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A New Supervision-Based Adaptive Control for Pressure Process Rig to Overcome Bursting Phenomena under Low Persistently Excitation

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Abstract: A modified supervision algorithm for an indirect adaptive control of a pressure process is presented. The aim is to improve control performance under low persistently excitation that is common in the industrial process in which adaptive controllers are known fail to guarantee the boundedness of parameter estimates and the other closed-loop signals. The proposed supervision algorithm is designed to detect low sufficiently conditions quickly as possible, analyse monitoring signals using recursive computation and make a logic decision for updating the controller parameters or freezing the adaptation. The control performance is verified through experiments on a lab-scale pressure process rig and compared with the original adaptive controller.

Key words: Supervision, adaptive control, pressure process, bursting phenomena, closed loop, signal

INTRODUCTION

In industrial processes, the control systems may often have to cope with complex processes having significant uncertainties, varying parameters or operating conditions that change drastically. The challenge for control engineers is to develop control system techniques able to achieve good performance in terms of speed, accuracy and stability. During a few decades, adaptive control techniques have been applied to systems presenting slow time-varying dynamics or nonlinear behavior. In order to retain good performance at different operating conditions, an adaptive controller adjusts its parameters based on a linearized system model which is identified using a parameter estimation algorithm.

However, a major disadvantage of the conventional continuous adaptation techniques is their inability to perform satisfactorily mainly under low persistently excitation. In this case, undesirable transients, such as bursting phenomena may typically arise due to slow adaptation. Simulation studies and practical applications have shown that the boundedness of parameter estimates and the other closed-loop signals cannot be guaranteed by the original adaptive control algorithms. It has been investigated that control performance deteriorates proportionally with the size of the parametric uncertainty set (Tsakalis, 1996).

In order to improve reliability, safety and economy, a supervision function is required in the adaptive control of technical systems. In recent years, adaptive Switching Supervisory Control (SSC) have been introduced for tackling the absence of sufficient excitation. The idea of SSC is to construct a family of controllers and switch among candidate controllers managed by a supervision level. A great number of studies on the topic of SSC have appeared in the literature. Angeli and Mosca (2004), introduced integration of controller falsification and inference criteria in a new supervisory switching logic. This class of SSC has no prior information required and applicable to a wide class of linear and nonlinear plants. In the research of Alonso-Quesada *et al*. (2005), uncertainties in model dynamics are considered in designing multiple estimators. A supervisory quality index is used online to decide a pair of activated estimator-controller. Another estimator-based supervisory control is designed based on certainty equivalence and has shown its capability of overcoming limitations that are characteristic of conventional adaptive control algorithm (Hespanha *et al*., 2003). The signals to be monitored in SSC are defined as appropriate integral norms of the estimation errors in (Hespanha *et al*., 2003) and analysis of SSC scheme is carried out in the time domain where Lyapunov-like dissipationin equalities are used. The problems of stabilizing uncertain nonlinear systems in the presence of disturbances via switching supervisory control are also investigated by Vu *et al*. (2007). Ye proposed multiple model adaptive control of nonlinear systems in parametric-strict-feedback form

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and proved that the closed-loop switching system is global asymtotic stable (Ye, 2008). Another research of Medjaher *et al.* (2006) has shown a good control performance of a supervision system for a complex steam generator using application of a bond graph model based FDI approach.

Instead of using logic-based switching between controllers, another supervisory control algorithm deals with updating the model parameters or freezing adaptation based on current data and possibly the state of the supervisor automaton. The supervision algorithm updates controller parameters when a persistently excitation becomes available. A variety of simulations and practical studies have demonstrated that it can improve transient performance significantly and overcome the bursting phenomena. Comprehensive work concerning the design of a set of supervisory rules for an improved adaptive GPC and feedforward control is presented by Chen (2002). A supervisory scheme which prohibit adaptation during load disturbance, signal saturation and oscillation is demonstrated by Hagglund and Astrom (2000). Related approaches are also found using different methods, such as a discrete hybrid automaton (Pregelj *et al.*, 2007), an integration of auto-tuners and artificial intelligence techniques (Quek and Wahab, 2000) and fuzzy logic (Asep *et al.*, 1996).

