H_∞ fuzzy state-feedback controller are provided in the following Lemmas. The Lyapunov approach is used to derived these sufficient conditions.

Next, let us introduce the first lemma for the following a state space form of the fuzzy system model with the controller is given by:

$$\begin{aligned}\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big[&[(A_i + B_{2_i}K_j)+\\ &(\Delta A_i + \Delta B_{2_i}K_j)]x(t)+\\ &[B_{1_i} + \Delta B_{1_i}]w(t)\Big],\ x(0) = 0\end{aligned} \quad (6)$$

Lemma 1: Consider the Eq. 6. Given a prescribed H_∞ performance $\gamma>0$ and a positive constant δ if there exist a matrix $P = P^T$ and matrices Y_j, $j = 1, 2, \ldots, r$, satisfying the following linear matrix inequalities:

$$P>0 \quad (7)$$

$$\Omega_{ii} < 0,\ i = 1, 2, \ldots, r \quad (8)$$

$$\Omega_{ij}+\Omega_{ji}<0, \quad i<j\leq r \quad (9)$$

Where:

$$\Omega_{ij} = \begin{pmatrix}\begin{pmatrix}A_iP + PA_i^T +\\ B_{2_i}Y_j + Y_j^TB_{2_i}^T\end{pmatrix} & (*)^T & (*)^T\\ \tilde{B}_{1_i}^T & -\gamma I & (*)T\\ \tilde{C}_{1_i}P + \tilde{D}_{12_i}Y_j & 0 & -\gamma I\end{pmatrix} \quad (10)$$

With;

$$\tilde{B}_{1_i} = \begin{bmatrix}\delta I & I & \delta I & B_{1_i}\end{bmatrix}$$

$$\tilde{C}_{1_i} = \begin{bmatrix}\frac{\gamma\rho}{\delta}H_{1_i}^T & 0 & \sqrt{2}\lambda\rho H_{4_i}^T & \sqrt{2}\lambda C_{1_i}^T\end{bmatrix}^T$$

$$\tilde{D}_{12_i} = \begin{bmatrix}0 & \frac{\gamma\rho}{\delta}H_{3_i}^T & \sqrt{2}\lambda\rho H_{6_i}^T & \sqrt{2}\lambda D_{12_i}^T\end{bmatrix}^T$$

$$\lambda = \left(1+\rho^2\sum_{i=1}^{r}\sum_{j=1}^{r}\left[\| H_{2_i}^TH_{2j} \|\right]\right)^{\frac{1}{2}}$$

then the inequality of Eq. 3 holds. Furthermore, a suitable choice of the fuzzy controller is:

$$u(t)=\sum_{j=1}^{r}\mu_j K_j x(t) \quad (11)$$

Where:

$$K_j = Y_jP^{-1} \quad (12)$$

Proof: Using assumption 1, the closed-loop fuzzy system (Eq. 6) can be expressed as follows:

$$\dot{x}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\left([A_i + B_{2_i}K_j]x(t)+\tilde{B}_{1_i}\tilde{w}(t)\right) \quad (13)$$

Where:

$$\tilde{B}_{1_i} = \begin{bmatrix}\delta I & I & \delta I & B_{1_i}\end{bmatrix}$$

and the disturbance $\tilde{w}(t)$ is:

$$\tilde{w}(t)=\begin{bmatrix}\frac{1}{\delta}F(x(t), t)H_{1_i}x(t)\\ F(x(t), t)H_{2_i}w(t)\\ \frac{1}{\delta}F(x(t), t)H_{3_i}K_jx(t)\\ w(t)\end{bmatrix} \quad (14)$$

Let consider a Lyapunov function:

$$V(x(t)) = \gamma x^T(t)Qx(t)$$

where, $Q = P^{-1}$. Differentiate $V(x(t))$ along the closed-loop system (Eq. 13) yields:

$$\begin{aligned}\dot{V}(x(t)) &= \gamma\dot{x}^T(t)Qx(t)+\gamma x^T(t)Q\dot{x}(t)\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\big(\gamma x^T(t)(A_i+B_{2_i}K_j)^T Qx(t)+\\ &\quad \gamma x^T(t)Q(A_i+B_{2_i}K_j)x(t)+\\ &\quad \gamma\tilde{w}^T(t)\tilde{B}_{1_i}^T Qx(t)+\gamma x^T(t)Q\tilde{B}_{1_i}\tilde{w}(t)\big)\end{aligned} \quad (15)$$

Adding and subtracting:

$$-\tilde{z}^T(t)\tilde{z}(t)+\gamma^2\sum\nolimits_{i=1}^{r}\sum\nolimits_{j=1}^{r}\sum\nolimits_{m=1}^{r}\sum\nolimits_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n[\tilde{w}^T(t)\tilde{w}(t)]$$

to and from Eq. 15, we get:

$$\begin{aligned}\dot{V}(x(t)) &= \gamma\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\\ &\Big([x^T(t)\ \ \tilde{w}^T(t)]\times\\ &\left[\begin{pmatrix}(A_i+B_{2_i}K_j)^T Q+\\ Q(A_i+B_{2_i}K_j)+\\ (\tilde{C}_{1_i}+\tilde{D}_{12_i}K_j)^T(\tilde{C}_{1_m}+\tilde{D}_{12_m}K_n)\\ \tilde{B}_{1_i}^{T'}Q\end{pmatrix}\begin{matrix}(*)^T\\ \\ -\gamma I\end{matrix}\right]\times\\ &\begin{bmatrix}x(t)\\ \tilde{w}(t)\end{bmatrix}\Big)-\tilde{z}^T(t)\tilde{z}(t)+\\ &\gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\left[\tilde{w}^T(t)\tilde{w}(t)\right]\end{aligned} \quad (16)$$

$$\tilde{z}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\left[\tilde{C}_{1_i}+\tilde{D}_{12_i}K_j\right]x(t) \quad (17)$$

with;

$$\tilde{C}_{1_i}=\left[\frac{\gamma\rho}{\delta}H_{1_i}^T\ \ 0\ \ \sqrt{2}\lambda\rho H_{4_i}^T\ \ \sqrt{2}\lambda C_{1_i}^T\right]^T$$

and;

$$\tilde{D}_{12_i}=\left[0\ \ \frac{\gamma\rho}{\delta}H_{3_i}^T\ \ \sqrt{2}\lambda\rho H_{6_i}^T\ \ \sqrt{2}\lambda D_{12_i}^T\right]^T$$

Pre and post multiply Eq. 8-9 by:

$$\begin{pmatrix}Q & 0 & 0\\ 0 & I & 0\\ 0 & 0 & I\end{pmatrix}$$

yields;

$$\begin{pmatrix}\begin{pmatrix}(A_i+B_{2_i}K_i)^T Q+\\ Q(A_i+B_{2_i}K_i)\end{pmatrix} & (*)^T & (*)^T\\ \tilde{B}_{1_i}^T Q & -\gamma I & (*)^T\\ \tilde{C}_{1_i}+D_{12_i}K_i & 0 & -\gamma I\end{pmatrix}<0 \quad (18)$$

i =1, 2, ... , r and:

$$\left\{\begin{pmatrix}\begin{pmatrix}(A_i+B_{2_i}K_j)^T Q+\\ Q(A_i+B_{2_i}K_j)\end{pmatrix} & (*)^T & (*)^T\\ \tilde{B}_{1_i}^T Q & -\gamma I & (*)^T\\ \tilde{C}_{1_i}+\tilde{D}_{12_i}K_j & 0 & -\gamma I\end{pmatrix}+\begin{pmatrix}\begin{pmatrix}(A_j+B_{2_j}K_i)^T Q+\\ Q(A_j+B_{2_j}K_i)\end{pmatrix} & (*)^T & (*)^T\\ \tilde{B}_{1_j}^T Q & -\gamma I & (*)^T\\ \tilde{C}_{1_j}+\tilde{D}_{12_j}K_i & 0 & -\gamma I\end{pmatrix}\right\}<0 \quad (19)$$

i<j≤r, respectively. Applying the Schur complement on Eq. 18-19 and rearranging them, we have:

$$\left\{\begin{pmatrix}\begin{pmatrix}(A_i+B_{2_i}K_i)^T Q+\\ Q(A_i+B_{2_i}K_i)+\\ (\tilde{C}_{1_i}+\tilde{D}_{12_i}K_i)^T(\tilde{C}_{1_i}+\tilde{D}_{12_i}K_i)\\ \tilde{B}_{1_i}^{T^\gamma}Q\end{pmatrix} & \begin{matrix}(*)^T\\ \\ -\gamma I\end{matrix}\end{pmatrix}\right\}<0 \quad (20)$$

i = 1, 2, ... , r and:

$$\left\{\begin{pmatrix}\begin{pmatrix}(A_i+B_{2_i}K_j)^T Q+\\ Q(A_i+B_{2_i}K_j)+\\ (\tilde{C}_{1_i}+\tilde{D}_{12_i}K_j)^T(\tilde{C}_{1_i}+\tilde{D}_{12_i}K_i)\\ \tilde{B}_{1_i}^{T'}Q\end{pmatrix} & \begin{matrix}(*)^T\\ \\ -\gamma I\end{matrix}\end{pmatrix}+\begin{pmatrix}\begin{pmatrix}(A_j+B_{2_j}K_i)^T Q+\\ Q(A_j+B_{2_j}K_i)+\\ (\tilde{C}_{1_j}+\tilde{D}_{12_j}K_i)^T(\tilde{C}_{1_j}+\tilde{D}_{12_j}K_i)^T\\ \tilde{B}_{1_j}^{T'}Q\end{pmatrix} & \begin{matrix}(*)^T\\ \\ -\gamma I\end{matrix}\end{pmatrix}\right\}<0 \quad (21)$$

i<j≤ r, respectively. Using Eq. 20-21 and the fact that:

