

## A New Modeling Approach for Queuing Vehicles in Front of Emergency Vehicle at a Traffic Intersection

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**Abstract:** One of the traffic problems in the transportation system is the emergency vehicles management. It is necessary to know the characteristics of the traffic flow on an intersection when an emergency vehicle is passing through on the road. This research is aimed to determine an appropriate model of the G/G/1 Model to represent the physical phenomena of the normal vehicle traffic on a single lane intersection in the presence of an emergency vehicle. The approach is performed by modeling the travel speed of the queue tail with the G/G/1 approximation. There are two scenarios when pre-emption signal is detected by the control system. The first scenario is during red signal and the second scenario is during green signal. The paper evaluates the G/G/1 Model developed by using Kraemer-Lagenbach-Belz, Kingman and Whitt approximation. A criterion of the G/G/1 Model approximations is to achieve the smallest Theil coefficient value of the queue tail travel speed. Theil coefficient indicates how close the speed of the model to the real speed. If the travel speed of the model approaches the real speed then Theil coefficient value is low or towards 0. Based on queuing theory, it can be constructed the traffic flow model at a traffic intersection in which the emergency vehicle is present. The experiment shows the appropriate G/G/1 Queuing Model for traffic signal intersection scenario is the Kingman approximation.

**Key words:** Emergency vehicle, traffic system, modeling, queuing, G/G/1, Theil coefficient

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### INTRODUCTION

Transportation plays a leading role in today's modern society. Currently, traffic congestion gets worse and becomes an increasingly critical problem, especially in major cities of Indonesia. Methods to increase the transport capacity of the existing transportation infrastructure have become an attention of many researchers around the world today). Many researchers propose Intelligent Transportation System (ITS) to solve the traffic problems. ITS is a synergy of information technology, real-time control and network communications.

The common problems in the traffic arrangements in modern big cities are to control traffic signals. Traffic signals can resolve conflicts between traffic flow at crossroads or intersections. One of the traffic problems is traffic management of Emergency Vehicles (EVs) such as fire trucks, ambulances and so forth. Traffic signals for handling EVs require a pre-emptive control. The pre-emptive control design provides the highest priority to Evs to cross the intersection. Many

researchers have examined the appropriate method for the pre-emptive control such as by Goel *et al.* (2012), Kuang and Xu (2012) and He *et al.* (2011). Two benefits associated with traffic signal pre-emption are as follow: first, response time/travel time for EV is shortened. Second, it improves the safety and reliability of vehicles receiving pre-emption right of way.

In order to keep the disturbance of EV travel time small, it is necessary to clear the jam or congestion of other vehicles in front of the EV. Research on the pre-emptive controlling by considering clearing the other vehicle congestion in front of the EV has been done by Wang *et al.* (2013). However, those researchers do not discuss how to clear the congestion of other vehicles in front of EVs in a the short time. Discharging the congestion is crucial if the road has only one lane. Prior research further about the congestion discharge methods, it is necessary to know the characteristics of the traffic flow on a single road when EVs are present.

Research by Sumaryo has analyzed the characteristics of traffic flow on a single road section

when the EV is present. Traffic flow conditions are modeled by using Queuing Model: M/M/1, M/G/1 and G/G/1. The performance measurement used in the paper is the average travel time of regular vehicles (normal traffic) across the road lane to with the condition before and after the presence of the EVs. The other performance measurement is the travel time of EVs to pass through the road. Then, the EV travel time is compared to the expected travel time of the EVs to pass through the road. The G/G/1 Model shows the characteristics of the normal vehicles travel time and the EVs travel time that are lower than those in the M/M/1 and the M/G/1 Model. In that research, the analysis is performed based on the results of the simulation experiments (based on event simulation). So that the G/G/1 Queuing Model is only characterized by the arrival process and the service time.