In this study, a supervision-based control algorithm which consists of a basic adaptive controller, a less-excited-detection module, a monitoring signal module and a logic decision module, is presented. The supervisory algorithm is designed to guarantee stability of the pressure process and to avoid the bursting phenomena. The main contribution of this study is to present, a modified detection of low persistently exciting from Isermann (1997)'s research whereby prediction error, variance of prediction error and variances of estimated parameters are calculated recursively. Due to its less computing power, the proposed algorithm is suitable for controlling the pressure process which presenting fast dynamic response. Experiments are carried out in a lab-scale pressure process rig to verify the proposed algorithm. The performance of the supervision algorithm is finally evaluated experimentally showing, its ability to overcome bursting phenomena under a low persistently excited set point.

MATERIALS AND METHODS

In order to ensure the closed-loop stabilization, a relative adaptation dead-zone is usually included in estimation algorithm. Such a dead-zone switches off the controller parameter adaptation when the estimation error is sufficiently small. A different scheme of freezing the adaptation is to use the supervisory control as shown in Fig. 1. The main idea of the proposed method is that the

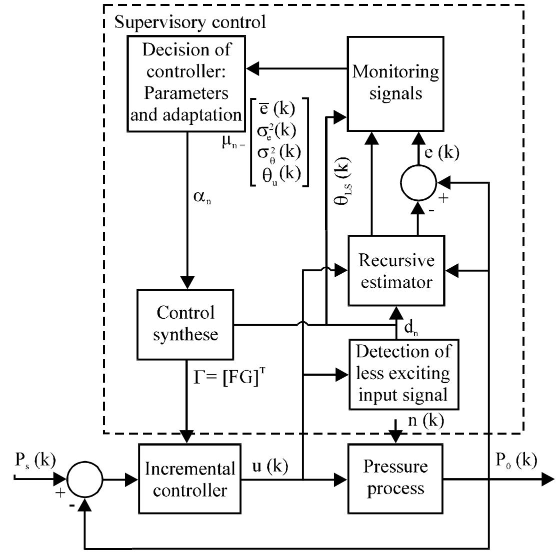

Fig. 1: Scheme of supervision-based adaptive control for pressure process

updating of new process model and new controller parameter is only performed when the data has a rich information about the process and the estimator has an accurate model. The excitation of input signal u (k) is verified at each time interval in order to enable the parameter estimation calculation d_n. The quality of process model is monitored in terms of recursive formula of mean and variance for parameter estimate and the prediction error. Moreover, the trend of prediction error signal is, also examined to decide using the current model parameter estimate or the previous one. A logic decision α_n enables the controller parameter calculation based on quality of the process model, otherwise the incremental controller uses the previous values. Finally, updating the new controller parameter Γ is performed by solving the diophantine equation and sent to the incremental controller for calculating the current control signal u (k).

The proposed supervisory control integrates the original adaptive controller and a supervision layer which comprises 5 subsystems:

Detection of less exciting input signal: The first subsystem is a detection of less exciting condition. This is a dynamical system whose input is the input of the process u (k) and whose output is the status of autocorrelation matrix $R_{n,k} \in \Re^{n \times n}$. The process input u (k) is said to be persistently exciting of order n if the following autocorrelation limit exists:

$$r_{uu,k}(\tau) = \lim_{k \to \infty} \frac{1}{k} \sum_{i=1}^{k} u(i)u(i-\tau) \quad (1)$$