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n M_{ij}^T N_{mn}\\ &\le\frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\left[M_{ij}^T M_{ij}+N_{ij}N_{ij}^T\right]\end{aligned} \quad (22)$$

it is obvious that we have:

$$\left(\left(\begin{matrix}\dfrac{\begin{matrix}(A_i + B_{2_i}K_j)^T Q + \\ Q(A_i + B_{2_i}K_j) + \\ (\tilde{C}_{1_i} + \tilde{D}_{12_i}K_j)^T(\tilde{C}_{1_i} + \tilde{D}_{12_i}K_j)\end{matrix}}{\tilde{B}_{1_i}^{T'} Q} \end{matrix}\right) \begin{matrix}(*)^T \\ \\ -\gamma I\end{matrix}\right) < 0 \quad (23)$$

where, i, j = 1, 2, ... , r. Since Eq. 23 is <0 and the fact that $\mu_i \geq 0$ and $\sum_{i=1}^{r} \mu_i = 1$ then Eq. 16 becomes:

$$\dot{V}(x(t)) \leq -\tilde{z}^T(t)\tilde{z}(t) + \gamma^2 \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\left[\tilde{w}^T(t)\tilde{w}(t)\right] \quad (24)$$

Integrate both sides of Eq. 24 yields:

$$\int_0^{T_f}\dot{V}(x(t))dt \leq \int_0^{T_f}\left[-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\left[\tilde{w}^T(t)\tilde{w}(t)\right]dt\right.$$

$$V(x(T_f)) - V(x(0)) \leq \int_0^{T_f}\left[-\tilde{z}^T(t)\tilde{z}(t) + \gamma^2\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\left[\tilde{w}^T(t)\tilde{w}(t)\right]dt\right.$$

Using the fact that x(0) = 0 and $V(x(T_f)) \geq 0$ for all $T_f \neq 0$, we get:

$$\int_0^{T_f}\left[-\tilde{z}^T(t)\tilde{z}(t)dt \leq \gamma^2\left[\int_0^{T_f}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{m=1}^{r}\sum_{n=1}^{r}\mu_i\mu_j\mu_m\mu_n\left[\tilde{w}^T(t)\tilde{w}(t)\right]dt\right] \right. \quad (25)$$

Putting $\tilde{z}(t)$ and $\tilde{w}(t)$, respectively given in Eq. 17 and 14 into Eq. 25 and using the fact that:

$$\| F(x(t),t) \| \leq \rho$$

$$\lambda^2 = \left(1 + \rho^2 \sum_{i=1}^{r}\sum_{j=1}^{r}\left[\| H_{2_i}^T H_{2_j} \|\right]\right)$$

and Eq. 22, we have:

$$\int_0^{T_f}\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j \times \Big(2\lambda^2 x^T(t)\left[C_{1_i} + D_{12_i}K_j\right]^T\left[C_{1_i} + D_{12_i}K_j\right]x(t) + 2\lambda^2\rho^2 x^T(t)\left[H_{4_i} + H_{6_i}K_j\right]^T\left[H_{4_i} + H_{6_i}K_j\right]x(t)\Big)dt \leq \gamma^2\lambda^2\left[\int_0^{T_f} w^T(t)w(t)dt\right] \quad (26)$$

Adding and subtracting:

$$\lambda^2 z^T(t)z(t) = \lambda^2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j \times \left(x^T(t)\begin{bmatrix}C_{1_i} + F(x(t),t)H_{4_i} + \\ D_{12_i}K_j + F(x(t),t)H_{6_i}K_j\end{bmatrix}^T\begin{bmatrix}C_{1_i} + F(x(t),t)H_{4_i} + \\ D_{12_i}K_j + F(x(t),t)H_{6_i}K_j\end{bmatrix}x(t)\right)$$

to and from Eq. 26, one obtains:

$$\begin{aligned}&\int_0^{T_f}\Big\{\lambda^2 z^T(t)z(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j \times \\ &\Big(2\lambda^2 x^T(t)\left[C_{1_i} + D_{12_i}K_j\right]^T\left[C_{1_i} + D_{12_i}K_j\right]x(t) + \\ &2\lambda^2\rho^2 x^T(t)\left[H_{4_i} + H_{6_i}K_j\right]^T\left[H_{4_i} + H_{6_i}K_j\right]x(t) - \\ &\lambda^2 x^T(t)\left[C_{1_i} + F(x(t),t)H_{4_i} + D_{12_i}K_j + F(x(t),t)H_{6_i}K_j\right]^T \\ &\left[C_{1_i} + F(x(t),t)H_{4_i} + D_{12_i}K_j + F(x(t),t)H_{6_i}K_j\right]x(t)\Big)\Big\}dt \\ &\leq \gamma^2\lambda^2\left[\int_0^{T_f} w^T(t)w(t)dt\right]\end{aligned} \quad (27)$$

Using the triangular inequality and the fact that $\|F(x(t), t)\| \leq \rho$, we have:

$$\begin{aligned}&\lambda^2\sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big(x^T(t) \times \\ &\left[C_{1_i} + F(x(t),t)H_{4_i} + D_{12_i}K_j + F(x(t),t)H_{6_i}K_j\right]^T \\ &\left[C_{1_i} + F(x(t),t)H_{4_i} + D_{12_i}K_j + F(x(t),t)H_{6_i}K_j\right]x(t)\Big) \\ &\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i\mu_j\Big(\Big\{2\lambda^2 x^T(t)\left[C_{1_i} + D_{12_i}K_j\right]^T\left[C_{1_i} + D_{12_i}K_j\right]x(t)\Big\} + \\ &2\lambda^2\rho^2 x^T(t)\left[H_{4_i} + H_{6_i}K_j\right]^T\left[H_{4_i} + H_{6_i}K_j\right]x(t)\Big)\end{aligned} \quad (28)$$

Using Eq. 28 on Eq. 27, we obtain:

$$\int_0^{T_f} z^T(t)z(t) \leq \gamma^2\int_0^{T_f} w^T(t)w(t)dt \quad (29)$$

Hence, the inequality of Eq. 3 holds. Note that from Lemma 1, we can obtain Lemma 2 which corresponds to the system (Eq. 1) as follows:

Lemma 2: Consider Eq. 1. Given a prescribed H_∞ performance $\gamma>0$ and a positive constant δ if there exist a matrix $P_\varepsilon = P_\varepsilon^T$ and matrices Y_j, j = 1, 2, ..., r, satisfying the following ε-dependent linear matrix inequalities:

$$P_\varepsilon > 0 \quad (30)$$

$$\Psi_{ii}(\varepsilon) < 0,\ i = 1, 2, \dots, r \quad (31)$$

$$\Psi_{ij}(\varepsilon) + \Psi_{ji}(\varepsilon) < 0,\ i < j \le r \quad (32)$$

Where:

$$\Psi_{ij}(\varepsilon) = \begin{pmatrix} \begin{pmatrix} A_i E_\varepsilon^{-1} P_\varepsilon + \\ E_\varepsilon^{-1} P_\varepsilon A_i^T + \\ B_{2_i} Y_j + \\ Y_j^T B_{2_i}^T \end{pmatrix} & (*)^T & (*)^T \\ \tilde{B}_{1_i}^T & -\gamma I & (*)^T \\ \tilde{C}_{1_i} E_\varepsilon^{-1} P_\varepsilon + \tilde{D}_{12i} Y_j & 0 & -\gamma I \end{pmatrix} \quad (33)$$

with;

$$\tilde{B}_{1_i} = \begin{bmatrix} \delta I & I & \delta I & B_{1_i} \end{bmatrix}$$

$$\tilde{C}_{1_i} = \begin{bmatrix} \frac{\gamma\rho}{\delta} H_{1_i}^T & 0 & \sqrt{2}\lambda\rho H_{4_i}^T & \sqrt{2}\lambda C_{1_i}^T \end{bmatrix}^T$$

$$\tilde{D}_{12_i} = \begin{bmatrix} 0 & \frac{\gamma\rho}{\delta} H_{3_i}^T & \sqrt{2}\lambda\rho H_{6_i}^T & \sqrt{2}\lambda D_{12_i}^T \end{bmatrix}^T$$

$$\lambda = \left(1 + \rho^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \left[\| H_{2_i}^T H_{2_j} \| \right] \right)^{\frac{1}{2}}$$

then the inequality of Eq. 3 holds. Furthermore, a suitable choice of the fuzzy controller is:

$$u(t) = \sum_{j=1}^{r} \mu_j K_j(\varepsilon) x(t) \quad (34)$$

Where:

$$K_j(\varepsilon) = Y_j P_\varepsilon^{-1} E_\varepsilon \quad (35)$$

Proof: The proof can be carried out the same technique used in Lemma 1.