In contrast to what is done by Sumaryo, this study uses the G/G/1 analytical Model. G/G/1 Model does not have a closed form expression for the expected waiting time  $T_q$ . For G/G/1 Model, the total time in the system is determined by approximations. There are several G/G/1 Model approximations: Kreamer-Lagenbach-Belz (KLB) approximation, Kingman (K) approximation and Whitt (W) approximation. The contribution of this study is to determine the appropriate model approximations (KLB, K, W) for the G/G/1 Model in order to represent the physical phenomena of the normal vehicle traffic on a single lane intersection in the presence of EV. The physical phenomena are based on the research by Wang *et al.* (2013). There are two scenarios when pre-emption signal is detected by the observed control system. The first scenario is during red signal and the second scenario is during green signal. Criteria for the G/G/1 Model approximation is the Theil coefficient (Van Woensel and Vandaele, 2006) value of the queue tail travel speed. Theil coefficient indicate how close the speed of the model to the real speed.

## MATERIALS AND MEHODS

**The G/G/1 Queuing Models:** Research by Van Woensel and Vandaele (2006) has concluded that one single Queuing Model for representing the whole day of traffic flow is inadequate. It needs to use different Queuing Models for different periods of the day. Research by Van Woensel and Vandaele (2006) has also concluded that the M/M/1 Queuing Model is considered as a base case, due to its specific assumptions regarding the arrival and service processes in which it is not useful to describe the real-life situations. Relaxing the specification for the service process of the M/M/1 Queuing Model, it leads

to the M/G/1 Queuing Model (generally distributed service rate). Relaxing both assumptions for the arrival and service processes result in the G/G/1 Queuing Model (Van Woensel and Vandaele, 2007). The G/G/1 Queuing Model does not make any specific assumption on the interarrival and service time distributions (Buzacott and Shanthikumar, 1993).

The G/G/1 Queuing Model assumes both arrival time and service time following a general distribution with expected arrival time  $1/\lambda$  and its standard deviation  $\sigma_a$  and with expected service times  $1/\mu$  and its standard deviation  $\sigma_s$ . The G/G/1 Queuing Model has no closed form expression for the expected waiting time in the system  $T_q$ . The determination of the total time in the system  $T$  for the G/G/1 Queuing Models relies on approximations. In this research, the following are used: the Kreamer-Lagenbach-Belz, the Kingman and the Whitt approximation.

The expected total time in the system  $T$  equals to the sum of the waiting time (because of congestion)  $T_q$  and the service time  $T_p$ :

$$T = T_q + T_p, T_p = \frac{1}{\mu} \quad (1)$$

where,  $\mu$  is the service rate (veh/h), after the total time in the system  $T$  is known, the effective speed  $s$  can be obtained as follows:

$$s = \frac{1/C}{T} \quad (2)$$

where,  $1/C$  is the length of the road segment. From (Woensel, 2003) it follows that:

$$\lambda = E \cdot SN, \mu = C \cdot SN, \rho = \frac{\lambda}{\mu} \quad (3)$$

Where:

$\lambda$  = The arrival rate (veh/h)

$E$  = The traffic density (veh/km)

$C$  = The maximum traffic density (veh/km)

$SN$  = The nominal speed or the free flow speed (km/h)

$\rho$  = The traffic intensity

The value of  $T$  depends on the G/G/1 Queuing Model approximations.  $T$  equals to  $T_{KLB}$  if Kreamer-Lagenbach-Belz approximation is used to  $T_q$  if Kingman approximation and to  $T_w$  if Whitt approximation. The equations of expected total time in the system of G/G/1 Queuing Model approximations are shown in Table 1.