And the matrix $R_{n,k}$:

$$R_{n,k} = \begin{bmatrix} r_{uu,k}(0) & r_{uu,k}(1) & \cdots & r_{uu,k}(n-1) \\ r_{uu,k}(-1) & r_{uu,k}(0) & \cdots & r_{uu,k}(n-2) \\ \vdots & \vdots & \ddots & \vdots \\ r_{uu,k}(1-n) & r_{uu,k}(2-n) & \cdots & r_{uu,k}(0) \end{bmatrix} \quad (2)$$

is nonsingular (Ljung, 1987). The argument k is used here to indicate the dependence of r_{uu} (τ) on time. The autocorrelation function at time k-1 can be written as:

$$r_{uu,k-1}(\tau) = \frac{1}{k-1}\sum_{i=1}^{k-1} u(i)u(i-\tau) \quad (3)$$

According to definitions of Eq. 1 and 3, the autocorrelation function at the current time k can be calculated based on the previous value of autocorrelation function for a new data of u (k). This leads to the recursive formula for updating the autocorrelation function:

$$r_{uu,k}(\tau) = \frac{1}{k}\left[(k-1)r_{uu,k-1}(\tau) + u(k)u(k-\tau)\right] \quad (4)$$

This subsystem will give the logical output d_n regarding to exciting condition of the input signal:

$$d_n = \begin{cases} 1, \text{ if } \left|R_{n,k}\right| > \varepsilon_1 \\ 0, \text{ otherwise} \end{cases} \quad (5)$$

The input signal u (k) is said to be persistently exciting if $d_n = 1$, otherwise u (k) less exciting.

Recursive estimator: In case that the input signal is sufficiently excited then the model parameters are estimated using Recursive Least Squares (RLS) algorithm described in the following form (Ljung, 1987):

$$P(k) = \lambda^{-1}P(k-1)\left[I - \frac{\varphi(k)\varphi^T(k)P(k-1)}{\lambda + \varphi^T(k)P(k-1)\varphi(k)}\right]$$
$$\hat{\theta}(k) = \hat{\theta}(k-1) + P(k)\varphi(k)\left(P_o(k) - \varphi^T(k)\hat{\theta}(k-1)\right) \quad (6)$$

RLS algorithm is especially suited for recursive parameter estimation due to small computational power and their robustness. In order to ensure that the covariance matrix P is positive definite in the RLS calculation and to make the algorithm more numerically robust, the U-D factorization technique is adopted.

The pressure P_o (k)$\in\Re^1$ is considered as a controlled variable. It is assumed that the pressure process has an Autoregressive Exogenuous (ARX) model in incremental form:

$$\bar{A}(z^{-1})P_o(k) = B(z^{-1})\Delta u(k-1) + \Delta n(k) \quad (7)$$

Where, $\bar{A}(z^{-1}) = \Delta A(z^{-1})$ and:

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_{na} z^{-na}$$
$$B(z^{-1}) = b_0 + b_1 z^{-1} + \ldots + a_{nb} z^{-nb} \quad (8)$$

Where:

u (k)$\in\Re^1$	=	Control signal
n (k)$\in\Re^1$	=	Sequence of uncorrelated noise with zero mean value
z^{-1}	=	The one-step delay operator
Δ = (1-z^{-1})	=	An integrator

It is assumed that polynomial degree of n_a and n_b is known. Separating the integrator term from the model, the pressure process can be written in a linear difference equation with constant parameters and as follow:

$$P_o(k) = a_1 P_o(k-1) - \cdots - a_{na} P_o(k-n_a) + b_0 u(k-1) + \cdots + b_{nb} u(k-n_b) + e(k) \quad (9)$$

Or in a vector form:

$$P_o(k) = \varphi^T(k)\theta + e(k)$$

where;

$$\varphi^T(k) = \left[-P_o(k-1), \cdots, -P_o(k-n_a)\ u(k-1), \cdots, u(k-n_b)\right]$$
$$\theta = \left[a_1, \cdots, a_{na}, b_0, \cdots, b_{nb}\right]^T \quad (10)$$

Based on the sampled data of input signal u (k) and output signal P_o (k), the model parameters θ are calculated at each sampling period using RLS formula in Eq. 6.