Remark 1: The LMIs given in Lemma 2 become ill-conditioned when is sufficiently small which is always the case for the multiple time-scale systems. In general, these ill-conditioned LMIs are very difficult to solve. Thus, to alleviate these ill-conditioned LMIs, we have the following theorem which does not depend on.

Theorem 1: Consider Eq. 1. Given a prescribed H_∞ performance γ>0 and a positive constant δ if there exist a matrix P and matrices Y_j, j = 1, 2, ..., r satisfying the following independent inear matrix inequalities:

$$EP = P^T E,\ PD = DP,\ EP + PD > 0 \quad (36)$$

$$\psi_{ii} < 0,\ i = 1, 2, \dots, r \quad (37)$$

$$\psi_{ij} + \psi_{ji} < 0,\ i < j \le r \quad (38)$$

Where:

$$E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix},\ D = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

and;

$$\Psi_{ij} = \begin{pmatrix} \begin{pmatrix} (A_i P + P^T A_i^T) + \\ (B_{2_i} Y_j + Y_j^T B_{2_i}^T) \end{pmatrix} & (*)^T & (*)^T \\ \tilde{B}_{1_i}^T & -\gamma I & (*)^T \\ \tilde{C}_{1_i} P + \tilde{D}_{12_i} Y_j & 0 & -\gamma I \end{pmatrix} \quad (39)$$

with;

$$\tilde{B}_{1_i} = \begin{bmatrix} \delta I & I & \delta I & B_{1_i} \end{bmatrix}$$

$$\tilde{C}_{1_i} = \begin{bmatrix} \frac{\gamma\rho}{\delta} H_{1_i}^T & 0 & \sqrt{2}\lambda\rho H_{4_i}^T & \sqrt{2}\lambda C_{1_i}^T \end{bmatrix}^T$$

$$\tilde{D}_{12_i} = \begin{bmatrix} 0 & \frac{\gamma\rho}{\delta} H_{3_i}^T & \sqrt{2}\lambda\rho H_{6_i}^T & \sqrt{2}\lambda D_{12_i}^T \end{bmatrix}^T$$

$$\lambda = \left(1 + \rho^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \left[\| H_{2_i}^T H_{2_j} \| \right] \right)^{\frac{1}{2}}$$

then there exists a sufficiently $\hat{\varepsilon} > 0$ small such that the inequality of Eq. 3 holds for $\varepsilon \in (0, \hat{\varepsilon}]$. Furthermore, a suitable choice of the fuzzy controller is:

$$u(t) = \sum_{j=1}^{r} \mu_j K_j x(t) \quad (40)$$

Where:

$$K_j = Y_j P^{-1} \quad (41)$$

Proof: Suppose there exists a matrix P such that the inequality of Eq. 36 holds then P is of the following form:

$$P = \begin{pmatrix} P_1 & 0 \\ P_2^T & P_3 \end{pmatrix} \quad (42)$$

with $P_1 = P_1^T > 0$ and $P_3 = P_3^T > 0$. Let:

$$P_\varepsilon = E_\varepsilon (P + \varepsilon \tilde{P}) \quad (43)$$

with;

$$\tilde{P} = \begin{pmatrix} 0 & P_2 \\ 0 & 0 \end{pmatrix} \quad (44)$$

Substituting Eq. 42 and 44 into Eq. 43, we have:

$$P_\varepsilon = \begin{pmatrix} P_1 & \varepsilon P_2 \\ \varepsilon P_2^T & \varepsilon P_3 \end{pmatrix} \tag{45}$$

Clearly, $P_\varepsilon = P_\varepsilon^T$ and there exists a sufficient small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, $P_\varepsilon > 0$. Using the matrix inversion lemma, we learn that:

$$P_\varepsilon^{-1} = \left[P^{-1} + \varepsilon M_\varepsilon\right] E_\varepsilon^{-1} \tag{46}$$

Where:

$$M_\varepsilon = -P^{-1}\tilde{P}(I + \varepsilon P^{-1}\tilde{P})^{-1}P^{-1}$$

Substituting Eq. 43 and 46 into Eq. 33, we obtain:

$$\Psi_{ij} + \psi_{ij} \tag{47}$$

Where, the ε-independent linear matrix Ψ_{ij} is defined in Eq. 39 and the ε-dependent linear matrix is:

$$\psi_{ij} = \varepsilon \begin{pmatrix} \begin{pmatrix} A_i\tilde{P} + \\ \tilde{P}^T A_i^T + \\ B_{2_i} Y_{\varepsilon_j} + \\ Y_{\varepsilon_j}^T B_{2_i}^T \end{pmatrix} & (*)^T & (*)^T \\ 0 & 0 & (*)^T \\ \tilde{C}_{1_i}\tilde{P} + \tilde{D}_{12_i} Y_{\varepsilon_j} & 0 & 0 \end{pmatrix} \tag{48}$$

where, $Y_\varepsilon = K_j M_\varepsilon^{-1}$. Note that the ε-dependent linear matrix tends to zero when ε approaches zero. Employing Eq. 36-39 and using the fact that for any given negative definite matrix W, there exists an $\varepsilon > 0$ such that $W + \varepsilon I < 0$, one can show that there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$ Eq. 31 and 32 hold. Since Eq. 30-32 hold, using Lemma 1, the inequality of Eq. 3 holds for $\varepsilon \in (0, \hat{\varepsilon}]$.

NON-FRAGILE H_∞ OUTPUT FEEDBACK CONTROL DESIGN

The nature of the information of the state available to the controller has a major effect on the complexity of the designing problem and of the resulting controller. The state-feedback control design problem is an easier problem in which all information are available. However, in most real physical systems, the state is not perfectly known and so we must estimate it. The process of estimating the system state from the measurement output that are available is called the estimator design. By utilizing the state estimator, the output feedback problem is converted to the state-feedback problem for a new problem.

This new problem employs the estimated state as its own state variable and the solution of the new state-feedback problem leads to the solution of the dynamic output feedback control problem. Basically, the dynamic output feedback is a coupling of control and estimation. This study aims at designing a full order dynamic non-fragile H_∞ fuzzy output feedback controller of the form:

$$\begin{aligned} E_\varepsilon \dot{\hat{x}}(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} \hat{\mu}_i \hat{\mu}_j \left[\hat{A}_{ij}(\varepsilon)\hat{x}(t) + \hat{B}_{iy}(t)\right] \\ u(t) &= \sum_{i=1}^{r} \hat{\mu}_i \hat{C}_i \hat{x}(t) \end{aligned} \tag{49}$$

where, $\hat{x}(t) \in \Re^n$ is the controller's state vector, $\hat{A}_{ij}$, $\hat{B}_i$ and $\hat{C}_i$ are parameters of the controller which are to be determined and $\hat{\mu}_i$ denotes the normalized time-varying fuzzy weighting functions for each rule (i.e., $\hat{\mu}_i \geq 0$ and $\sum_{i=1}^{r} \hat{\mu}_i = 1$ such that the inequality of Eq. 3 holds.

Clearly in real control problems, all of the premise variables are not necessarily measurable. Thus in this study, we consider the designing of the non-fragile H_∞ output feedback control into two cases as follows. In this study, we consider the case where the premise variable of the fuzzy model μ_i is measurable while in this study, the premise variable which is assumed to be unmeasurable is considered.

Case 1-ν(t) is available for feedback: The premise variable of the fuzzy model ν(t) is available for feedback which implies that μ_i is available for feedback. Thus, we can select the controller that depends on μ_i as follows:

$$\begin{aligned} E_\varepsilon \dot{\hat{x}}(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j \left[\hat{A}_{ij}(\varepsilon)\hat{x}(t) + \hat{B}_{iy}(t)\right] \\ u(t) &= \sum_{i=1}^{r} \mu_i \hat{C}_i \hat{x}(t) \end{aligned} \tag{50}$$

Before presenting the next results, the following lemma is recalled.