Table 1: Expected total time in the system

Approximations	Expected total time in the system
Kraemer-Lagenbach-Belz	$T_{KLB} = \frac{\rho}{(1-\rho)} \cdot \frac{(C_a^2 + C_s^2)}{2} \cdot \frac{1}{C \cdot SN} \cdot e^{\frac{-2(1-\rho)(C_a^2 + C_s^2)}{3\rho(C_a^2 + C_s^2)}} + \frac{1}{C \cdot SN}; C_a^2 \leq 1$ $T_{KLB} = \frac{\rho}{(1-\rho)} \cdot \frac{(C_a^2 + C_s^2)}{2} \cdot \frac{1}{C \cdot SN} \cdot e^{\frac{-(1-\rho)(C_a^2 + C_s^2)}{(1+\rho)(C_a^2 + C_s^2)}} + \frac{1}{C \cdot SN}; C_a^2 > 1$ <p>where, <math>C_a^2</math> = squared coefficient of variation of inter-arrival times; <math>C_s^2</math> = squared coefficient of variation of service time; <math>\rho = \lambda/\mu</math>; <math>\lambda^{-1}</math> = mean inter-arrival time; <math>\mu^{-1}</math> = mean service rate</p>
Kingman	$T_K = \left[ \frac{\rho(\sqrt{2(m+1)}-1)}{m(1-\rho)} \right] \frac{(C_a^2 + C_s^2)}{2} \cdot \frac{1}{C \cdot SN} + \frac{1}{C \cdot SN}$ <p><math>m</math> is the number of servers</p>
Whitt	$T_w = \phi \cdot \frac{(C_a^2 + C_s^2)}{2} \cdot \left( \frac{1}{C \cdot SN} \right) \cdot T_{qM/M/m} + \frac{1}{C \cdot SN}$ $\phi = \left( \frac{4(C_a^2 - C_s^2)}{4C_a^2 - 3C_s^2} \right) \cdot \phi_1 + \left( \frac{C_s^2}{4C_a^2 - 3C_s^2} \right) \cdot \psi, \text{ if } C_a^2 \geq C_s^2$ $\phi = \left( \frac{C_s^2 - C_a^2}{2C_a^2 - 2C_s^2} \right) \cdot \phi_2 + \left( \frac{C_s^2 + 3C_a^2}{2C_a^2 - 2C_s^2} \right) \cdot \psi, \text{ if } C_a^2 < C_s^2$ $\Psi = \begin{cases} 1 & \text{if } \frac{(C_a^2 + C_s^2)}{2} \geq 1 \\ \phi_4 \left( \frac{(C_a^2 + C_s^2)}{2} \right) & \text{if } 0 \leq \frac{(C_a^2 + C_s^2)}{2} \leq 1 \end{cases}$ $\phi_1 = 1 + \min \left\{ 0.24; (1-\rho)(m-1) \left( \frac{\sqrt{4+5m-2}}{16 \cdot m \cdot \rho} \right) \right\}$ $\phi_2 = \phi_3 \cdot e^{\frac{-2(1-\rho)}{3\rho}}$ $\phi_3 = 1 - 4 \cdot \min \left\{ 0.24; (1-\rho)(m-1) \left( \frac{\sqrt{4+5m-2}}{16 \cdot m \cdot \rho} \right) \right\}$ $\phi_4 = \min \left\{ 1, \frac{\phi_1 + \phi_2}{2} \right\}$ $T_{qM/M/m} = \xi \cdot \left( \frac{(m\rho)^m}{m!(1-\rho)} \right) \cdot \left( \frac{1}{m \cdot (1-\rho)} \right) \cdot \left( \frac{1}{\mu} \right)$ $\xi = \left[ \left( \frac{(m\rho)^m}{m!(1-\rho)} \right) + \sum_{t=0}^{m-1} \frac{(m\rho)^t}{t!} \right]^{-1}$

The Kraemer-Lagenbach-Belz (KLB) approximation behaves very well under a broad range of circumstances (Buzacott and Shanthikumar, 1993). The only drawback is that it can only be used for G/G/1 Queueing Models or single servers. Kingman approximation (K) or heavy-traffic approximation for the G/G/1 and G/G/m Queueing Model show that the expected time in the queue  $T_q$  is exponentially distributed as  $\rho \rightarrow 1$  (Whitt, 1993) (but it remains strictly less than one, preserving stability). The Kingman approximation is only useful in limited cases depending upon a specific setting of parameters (i.e.,  $\rho$  closer to 1). Whitt proposed another approximation that performs well for a broad range of circumstances and multiple servers (as opposed to the single server results

of the Kraemer-Lagenbach-Belz approximation) (Whitt, 1993). The basic idea of Whitt approximation is to start from the Kingman (heavy traffic) approximation but by including a correction factor  $\phi$ . This correction factor is a function of the traffic intensity  $\rho$ , the coefficient of variation of inter-arrival times and service times and the number of servers  $m$ .