Monitoring signals: The aim of this function is to evaluate the performance of estimator results using recursive formulation in terms of following several quantities. Mean value of prediction error e (k):

$$\bar{e}(k) = E\{e(k)\} = \frac{1}{k}\sum_{i=1}^{k} e(i) \quad (11)$$

Or in a recursive form:

$$\bar{e}(k) = \frac{1}{k}\left[\sum_{i=1}^{k-1} e(i) + e(k)\right] = \frac{(k-1)\bar{e}(k-1) + e(k)}{k} \quad (12)$$

Variance value of the prediction error σ_e^2 (k):

$$\sigma_e^2(k) = E^2\{e(k) - E\{e(k)\}\} = \frac{\sum_{i=1}^{k}\left(e(i) - \bar{e}(k)\right)^2}{k-1} \quad (13)$$

The last term in the sum can be separated into:

$$(k-1)\sigma_e^2(k) = \sum_{i=1}^{k-1}\left(e(i) - \bar{e}(k)\right)^2 + \left(e(k) - \bar{e}(k)\right)^2$$

Define the recursive mean equation in Eq. 11 as:

$$\bar{e}(k) = \alpha\bar{e}(k-1) + \beta \quad (14)$$

Where:

$$\alpha = \frac{k-1}{k},\ \beta = \frac{e(k)}{k} \quad (15)$$

Then, the current variance value can be written as:

$$(k-1)\sigma_e^2(k) = \sum_{i=1}^{k-1}\left(e(i) - \left(\alpha\bar{e}(k-1) + \beta\right)\right)^2 + \left(e(k) - \bar{e}(k)\right)^2 \quad (16)$$

After simplifying the earliar equation as described in Appendix, the current value of variance is now only dependent on the previous variance value, the current and the previous of mean value and the current data e (k) as follows:

$$\sigma_e^2(k) = \frac{k-2}{k-1}\sigma_e^2(k-1) + \frac{\left(e(k)\bar{e}(k-1)\right)^2}{k^2} + \ldots + \frac{\left(e(k)\bar{e}(k)\right)^2}{k-1} \quad (17)$$

Variances of parameter estimation σ_θ^2 (k):

$$\sigma_\theta^2(k) = \sqrt{\sigma_{\theta_1}^2(k) + \ldots + \sigma_{\theta_n}^2} \quad (18)$$

Where:

$$\theta = \left[\theta_1 \cdots \theta_n\right]^T = \left[a_1 \cdots a_{na} \quad b_0 \cdots b_{nb}\right]^T$$

and;

$$\sigma_{\theta_i}^2(k) = \frac{k-2}{k-1}\sigma_{\theta_i}^2(k-1) + \frac{\left(\theta_i(k) - \bar{\theta}_i(k-1)\right)^2}{k^2} + \cdots + \frac{\left(\theta_i(k) - \bar{\theta}_i(k)\right)^2}{k-1}$$

Checking the quality of estimated model. The model parameter estimate is said to be an accurate model if the variance of parameter estimate is less than ε_2. The verification of the quality model will give a binary number as its output as follows:

$$q_n(k) = \begin{Bmatrix} 1, \text{ if } \sigma_\theta^2 < \varepsilon_2 \\ 0, \text{ otherwise} \end{Bmatrix} \quad (19)$$

Decision of controller parameter adaptation: The objective of this supervision is to make a logic decision for updating the controller parameters. Due to most of controller design based on the process model then the new controller parameters are determined when the estimator has an improvement in the quality of process model which is verified by the third subsystem of the supervisor.