Lemma 3: Consider Eq. 1. Given a prescribed H_∞ performance γ and a positive constant δ if there exist matrices $X_\varepsilon = X_\varepsilon^T$, $Y_\varepsilon = Y_\varepsilon^T$, $B_i(\varepsilon)$ and $C_i(\varepsilon)$, i = 1, 2, ... , r satisfying the following ε-dependent linear matrix inequalities:

$$\begin{bmatrix} X_\varepsilon & I \\ I & Y_\varepsilon \end{bmatrix} > 0 \tag{51}$$

$$X_\varepsilon > 0 \tag{52}$$

$$Y_\varepsilon > 0 \tag{53}$$

$$\Psi_{11_{ii}}(\varepsilon) < 0,\ i = 1, 2, ..., r \tag{54}$$

$$\Psi_{22_{ii}}(\varepsilon) < 0,\ i = 1, 2, ..., r \tag{55}$$

$$\psi_{11_{ij}}(\varepsilon) + \psi_{11_{ji}}(\varepsilon) < 0,\ i < j \le r \tag{56}$$

$$\psi_{22_{ij}}(\varepsilon) + \psi_{22_{ji}}(\varepsilon) < 0,\ i < j \le r \tag{57}$$

Where:

$$\psi_{11_{ij}}(\varepsilon) = \begin{pmatrix} \begin{pmatrix} E_\varepsilon^{-1} A_i Y_\varepsilon + \\ Y_\varepsilon A_i^T E_\varepsilon^{-1} + \\ E_\varepsilon^{-1} B_{2_i} C_j(\varepsilon) E_\varepsilon^{-1} + \\ E_\varepsilon^{-1} C_i^T(\varepsilon) B_{2_j}^T E_\varepsilon^{-1} + \\ \gamma^{-2} E_\varepsilon^{-1} \tilde{B}_{1_i} \tilde{B}_{1_j}^T E_\varepsilon^{-1} \end{pmatrix} & (*)^T \\ \left[Y_\varepsilon \tilde{C}_{1_i}^T + E_\varepsilon^{-1} C_i^T(\varepsilon) \tilde{D}_{12_j}^T \right]^T & -I \end{pmatrix} \tag{58}$$

$$\psi_{22_{ij}}(\varepsilon) = \begin{pmatrix} \begin{pmatrix} A_i^T E_\varepsilon^{-1} X_\varepsilon + \\ X_\varepsilon E_\varepsilon^{-1} A_i + \\ B_i(\varepsilon) C_{2_j} + \\ C_{2_i}^T B_j^T(\varepsilon) + \\ \tilde{C}_{1_i}^T \tilde{C}_{1_j} \end{pmatrix} & (*)^T \\ \left[X_\varepsilon E_\varepsilon^{-1} \tilde{B}_{1_i} + B_i(\varepsilon) \tilde{D}_{21_j} \right]^T & -\gamma^2 I \end{pmatrix} \tag{59}$$

with;

$$\tilde{B}_{1_i} = \left[\delta I \ \ I \ \ \delta I \ \ 0 \ \ B_{1_i} \ \ 0 \right]$$
$$\tilde{C}_{1_i} = \left[\frac{\gamma\rho}{\delta} H_{1_i}^T \ \ 0 \ \ \frac{\gamma\rho}{\delta} H_{5_i}^T \ \ \sqrt{2}\lambda\rho H_{4_i}^T \ \ \sqrt{2}\lambda C_{1_i}^T \right]^T$$
$$\tilde{D}_{12_i} = \left[0 \ \ \frac{\gamma\rho}{\delta} H_{3_i}^T \ \ 0 \ \ \sqrt{2}\lambda\rho H_{6_i}^T \ \ \sqrt{2}\lambda D_{12_i}^T \right]^T$$
$$\tilde{D}_{21_i} = \left[0 \ \ 0 \ \ 0 \ \ \delta I \ \ D_{21_i} \ \ I \right]$$

and;

$$\lambda = \left(1 + \rho^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \left[\| H_{2_i}^T H_{2_j} \| + \| H_{7_i}^T H_{7_j} \| \right] \right)^{\frac{1}{2}}$$

then the system (Eq. 1) has the prescribed H_∞ performance $\gamma > 0$. Furthermore, a suitable controller is of the form (Eq. 50) with:

$$\begin{aligned} \hat{A}_{ij}(\varepsilon) &= E_\varepsilon \left[Y_\varepsilon^{-1} - X_\varepsilon \right]^{-1} M_{ij}(\varepsilon) Y_\varepsilon^{-1} \\ \hat{B}_i &= E_\varepsilon \left[Y_\varepsilon^{-1} - X_\varepsilon \right]^{-1} B_i(\varepsilon) \\ \hat{C}_i &= C_i(\varepsilon) E_\varepsilon^{-1} Y_\varepsilon^{-1} \end{aligned} \tag{60}$$

Where:

$$\begin{aligned} M_{ij}(\varepsilon) = & -A_i^T E_\varepsilon^{-1} - X_\varepsilon E_\varepsilon^{-1} A_i Y_\varepsilon - X_\varepsilon E_\varepsilon^{-1} B_{2_i} \hat{C}_j Y_\varepsilon - \\ & \left[Y_\varepsilon^{-1} - X_\varepsilon \right] E_\varepsilon^{-1} \hat{B}_i C_{2_j} Y_\varepsilon - \tilde{C}_{1_i}^T \left[\tilde{C}_{1_j} Y_\varepsilon + \hat{D}_{12_j} \tilde{C}_j Y_\varepsilon \right] - \\ & \gamma^{-2} \left\{ X_\varepsilon E_\varepsilon^{-1} \tilde{B}_{1_i} + \left[Y_\varepsilon^{-1} - X_\varepsilon \right] E_\varepsilon^{-1} \hat{B}_i \tilde{D}_{21_i} \right\} \tilde{B}_{1_j}^T E_\varepsilon^{-1} \end{aligned} \tag{61}$$

Proof: The proof can be carried out the same technique used in Lemma 1.

Remark 2: The LMIs given in Lemma 3 may become ill-conditioned when ε is suffi-ciently small which is always the case for the multiple time-scale systems. In general, these ill-conditioned LMIs are very difficult to solve. Thus to alleviate these ill-conditioned LMIs, we have the following ε-independent well-posed LMI-based suf-ficient conditions for the uncertain fuzzy multiple time-scale systems to obtain the prescribed H_∞ performance.

Theorem 2: Consider Eq. 1. Given a prescribed H_∞ performance γ>0 and a positive constant δ if there exist matrices X_0, Y_0, B_0 and C_0, i = 1, 2,..., r satisfying the following ε-independent linear matrix inequalities:

$$\begin{bmatrix} X_0 E + D X_0 & I \\ I & Y_0 E + D Y_0 \end{bmatrix} > 0 \tag{62}$$

$$E X_0^T = X_0 E,\ X_0^T D = D X_0,\ X_0 E + D X_0 > 0 \tag{63}$$

$$E Y_0^T = Y_0 E,\ Y_0^T D = D Y_0,\ Y_0 E + D Y_0 > 0 \tag{64}$$

$$\psi_{11_{ii}} < 0,\ i = 1, 2, ..., r \tag{65}$$

$$\psi_{22_{ii}} < 0,\ i = 1, 2, ..., r \tag{66}$$

$$\psi_{11_{ij}} + \psi_{11_{ji}} < 0,\ i < j \le r \tag{67}$$

$$\psi_{22ij} + \psi_{22ji} < 0,\ i < j \le r \tag{68}$$

Where:

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\Psi_{11_{ij}} = \begin{pmatrix} \begin{pmatrix} A_i Y_0^T + Y_0 A_i^T + \\ B_{2_i} C_{0_j} + C_{0_i}^T B_{2_j}^T + \\ \gamma^{-2}\tilde{B}_{1_i}\tilde{B}_{1_j}^T \end{pmatrix} & (*)^T \\ \left[Y_0 \tilde{C}_{1_i}^T + C_{0_i}^T \tilde{D}_{12_j}^T \right]^T & -I \end{pmatrix} \quad (69)$$

$$\Psi_{22_{ij}} = \begin{pmatrix} \begin{pmatrix} A_i^T X_0^T + X_0 A_i + \\ B_{0_i} C_{2_j} + C_{2_i}^T B_{0_j}^T + \\ \tilde{C}_{1_i}^T \tilde{C}_{1_j} \end{pmatrix} & (*)^T \\ \left[X_0 \tilde{B}_{1_i} + B_{0_i} \tilde{D}_{21_j} \right]^T & -\gamma^2 I \end{pmatrix} \quad (70)$$

with;

$$\tilde{B}_{1_i} = \left[\delta I \quad I \quad \delta I \quad 0 \quad B_{1_i} \quad 0 \right]$$

$$\tilde{C}_{1_i} = \left[\frac{\gamma\rho}{\delta} H_{1_i}^T \quad 0 \quad \frac{\gamma\rho}{\delta} H_{5_i}^T \quad \sqrt{2}\lambda\rho H_{4_i}^T \quad \sqrt{2}\lambda C_{1_i}^T \right]^T$$