**The physical phenomena of the normal traffic intersection with the presence of EV:** Research by Wang *et al.* (2013) is based on physical phenomena of the Queueing Model. The paper proposes that there is a device (on board unit) on the EV which always emits the

emergency pre-emption request en route. The software on the Road Side Unit (on the road intersection) processes the request and provides a priority green light through normal traffic operations for the emergency vehicle that approach the intersection. Let  $T_E$  represents the expected travel time EV from current position to the intersection and  $T_Q$  denotes the estimated clearance time of the normal vehicles (stop or in a continuous flow) in front of EV (Wang *et al.*, 2013). To ensure that EV gets through the intersection with nonstop and no speed decreasing and avoiding too much green loss at the same time, the relationship between  $T_E$  and  $T_Q$  has the following forms (Wang *et al.*, 2013):

$$T_E = T_Q + t_\epsilon \quad (4)$$

$$T_E = \frac{L_E}{V_E} \quad (5)$$

$$T_Q = \frac{L_Q}{V_Q} \quad (6)$$

where,  $t_\epsilon$  indicates the transitional period which consists of traffic signal switch time and a headway in a free flow condition.  $V_E$  indicates the expected speed of EV that can be obtained from observation and depends on the road conditions. The value of  $V_E$  is from 40 km h<sup>-1</sup> until 80 km h<sup>-1</sup>.  $L_E$  indicates the EV distance to the intersection when the pre-emption request is received by Road Side Unit. Research by Wang *et al.* (2013) conservatively hypothesize that all vehicles in front of EV are stopped, waiting in a queue. It is defined the queue tail travel speed  $V_Q$  of the vehicle from the start position until passing the stop line (Wang *et al.*, 2013). Let,  $L_Q$  be defined as the distance from stop line to the queue tail then (Wang *et al.*, 2013):

$$L_Q = \bar{d} \cdot N \quad (7)$$

Where:

$\bar{d}$  = The average spacing of a vehicle occupied in a queue

$N$  = The number of normal vehicle in front of EV

From the observation of the research (Wang *et al.*, 2013), the value of  $\bar{d}$  is 8 m. In the different location, the average spacing  $\bar{d}$  may vary by the type of the vehicle, driving behaviors and so on.

The real traffic field test data by Wang *et al.* (2013) is utilized to obtain the  $L_Q$  and  $V_Q$  relationship. Based on the Fig. 1, it results in the equation in the following form (Wang *et al.*, 2013):

$$V_Q = 1.0052\ln(L_Q) + 0.6131 \quad (8)$$

Equation 7 and 8 represent the scenario that the initial conditions of all vehicles in front of the EV are in a stop and wait position in a queue of the observed road. The signal light of the intersection at that initial condition is red.

Figure 2 shows the trajectory graph of the EV and queue tail of the normal vehicles that the initial condition is stopped (road is observed to have a red signal) if no pre-emptive signal phase. The beginning of the trajectory curve indicates the EV pre-emption request is received by the Signal Control System. Figure 2 also shows that the EV trajectory intersects the tail queue trajectory of normal vehicles in front of the EV. The EV must slow down so that the EV does not collide with normal vehicles in front of it.

Figure 3 shows a condition when if an EV pre-emption request is obtained at the time of the traffic signal is green. Thus, the queue tail trajectory curve of the vehicles in front of the EV is a straight line. The straight line indicates that the tail queue speed  $V_Q$  is constant. This is contrast to the previous case when the current signal is red and the initial conditions of a normal vehicle in front of the EV is in stop condition, then the value of  $V_Q$  is in accordance with Eq. 8.