The variance of model parameter estimation should be small enough not exceeding a threshold value ε_2 for several sampling steps, σ_e^2 (k-j)$\leq\varepsilon_2$ for j = 0, ..., M-1, where M is a window size containing the status of parameter estimation variance.

$$\alpha_n = \left\lceil \frac{q_n(k)q_n(k-1)\cdots q_n(k-M+1)}{M\,\text{bits}} \right\rceil \quad (20)$$

In case that there is a change of reference signal, disturbance or characteristic of the process then the controller parameters will be not updated until the variance of model parameter is less than ε_2 for M-1 steps of sampling periods ahead. Finally, the last step is the calculation of model poles and zeros. In order to avoid cancellation between the zeros of process and the poles of controller that can cause a large control signal, it is necessary to ensure that all poles and zeros of the process model have to be inside the unit circle. This leads to a decision of a new controller parameter set. Otherwise, the adaptive controller still uses the last set of controller parameters.

Control synthese: Instead of using two-degree of freedom structure, an incremental controller is selected to regulate the pressure process which shows fast transient response. An incremental control algorithm introduces a digital integrator into the loop with ability of tracking the steady state reference set point despite the presence of unmodelled disturbances.

The control signal u (k) is defined by following equation:

$$\Delta u(k) = -\frac{G\left(z^{-1}\right)}{F\left(z^{-1}\right)}\left(P_o(k) - P_s(k)\right) \quad (21)$$

Where, the controller parameters in polynomial F and G are determined by solving the diophantine equation as:

$$\bar{A}\left(z^{-1}\right)F\left(z^{-1}\right) + z^{-1}B\left(z^{-1}\right)G\left(z^{-1}\right) = T\left(z^{-1}\right) \quad (22)$$

With polynomial T specifies a set of closed loop poles and:

$$F(z^{-1}) = 1 + f_1 z^{-1} + \cdots + f_{nf} z^{-nf}$$
$$G(z^{-1}) = g_0 + g_1 z^{-1} + \cdots + g_{ng} z^{-ng} \qquad (23)$$

For a unique solution, polynomials A and B have no common zeroes and the deg (F) and deg (G) should be selected as $n_f = n_b$ and $n_g = n_a$.

As a resume of designing the proposed controller for pressure process, the supervisory control algorithm can be stated in the following way:

Step 1: Detection of less exciting input signal
check_autocorrelation_matrix.

Step 2: Estimating the model parameters
If (sufficiently_exciting)
estimate_ model_parameter_$\boldsymbol{\theta}_{LS}$(k);
else
no_estimation; no_adaptation;
goto step 6;
end

Step 3: Monitoring signals
If e (k) monoton_increase
no_update_process_model; break;
else
update_process_model; $\theta_s(k) \leftarrow \theta_{LS}(k)$
update_mean_of_prediction_error;
update_variance_of_prediction_error;
update_variance_of_theta;
end

Step 4: Decision of controller parameters adaptation
If (new_process_model)
if σ_e^2 (k-M)>ε_2 and σ_e^2 (k-j)≤ε_2
check_poles_and_zeros;
if $|z_i|<1$ and $|\bar{p}_i|<1$
new_controller_parameters; end
else
no_new_controller_parameters;
end
end

Step 5: Controller synthese
If (new_controller_parameters)
solve_diophantine_equation; end

Step 6: Repeat steps 1-5 for the next step of sampling period.

RESULTS AND DISCUSSION

The schematic diagram of the pressure process rig control is shown in Fig. 2. The gas comes from a pipeline supplied by a compressor into the orrifice through a control valve. The control task is to control the P_{out} outlet pressure by adjusting signal of the valve. The pressure measurement is realized by a differential pressure transmitter. The pressure process rig is connected to a PC computer through a Process Interface (feedback 38-200), 2 signal conditionings (V/I converter and I/V converter) and a Data Acquisition Card NI-PCI-6024E. The supervisory adaptive controller is programmed in C-MEX