$$\tilde{D}_{12_i} = \left[0 \quad \frac{\gamma\rho}{\delta} H_{3_i}^T \quad 0 \quad \sqrt{2}\lambda\rho H_{6_i}^T \quad \sqrt{2}\lambda D_{12_i}^T \right]^T$$

$$\tilde{D}_{21_i} = \left[0 \quad 0 \quad 0 \quad \delta I \quad D_{21_i} \quad I \right]$$

and;

$$\lambda = \left(1 + \rho^2 \sum_{i=1}^{r} \sum_{j=1}^{r} \left[\left\| H_{2_i}^T H_{2_j} \right\| + \left\| H_{7_i}^T H_{7_j} \right\| \right] \right)^{\frac{1}{2}}$$

then there exists a sufficiently small $\hat{\varepsilon} > 0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, the prescribed H_∞ performance $\gamma > 0$ is guaranteed. Furthermore, a suitable controller is of the form Eq. 50 with:

$$\begin{aligned} \hat{A}_{ij}(\varepsilon) &= \left[Y_\varepsilon^{-1} - X_\varepsilon \right]^{-1} M_{0_{ij}}(\varepsilon) Y_\varepsilon^{-1} \\ \hat{B}_i &= \left[Y_0^{-1} - X_0 \right]^{-1} B_{0_i} \\ \hat{C}_i &= C_{0_i} Y_0^{-1} \end{aligned} \quad (71)$$

Where:

$$\begin{aligned} M_{0_{ij}}(\varepsilon) = &- A_i^T - X_\varepsilon A_i Y_\varepsilon - X_\varepsilon B_{2_i} \\ &\hat{C}_j Y_\varepsilon - \left[Y_\varepsilon^{-1} - X_\varepsilon \right] \hat{B}_i C_{2_j} Y_\varepsilon - \\ &\tilde{C}_{1_i}^T [\tilde{C}_{1_j} Y_\varepsilon + \tilde{D}_{12_j} \hat{C}_j Y_\varepsilon] - \\ &\gamma^{-2} \{ X_\varepsilon \tilde{B}_{1_i} + [Y_\varepsilon^{-1} - X_\varepsilon] \hat{B}_i \tilde{D}_{21_i} \} \tilde{B}_{1_j}^T \end{aligned} \quad (72)$$

$$\begin{aligned} X_\varepsilon &= \{ X_0 + \varepsilon \tilde{X} \} E_\varepsilon \\ Y_\varepsilon^{-1} &= \{ Y_0^{-1} + \varepsilon N_\varepsilon \} E_\varepsilon \end{aligned} \quad (73)$$

with;

$$\tilde{X} = D(X_0^T - X_0)$$

and;

$$N_\varepsilon = D\left(\left(Y_0^{-1} \right)^T - Y_0^{-1} \right)$$

Proof: Suppose the inequalities of Eq. 62-64 hold then the matrices X_0 and Y_0 are of the following forms:

$$X_0 = \begin{pmatrix} X_1 & X_2 \\ 0 & X_3 \end{pmatrix}, \quad Y_0 = \begin{pmatrix} Y_1 & Y_2 \\ 0 & Y_3 \end{pmatrix}$$

with $X_1 = X_1^T > 0$, $X_3 = X_3^T > 0$, $Y_1 = Y_1^T > 0$ and $Y_3 = Y_3^T > 0$. Substituting X_0 and Y_0 into Eq. 73, respectively, we have:

$$X_\varepsilon = \left\{ X_0 + \varepsilon \tilde{X} \right\} E_\varepsilon = \begin{pmatrix} X_1 & \varepsilon X_2 \\ \varepsilon X_2^T & \varepsilon X_3 \end{pmatrix} \quad (74)$$

and;

$$\begin{aligned} Y_\varepsilon^{-1} &= \{ Y_0^{-1} - \varepsilon N_\varepsilon \} \\ E_\varepsilon &= \begin{pmatrix} Y_1^{-1} & -\varepsilon Y^{-1} Y_2 Y_3^{-1} \\ -\varepsilon (Y^{-1} Y_2 Y_3^{-1})^T & \varepsilon Y_3^{-1} \end{pmatrix} \end{aligned} \quad (75)$$

Clearly, $X_\varepsilon = X_\varepsilon^T$ and $Y_\varepsilon^{-1} = (Y_\varepsilon^{-1})^T$. Knowing the fact that the inverse of a symmetric matrix is a symmetric matrix, we learn that Y_ε is a symmetric matrix. Using the matrix inversion lemma, we can see that:

$$Y_\varepsilon = E_\varepsilon^{-1} \{ Y_0 + \varepsilon \tilde{Y} \} \quad (76)$$

where, $\tilde{Y} = Y_0 N_\varepsilon (I + \varepsilon Y_0 N_\varepsilon)^{-1} Y_0$. Employing the Schur complement, one can show that there exists a sufficiently small $\hat{\varepsilon}$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$, Eq. 52 and 53 hold. Now, we need to show that:

$$\begin{pmatrix} X_\varepsilon & 1 \\ 1 & Y_\varepsilon \end{pmatrix} > 0 \quad (77)$$

By the Schur complement, it is equivalent to showing that:

$$X_\varepsilon - Y_\varepsilon^{-1} > 0 \quad (78)$$

Substituting Eq. 74 and 75 into the left hand side of Eq. 78, we get:

$$\begin{bmatrix} X_1 - Y_1^{-1} & \varepsilon (X_2 + Y_1^{-1} Y_2 Y_3^{-1}) \\ \varepsilon (X_2 + Y_1^{-1} Y_2 Y_3^{-1})^T & \varepsilon (X_3 - Y_3^{-1}) \end{bmatrix} \quad (79)$$

The Schur complement of Eq. 62 is:

$$\begin{pmatrix} X_1 - Y_1^{-1} & 0 \\ 0 & X_3 - Y_3^{-1} \end{pmatrix} > 0 \quad (80)$$

According to Eq. 80, we learn that:

$$X_1 - Y_1^{-1} > 0,\ X_3 - Y_3^{-1} > 0 \quad (81)$$

Using Eq. 81 and the Schur complement, it can be shown that there exists a sufficiently small $\hat{\varepsilon}>0$ such that for $\varepsilon \in (0, \hat{\varepsilon})$, Eq. 51 holds. Next, employing Eq. 74-76, the controller's matrices in Eq. 60 can be re-expressed as follows:

$$\begin{aligned} B_i(\varepsilon) &= [Y_0^{-1} - X_0]\hat{B}_i + \varepsilon[N_\varepsilon - \tilde{X}]\hat{B}_i \\ &\triangleq B_{0_i} + \varepsilon B_{\varepsilon_i} \\ C_i(\varepsilon) &= \hat{C}_i Y_0^T + \varepsilon \hat{C}_i \tilde{Y}^T \\ &\triangleq C_{0_i} + \varepsilon C_{\varepsilon_i} \end{aligned} \quad (82)$$

Substituting Eq. 74-76 and 82 into Eq. 58 and 59 and pre-post multiplying Eq. 58 by:

$$\begin{pmatrix} E_\varepsilon & 0 \\ 0 & I \end{pmatrix}$$

we, respectively obtain:

$$\Psi_{11_{ij}} + \psi_{11_{ij}},\ \Psi_{22_{ij}} + \psi_{22_{ij}} \quad (83)$$

where, the ε-independent linear matrices ψ_{11}, and ψ_{22} are defined in Eq. 69 and 70, respectively and the ε-dependent linear matrices are:

$$\psi_{11_{ij}} = \varepsilon \begin{pmatrix} \begin{pmatrix} A_i\tilde{Y}^T + \tilde{Y}A_i^T + \\ B_{2_i}C_{\varepsilon_j} + C_{\varepsilon_i}^T B_{2_j}^T \end{pmatrix} & (*)^T \\ \left[\tilde{Y}\tilde{C}_{1_i}^T + C_{\varepsilon_i}^T \tilde{D}_{12_j}^T\right]^T & 0 \end{pmatrix} \quad (84)$$

$$\psi_{22_{ij}} = \varepsilon \begin{pmatrix} \begin{pmatrix} A_i^T\tilde{X} + \tilde{X}^T A_i + \\ B_{\varepsilon_i}C_{2_j} + C_{2_i}^T B_{\varepsilon_j}^T \end{pmatrix} & (*)^T \\ \left[\tilde{X}\tilde{B}_{1_i} + B_{\varepsilon_i}\tilde{D}_{21_j}\right]^T & 0 \end{pmatrix} \quad (85)$$

The ε-dependent linear matrices tend to zero when ε approaches zero. Employing Eq. 65-68 and knowing the fact that for any given negative definite matrix W, there exists an $\varepsilon>0$ such that $W+\varepsilon I<0$, one can show that there exists a sufficiently small $\hat{\varepsilon}>0$ such that for $\varepsilon \in (0, \hat{\varepsilon}]$ Eq. 54-57 hold. Since Eq. 51-57 hold, using Lemma 2, the inequality of Eq. 3 holds.