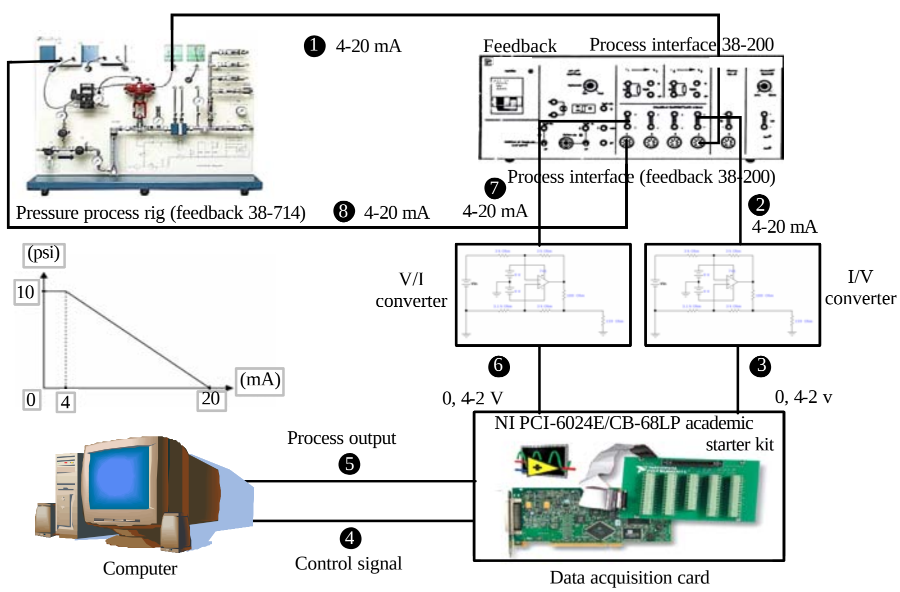

Fig. 2: Experimental set-up

language, compiled into the hex-code and runs in SIMULINK environment. The program gets the measured data through data acquisition card every 0.15 sec.

The determination of initial process model parameters is not critical as long as, it is performed in an open loop system identification. It is assumed that the characteristic of the pressure process is described by a second-order model, $n_a = 2$ and $n_b = 1$. The initial values of covariance matrix and parameter vector are $P(0) = I_4$, $\theta(0) = 0$ where I_4 denotes the 4th order identity matrix. The forgetting factor is chosen as $\lambda = 0.9$. During initialization phase, the input of pressure process is connected to a PRBS generator to generate 150 sample data then switched to a closed loop control scheme and taken over by the adaptive controller. The given set-point is kept constant during 3 time periods 22.5-90th, 91-360th and 361-500th sec.

The performance of adaptive controller without additional supervision functions for regulating the pressure process is shown in Fig. 3a, b. In the second time period of set-point, quite large variations of input and output signal appear but the controller is able to guide the pressure output back into the set-point with small variance. But after the second set-point change, the adaptive controller is not able to reduce the variations which indicate bursting in output and input of pressure process.

Figure 4a, b shows the control performance of the proposed supervisory adaptive controller. It is shown that

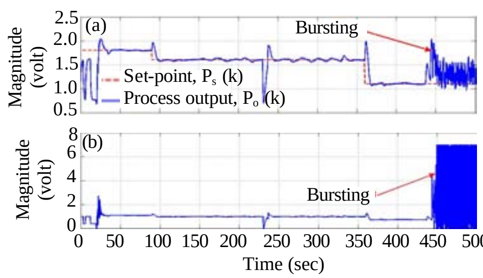

Fig. 3: The experimental results of orignal adaptive controller: a) Voltage of sensor output P_o (k); b) control signal u (k)

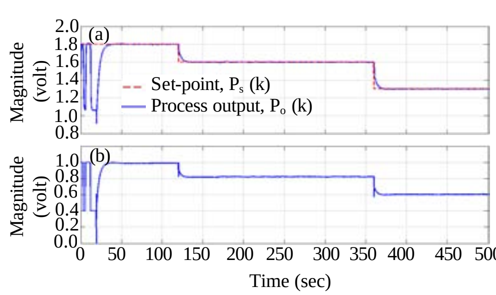

Fig. 4: The experimental results of supervision-based adaptive control: a) Voltage of sensor output P_o (k); b) Control signal u (k)

the proposed controller is able to provide a smooth response and also to avoid the bursting phenomena during long time period of set-point. The control algorithm is also capable of eliminating the steady state error despite of set point changing and also shows a rapid response guiding the output into the steady state condition.

After initialization phase of the process model identification, the estimator already gets an good model. The online estimator requires less time to freeze the calculation of new process model parameters as shown in Fig. 5. Since time of 35.7 sec, the supervision layer decides no updating of process model due to less exciting of input signal $|R_{n,k}|<\varepsilon_1$ (Fig. 6). The estimator gets an accurate process model at time 42.9 sec after checking the variance of model parameters less than ε_2 (Fig. 7). This will activate the bit status of updating new controller parameters. But

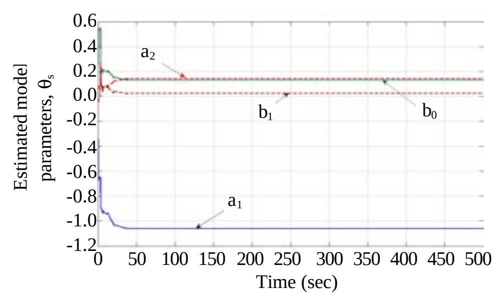

Fig. 5: Parameter estimate using supervisory control

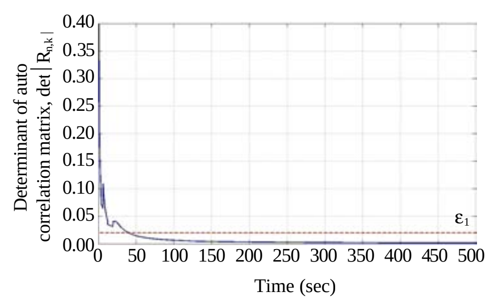

Fig. 6: Checking the excitation of input signal u (k)

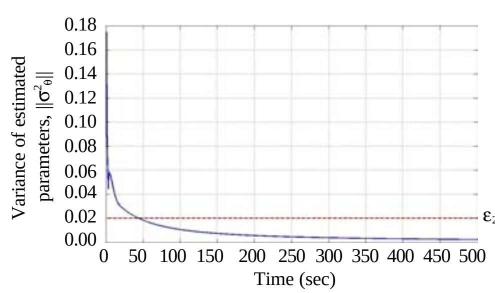

Fig. 7: Checking the quality of the process model using the varience of model parameters

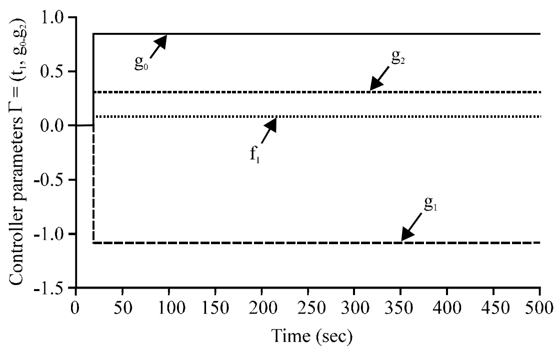

Fig. 8: Control parameters estimate using supervisory control

the requirement of sufficiently exciting of input signal is not fufilled at this time, therefore the controller still use the last set of controller parameters that are calculated after initialization phase as shown in Fig. 8.

CONCLUSION

In this research, a supervisory adaptive control has been proposed. It has been shown from experimental results that a better control performance for pressure process plant can be achieved using the proposed control algorithm compared to the original adaptive controller. The supervisory control is able to eliminate bursting phenomena during long time period of set point and may also serve as a safety factor of adaptive control. Due to recursive form of supervision functions, the computational effort will be increased by about 10%.