CASE II-ν(T) IS UNAVAILABLE FOR FEEDBACK

The output feedback fuzzy controller is assumed to be the same as the premise variables of the fuzzy system model. This actually means that the premise variables of fuzzy system model are assumed to be measurable. However in general, it is extremely difficult to derive an accurate fuzzy system model by imposing that all premise variables are measurable. In this study, we do not impose that condition, we choose the premise variables of the controller to be different from the premise variables of fuzzy system model of the plant. In here, the premise variables of the controller are selected to be the estimated premise variables of the plant. In the other words, the premise variable of the fuzzy model $\nu(t)$ is unavailable for feedback which implies μ_i is unavailable for feedback. Hence, we cannot select the controller which depends on μ_i. Thus, we select the controller as follows:

$$\begin{aligned} E_\varepsilon\dot{\hat{x}}(t) &= \sum_{i=1}^r \sum_{j=1}^r \hat{\mu}_i\hat{\mu}_j \left[\hat{A}_{ij}(\varepsilon)\hat{x}(t) + B_{iy}(t)\right] \\ u(t) &= \sum_{i=1}^r \hat{\mu}_i \hat{C}_i \hat{x}(t) \end{aligned} \quad (86)$$

Where, $\hat{\mu}_i$ depends on the premise variable of the controller which is different from μ_i. Let us re-express the system (Eq. 1) in terms of $\hat{\mu}_i$, thus the plant's premise variable becomes the same as the controller's premise variable. By doing so, the result given in the previous case can then be applied here. Note that it can be done by using the same technique as in subsection. After some manipulation, we get:

$$\begin{aligned} E_\varepsilon\dot{x}(t) &= \sum_{i=1}^r \hat{\mu}_i\left[A_i + \Delta\bar{A}_i\right]x(t) + \\ &\quad [B_{1_i} + \Delta\bar{B}_{1_i}]w(t) + \\ &\quad [B_{2_i} + \Delta\bar{B}_{2_i}]u(t),\ x(0)=0 \\ z(t) &= \sum_{i=1}^r \hat{\mu}_i\left[\left[C_{1_i} + \Delta\bar{C}_{1_i}\right]x(t) + \right. \\ &\quad \left.[D_{12_i} + \Delta\bar{D}_{12_i}]u(t)\right] \\ y(t) &= \sum_{i=1}^r \hat{\mu}_i\left[\left[C_{2_i} + \Delta\bar{C}_{2_i}\right]x(t) \right. \\ &\quad \left.[D_{21_i} + \Delta\bar{D}_{21_i}]w(t)\right] \end{aligned} \quad (87)$$

Where:

$$\begin{aligned} \Delta\bar{A}_i &= \bar{F}(x(t), \hat{x}(t), t)\bar{H}_{1_i} \\ \Delta\bar{B}_{1_i} &= \bar{F}(x(t), \hat{x}(t), t)\bar{H}_{2_i} \\ \Delta\bar{B}_{2_i} &= \bar{F}(x(t), \hat{x}(t), t)\bar{H}_{3_i} \\ \Delta\bar{C}_{1_i} &= \bar{F}(x(t), \hat{x}(t), t)\bar{H}_{4_i} \\ \Delta\bar{C}_{2_i} &= \bar{F}(x(t), \hat{x}(t), t)\bar{H}_{5_i} \\ \Delta\bar{D}_{12_i} &= \bar{F}(x(t), \hat{x}(t), t)\bar{H}_{6_i} \\ \Delta\bar{D}_{21_i} &= \bar{F}(x(t), \hat{x}(t), t)\bar{H}_{7_i} \end{aligned}$$

with;

$$\bar{H}_{1_i}=[H_{1_i}^TA_1^T...A_r^TH_{1_1}^T....H_{1_r}^T]^T$$
$$\bar{H}_{2_i}=[H_{2_i}^TB_{1_i}^T...B_{1_r}^TH_{2_1}^T....H_{2_r}^T]^T$$
$$\bar{H}_{3_i}=[H_{3_i}^TB_{2_i}^T...B_{2_r}^TH_{3_1}^T....H_{3_r}^T]^T$$
$$\bar{H}_{4_i}=[H_{4_i}^TC_{1_i}^T...C_{1_r}^TH_{4_1}^T....H_{4_r}^T]^T$$
$$\bar{H}_{5_i}=[H_{5_i}^TC_{2_i}^T...C_{2_r}^TH_{5_1}^T....H_{5_r}^T]^T$$
$$\bar{H}_{6_i}=[H_{6_i}^TD_{12_i}^T...D_{12_r}^TH_{6_1}^T....H_{6_r}^T]^T$$
$$\bar{H}_{7_i}=[H_{7_i}^TD_{21_i}^T...D_{21_r}^TH_{7_1}^T....H_{7_r}^T]^T$$

and;

$$\bar{F}(x(t),\hat{x}(t),t)=\begin{bmatrix}F(x(t),t)(\mu_1-\hat{\mu}_1)...(\mu_r-\hat{\mu}_r)\\ F(x(t),t)(\mu_1-\hat{\mu}_1)...\\ F(x(t),t)(\mu_r-\hat{\mu}_r)\end{bmatrix}$$

While:

$$\|\bar{F}(x(t),\hat{x}(t),t)\|\le\bar{\rho}$$

Where:

$$\bar{\rho}=\{3\rho^2+2\}^{\frac{1}{2}}$$

where, $\bar{\rho}$ is derived by utilizing the concept of vector norm in the basic system control theory and the fact that $\mu_i\ge 0$, $\hat{\mu}_i\ge 0$, $\sum_{i=1}^r\mu_i=1$ and $\sum_{i=1}^r\hat{\mu}_i=1$.

Note that the above technique is basically employed in order to obtain the plant's premise variable to be the same as the controller's premise variable, e.g., Nguang and Shi (2003). Now, the premise variable of the system is the same as the premise variable of the controller, thus we can apply the result given in case 1.

Theorem 3: Consider Eq. 1. Given a prescribed H_∞ performance $\gamma>0$ and a positive constant δ if there exist matrices X_0, Y_0, B_0 and C_0, i = 1, 2, ... , r satisfying the following ε-independent linear matrix inequalities:

$$\begin{bmatrix}X_0E+DX_0 & 1\\ 1 & Y_0E+DY_0\end{bmatrix}>0 \qquad (88)$$

$$EX_0^T=X_0E,\ X_0^TD=DX_0,\ X_0E+DX_0>0 \qquad (89)$$

$$EY_0^T=Y_0E,\ Y_0^TD=DY_0,\ Y_0E+DY_0>0 \qquad (90)$$

$$\Psi_{11_{ii}}<0, i=1,2,...,r \qquad (91)$$

$$\Psi_{22_{ii}}<0, i=1,2,...,r \qquad (92)$$

$$\Psi_{11_{ij}}+\Psi_{11_{ji}}<0,\ i<j\le r \qquad (93)$$

$$\Psi_{22_{ij}}+\Psi_{22_{ji}}<0, i<j\le r \qquad (94)$$

Where:

$$E=\begin{pmatrix}I & 0\\ 0 & 0\end{pmatrix},\ D=\begin{pmatrix}0 & 0\\ 0 & I\end{pmatrix}$$

$$\Psi_{11_{ij}}=\begin{pmatrix}\begin{pmatrix}A_iY_0^T+Y_0A_i^T+\\ B_{2_i}C_{0_j}+C_{0_i}^TB_{2_j}^T+\\ \gamma^{-2}\tilde{\bar{B}}_{1_i}\tilde{\bar{B}}_{1_j}^T\end{pmatrix} & (*)^T\\ \left[Y_0\tilde{\bar{C}}_{1_i}^T+C_{0_i}^T\tilde{\bar{D}}_{12_j}^T\right]^T & -I\end{pmatrix} \qquad (95)$$

$$\Psi_{22_{ij}}=\begin{pmatrix}\begin{pmatrix}A_i^TX_0^T+Y_0A_i+\\ B_{0_i}C_{2_j}+C_{2_i}^TB_{0_j}^T+\\ \tilde{\bar{C}}_{1_i}^T\tilde{\bar{C}}_{1_j}\end{pmatrix} & (*)^T\\ \left[X_0\tilde{\bar{B}}_{1_i}+B_{0_i}\tilde{\bar{D}}_{21_j}\right]^T & -\gamma^2I\end{pmatrix} \qquad (96)$$

$$\tilde{\bar{B}}_{1_i}=[\delta I\quad I\quad \delta I\quad 0\quad B_{1_i}\quad 0]$$
$$\tilde{\bar{C}}_{1_i}=\left[\frac{\gamma\bar{\rho}}{\delta}\bar{H}_{1_i}^T\quad 0\quad \frac{\gamma\bar{\rho}}{\delta}\bar{H}_{5_i}^T\quad \sqrt{2\bar{\lambda}}\bar{\rho}\bar{H}_{4_i}^T\quad \sqrt{2\bar{\lambda}}C_{1_i}^T\right]^T$$
$$\tilde{\bar{D}}_{12_i}=\left[0\quad \frac{\gamma\bar{\rho}}{\delta}\bar{H}_{3_i}^T\quad 0\quad \sqrt{2\bar{\lambda}}\bar{\rho}\bar{H}_{6_i}^T\quad \sqrt{2\bar{\lambda}}D_{12_i}^T\right]^T$$
$$\tilde{\bar{D}}_{21_i}=[0\quad 0\quad 0\quad \delta I\quad D_{21_i}\quad I]$$

and;

$$\bar{\lambda}=\left(1+\bar{\rho}^2\sum_{i=1}^{r}\sum_{j=1}^{r}\left[\left\|\bar{H}_{2_i}^T\bar{H}_{2_j}\right\|+\left\|\bar{H}_{7_i}^T\bar{H}_{7_j}\right\|\right]\right)^{\frac{1}{2}}$$

then there exists a sufficiently small $\hat{\varepsilon}>0$ such that for $\varepsilon\in(0,\hat{\varepsilon}]$, the prescribed H_∞ performance $\gamma>0$ is guaranteed. Furthermore, a suitable controller is of the form (Eq. 86) with:

$$\hat{A}_{ij}(\varepsilon)=[Y_\varepsilon^{-1}-X_\varepsilon]^{-1}M_{0_{ij}}(\varepsilon)Y_\varepsilon^{-1}$$
$$\hat{B}_i=[Y_0^{-1}-X_0]^{-1}B_{0_i} \qquad (97)$$
$$\hat{C}_i=C_{0_i}Y_0^{-1}$$

Where:

$$M_{0_{ij}}(\varepsilon)=-A_i^T-X_\varepsilon A_iY_\varepsilon-X_\varepsilon B_{2_i}\hat{C}_jY_\varepsilon-[Y_\varepsilon^{-1}-X_\varepsilon]\hat{B}_iC_{2_j}Y_\varepsilon-\tilde{\bar{C}}_{1_i}^T\{\tilde{\bar{C}}_{1_j}Y_\varepsilon+\tilde{\bar{D}}_{12_j}\hat{C}_jY_\varepsilon]-\gamma^{-2}\{X_\varepsilon\tilde{\bar{B}}_{1_i}+[Y_\varepsilon^{-1}-X_\varepsilon]\hat{B}_i\tilde{\bar{D}}_{21_i}\}\tilde{\bar{B}}_{1j}^T \qquad (98)$$

$$X_\varepsilon=\{X_0+\varepsilon\tilde{X}\}E_\varepsilon$$
$$Y_\varepsilon^{-1}=\{Y_0^{-1}+\varepsilon N_\varepsilon\}E_\varepsilon \qquad (99)$$

$$\tilde{X} = D(X_0^T - X_0),\ N_\varepsilon = D((Y_0^{-1})^T - Y_0^{-1})$$

Proof: Since Eq. 87 is of the form of Eq. 1, it can be shown by employing the proof for Theorem 2.

EXAMPLE

Consider the tunnel diode circuit shown in Fig. 2 where the tunnel diode is characterized by:

$$i_D\ (t) = -0.2v_D\ (t) - 0.01v_D^3(t)$$

Assume that ε is a parasitic inductance in the network. Let $x_1(t) = v_C(t)$ be the capacitor voltage and $x_2(t) = i_L(t)$ be the inductor current. Then, the circuit shown in Fig. 1 can be modelled by the following state equations:

$$\begin{aligned} C\dot{x}_1(t) &= 0.2x_1(t) + 0.01x_1^3(t) + x_2(t) \\ \varepsilon\dot{x}_2 &= -x_1(t) - Rx_2(t) + u(t) + 0.1w_2(t) \\ y(t) &= Jx(t) + 0.1w_1(t) \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{100}$$

Where:

- $u(t)$ = Control input
- $w_1(t)$ = Measurement noise
- $w_2(t)$ = Process noise which may represent un-modelled dynamics
- $y(t)$ = Measured output
- $z(t)$ = Controlled output
- J = Sensor matrix
- $x(t)$ = $[x_1^T(t)\ x_2^T(t)]^T$
- $w(t)$ = $[w_1^T(t)\ w_2^T(t)]^T$

The variables $x_1(t)$ and $x_2(t)$ are treated as the deviation variables (variables deviate from its desired trajectories). The parameters in the circuit are given by C = 100 mF and R = 1±0.3% Ω with these parameters (Eq. 100) can be rewritten as:

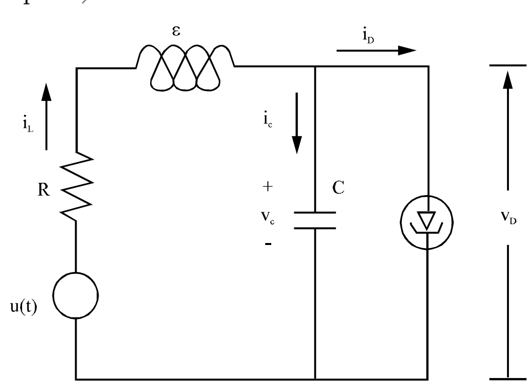

Fig. 2: A tunnel diode circuit

$$\begin{aligned} \dot{x}_1(t) &= 2x_1(t) + (0.1x_1^2(t) \times x_1(t) + 10x_2(t) \\ \varepsilon\dot{x}_2(t) &= -x_1(t) - (1 \pm \Delta R)x_2(t) + \\ &\quad u(t) + 0.1w_2(t) \\ y(t) &= Jx(t) + 0.1w_1(t) \\ z(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \end{aligned} \tag{101}$$

For the sake of simplicity, we will use as few rules as possible. Assuming that $|x_1(t)| \le 3$, the nonlinear network system (Eq. 101) can be approximated by the following TS fuzzy model:

Plant rule 1: If $x_1(t)$ is $M_1(x_1(t))$ then:

$$\begin{aligned} E_\varepsilon\dot{x}(t) &= [A_1 + \Delta A_1]x(t) + B_1w(t) + B_{2_1}u(t) \\ z(t) &= C_1x(t) \\ y(t) &= C_{2_1}x(t) + D_{21}w(t) \end{aligned}$$

Plant rule 2: If $x_1(t)$ is $M_2(x_1(t))$ then:

$$\begin{aligned} E_\varepsilon\dot{x}(t) &= [A_2 + \Delta A_2]x(t) + B_1w(t) + B_{2_2}u(t) \\ z(t) &= C_1x(t) \\ y(t) &= C_{2_2}x(t) + D_{21}w(t) \end{aligned}$$

Where:

$$x(0) = 0$$
$$x(t) = \left[x_1^T(t)\ x_2^T(t)\right]^T$$
$$w(t) = \left[w_1^T(t)\ w_2^T(t)\right]^T$$

$$A_1 = \begin{bmatrix} 2 & 10 \\ -1 & -1 \end{bmatrix},\quad A_2 = \begin{bmatrix} 2.9 & 10 \\ -1 & -1 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix},\quad B_{2_1} = B_{2_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad C_{2_1} = C_{2_2} = J$$
$$D_{21} = [0.1 \quad 0]$$

$$\Delta A_1 = F(x(t),t)H_{1_1},\quad \Delta A_2 = F(x(t),t)H_{1_2}$$

and;

$$E_\varepsilon = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}$$

The plot of the membership functions is the same as in Fig. 3. Now by assuming that in Eq. 2, $\|F(x(t), t)\| \le \rho = 1$ and since the values of R are uncertain but bounded within 30% of their nominal values in Eq. 100, we have:

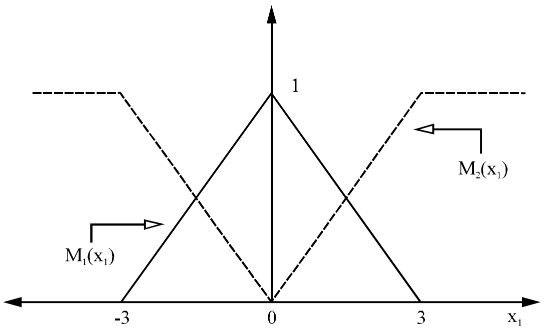

Fig. 3: Membership functions for the two fuzzy set

$$H_{l_1} = H_{l_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0.3 \end{bmatrix}$$

State-feedback controller design: Employing the results given in Lemma 1 and the Matlab LMI solver, it is easy to re-alize that when ε<0.03, the LMIs become ill-conditioned and the Matlab LMI solver yields the error message, Rank deficient. Using the LMI optimization algorithm and theorem 1 with $\gamma = 1$ and $\delta = 1$, we obtain:

$$X = \begin{bmatrix} 0.0943 & 0 \\ -0.7111 & 123.3692 \end{bmatrix}$$
$$Y_1 = [-0.0670 - 413.6464], \quad Y_2 = [-0.0447 - 414.8722]$$
$$K_1 = [-25.9930 - 3.3529], \quad K_2 = [-25.8312 - 3.3629]$$