ACKNOWLEDGEMENT

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APPENDIX

Proof: A recursive form of variance equation:

$$\begin{aligned}
\sigma_e^2(k) &= \sum_{i=1}^{k-1}\frac{\left(e(i)-e(k-1)\right)^2}{k-1}+\sum_{i=1}^{k-1}\frac{e(i)\left(2\bar{e}(k-1)-2\alpha\bar{e}(k-1)\right)}{k-1}+\sum_{i=1}^{k-1}\frac{2\alpha\beta\bar{e}(k-1)}{k-1}+\\
&\quad \sum_{i=1}^{k-1}\frac{\bar{e}^2(k-1)-\left(\alpha^2-1\right)}{k-1}-\cdots-2\beta\sum_{i=1}^{k-1}\frac{e(i)}{k-1}+\sum_{i=1}^{k-1}\frac{\beta^2}{k-1}+\frac{\left(e(k)-\bar{e}(k)\right)^2}{k-1}\\
&= \frac{k-2}{k-1}\sigma_e^2(k-1)+\bar{e}(k-1)\left(2\bar{e}(k-1)-2\alpha\bar{e}(k-1)\right)+2\alpha\beta\bar{e}(k-1)+\left(\alpha^2-1\right)\bar{e}^2(k-1)-\cdots-2\beta\bar{e}(k-1)+\beta^2+\frac{\left(e(k)-\bar{e}(k)\right)^2}{k-1}\\
&= \frac{k-2}{k-1}\sigma_e^2(k-1)+2\bar{e}^2(k-1)-2\alpha\bar{e}^2(k-1)+2\alpha\beta\bar{e}(k-1)+\alpha^2\bar{e}^2(k-1)-\bar{e}^2(k-1)-2\beta\bar{e}(k-1)+\cdots+\beta^2+\frac{\left(e(k)-\bar{e}(k)\right)^2}{k-1}\\
&= \frac{k-2}{k-1}\sigma_e^2(k-1)+\bar{e}^2(k-1)\left(2-2\alpha+\alpha^2-1\right)+\bar{e}(k-1)\left(2\alpha\beta-2\beta\right)+\beta^2+\frac{\left(e(k)-\bar{e}(k)\right)^2}{k-1}\\
&= \frac{k-2}{k-1}\sigma_e^2(k-1)+\bar{e}^2(k-1)(\alpha-1)^2+\bar{e}(k-1)\left(2\alpha\beta-2\beta\right)+\beta^2+\frac{\left(e(k)-\bar{e}(k)\right)^2}{k-1}
\end{aligned} \tag{24}$$

Substitute Eq. 15 into Eq. 24 yields the recursive form of updating the variance of prediction error based the previous value of the variance of prediction error, the previous and current value of mean value of prediction error and the current value of prediction error:

$$\begin{aligned}
\sigma_e^2(k) &= \frac{k-2}{k-1}\sigma_e^2(k-1)+\bar{e}^2(k-1)\left(\frac{k-1}{k}-1\right)^2+\bar{e}(k-1)\left(2\frac{k-1}{k}\frac{e(k)}{k}-2\frac{e(k)}{k}\right)+\frac{e^2(k)}{k^2}+\frac{\left(e(k)-\bar{e}(k)\right)^2}{k-1}\\
&= \frac{k-2}{k-1}\sigma_e^2(k-1)+\frac{\bar{e}^2(k-1)-2\bar{e}(k-1)e(k)+e^2(k)}{k^2}+\frac{\left(e(k)-\bar{e}(k)\right)^2}{k-1}\\
&= \frac{k-2}{k-1}\sigma_e^2(k-1)+\frac{\left(\bar{e}(k-1)-e(k)\right)^2}{k^2}+\frac{\left(e(k)-\bar{e}(k)\right)^2}{k-1}
\end{aligned} \tag{25}$$

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