The resulting fuzzy controller is:

$$u(t) = \sum_{j=1}^{2} \mu_j K_j x(t) \qquad (102)$$

Where:

$$\mu_1 = M_1(x_1(t))$$

and;

$$\mu_2 = M_2(x_1(t))$$

Output feedback controller design: Note that by employing the results given in Lemma 2 and the Matlab LMI solver, it is easy to realize that when ε<0.03, the LMIs become ill-conditioned and the Matlab LMI solver yields the error message, Rank deficient. Using the LMI optimization algorithm and theorem 2-3 with $\varepsilon = 0.01$, $\gamma = 1$ and $\delta = 1$, we obtain the following results:

Case I-ν(t) are available for feedback: In this case, $x_1(t) = \nu(t)$ is assumed to be available for feed back; for instance, J = [1 0]. This implies that μ_i is available for feedback:

$$X_0 = \begin{bmatrix} 0.3910 & 4.2648 \\ 0 & 26.0371 \end{bmatrix}$$

$$Y_0 = \begin{bmatrix} 135.2417 & -48.6420 \\ 0 & 143.3726 \end{bmatrix}$$

$$\hat{A}_{11}(\varepsilon) = \begin{bmatrix} -89.4198 & 9.2455 \\ -3.0543 & -18.6773 \end{bmatrix}$$

$$\hat{A}_{12}(\varepsilon) = \begin{bmatrix} -89.4196 & 9.2461 \\ -2.9811 & -18.5296 \end{bmatrix}$$

$$\hat{A}_{21}(\varepsilon) = \begin{bmatrix} -85.8526 & 9.2514 \\ -3.4361 & -18.6773 \end{bmatrix}$$

$$\hat{A}_{22}(\varepsilon) = \begin{bmatrix} -85.8523 & 9.2519 \\ -3.3628 & -18.5296 \end{bmatrix}$$

$$\hat{B}_1 = \begin{bmatrix} 96.6841 \\ -1.7141 \end{bmatrix}, \hat{B}_2 = \begin{bmatrix} 94.0516 \\ -1.3324 \end{bmatrix}$$

$$\hat{C}_1 = [-46.7003 \quad -177.7428]$$

$$\hat{C}_2 = [-45.9676 \quad -176.2654]$$

The resulting fuzzy controller is:

$$E_\varepsilon \dot{\hat{x}}(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} \mu_i \mu_j \hat{A}_{i_j}(\varepsilon)\hat{x}(t) + \sum_{i=1}^{2} \mu_i \hat{B}_{iy}(t)$$
$$u(t) = \sum_{i=1}^{2} \mu_i \hat{C}_i \hat{x}(t)$$

Where:

$$\mu_1 = M_1(x_1(t))$$

and;

$$\mu_2 = M_2(x_1(t))$$

Case 2 ν (t) are unavailable for feedback: In this case, $x_1(t) = \nu(t)$ is assumed to be unavailable for feedback for instance, J = [0 1]. This implies that μ_i is unavailable for feedback:

$$X_0 = \begin{bmatrix} 0.3519 & 5.1178 \\ 0 & 43.3952 \end{bmatrix}$$

$$Y_0 = \begin{bmatrix} 225.4028 & -96.9239 \\ 0 & 143.3726 \end{bmatrix}$$

$$\hat{A}_{11}(\varepsilon)=\begin{bmatrix}-93.0660 & 9.3158\\ -1.8731 & -19.0234\end{bmatrix}$$

$$\hat{A}_{12}(\varepsilon)=\begin{bmatrix}-93.0657 & 9.3167\\ -1.8291 & -18.8706\end{bmatrix}$$

$$\hat{A}_{21}(\varepsilon)=\begin{bmatrix}-90.5479 & 9.3235\\ -2.1021 & -19.0234\end{bmatrix}$$

$$\hat{A}_{22}(\varepsilon)=\begin{bmatrix}-90.5476 & 9.3245\\ -2.0581 & -18.8706\end{bmatrix}$$

$$\hat{B}_1=\begin{bmatrix}100.8526\\ -1.0284\end{bmatrix},\ \hat{B}_2=\begin{bmatrix}99.2575\\ -0.7994\end{bmatrix}$$

$$\hat{C}_1=[-28.0202 \quad -180.8999]$$

$$\hat{C}_2=[-27.5806 \quad -179.3730]$$

The resulting fuzzy controller is:

$$E_\varepsilon\dot{\hat{x}}(t)=\sum_{i=1}^{2}\sum_{j=1}^{2}\hat{\mu}_i\hat{\mu}_j\hat{A}_{ij}(\varepsilon)\hat{x}(t)+\sum_{i=1}^{2}\hat{\mu}_i\hat{B}_{iy}(t)$$

$$u(t)=\sum_{i=1}^{2}\hat{\mu}_i\hat{C}_i\hat{x}(t)$$

Where:

$$\hat{\mu}_1=M_1(\hat{x}_1(t))$$

$$\hat{\mu}_2=M_2(\hat{x}_1(t))$$

Remark 3: For a sufficiently small ε, both non-fragile fuzzy state and output feedback controllers guarantee that the L_2-gain, γ is less than the prescribed value. The disturbance input signal, w (t) which was used during the simulation is shown in Fig. 4. The ratio of the regulated output energy to the disturbance input noise energy obtained by using the H_∞ fuzzy controllers with ε = 0.01 is shown in Fig. 5.

After 5 sec, the ratio of the regulated output energy to the disturbance input noise energy tends to a constant value which is about 0.032 for the state-feedback controller and 0.10 for the output feedback controller in case 1 and 0.12 in case 2.

Thus for the state-feedback controller where, $\gamma = \sqrt{0.032} = 0.178$ and for output feedback controller in case 1 where, $\gamma = \sqrt{0.10} = 0.316$ and in case 2 where, $\gamma = \sqrt{0.12} = 0.346$, all are less than the prescribed value 1. Finally, Table 1 shows the result of the performance index γ with different values of ε.

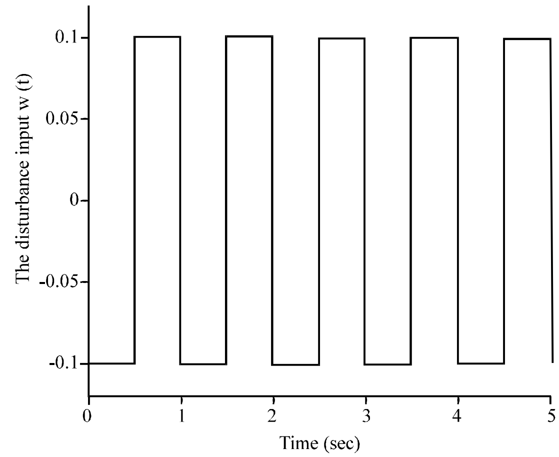

Fig. 4: The disturbance input noise, w (t)

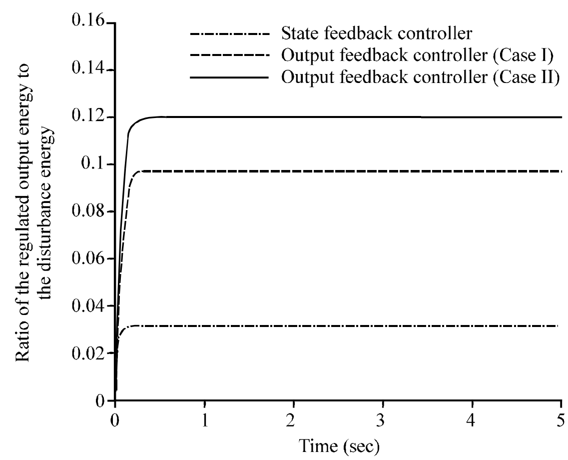

Fig. 5: The ratio of the regulated output energy to the disturbance noise energy

Table 1: The performance index γ of the system with different values of ε

ε	The performance index γ State-feedback	Output-feedback Case I	Case II
0.01	0.178	0.316	0.346
0.05	0.239	0.400	0.410
0.15	0.440	0.574	0.922
0.16	0.441	0.600	>1
0.28	0.500	0.989	>1
0.29	0.503	>1	>1
0.48	0.902	>1	>1
0.49	>1	>1	>1

CONCLUSION

This study has examined the problem of designing non-fragile fuzzy state and out-put feedback controllers for a TS fuzzy system with multiple time-scales. Sufficient conditions for the existence of non-fragile fuzzy controllers are derived in terms of a family of ε-independent linear matrix inequalities. The proposed approach does not involve the separation of states into slow and fast ones and it can be applied not only to standard but also to nonstandard multiple time-scale systems. A numerical simulation example has been presented to illustrate the effectiveness of the designs.

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