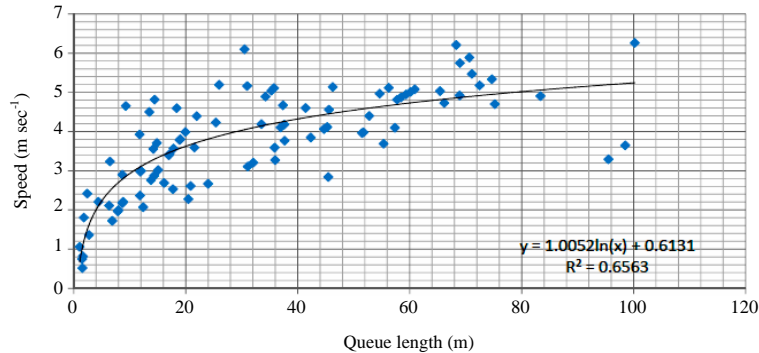


Fig. 1:  $L_Q$  and  $V_Q$  relationship (Wang *et al.*, 2013)

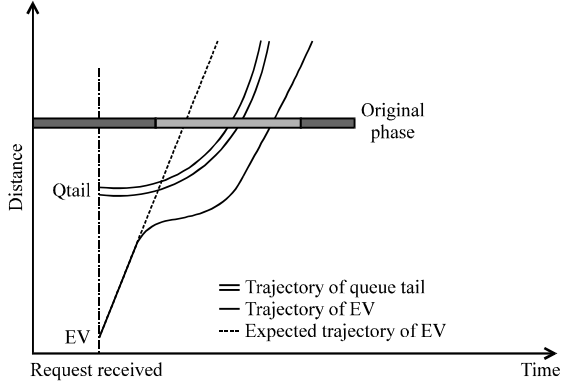


Fig. 2: Queue tail and EV trajectory of during a red signal scenario (Wang *et al.*, 2013)

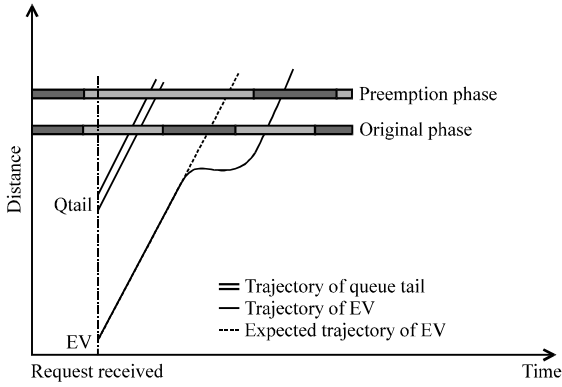


Fig. 3: Queue tail and EV trajectory of during a green signal scenario (edited) (Wang *et al.*, 2013)

From the previous discussions, it can be concluded that the relationship between  $V_{Qq}$ ,  $L_{Qq}$  and  $T_{Qq}$  are needed to be considered for queue modeling for the traffic flow in front of the EV.

### Modeling

**Scenario of modeling:** Equation 7 and 8 are used to model the physical phenomena of the normal traffic in front of the EV at the condition of red traffic signal and the pre-emptive signal is detected. The model can then be approximated by the following equation:

$$V_{Qq} = C_1 \ln(L_{Qq}) + C_2 \quad C_1, C_2: \text{constants} \quad (9)$$

$$L_{Qq} = \bar{d} \cdot N_q \quad (10)$$

Where:

$\bar{d}$  = The 8 m, according to experiment (Wang *et al.*, 2013)

$L_{q_q}$  = The distance from stop line to queue tail of the model

$V_{q_q}$  = The tail queue speed in front of the emergency vehicle of the model

$N_q$  = Amount of normal vehicles in the queue in front of the EV of the model

Meanwhile,  $N_q$  is defined according to the Little equation:

$$N_q = \lambda \cdot T \quad (11)$$

The equation states that the average number of customers in a queuing system equals to the product the average arrival rate of customers into the system  $\lambda$  with the average time customers in the system  $T$ . The little equation is generally applicable and does not refer to a specific distribution for the service time distribution or arrival distribution. Also, it does not refer to a certain number of servers and queue discipline in the system. The value of  $\lambda$  is in accordance with Eq. 3. The value of  $T$  is  $T_{KLB}$  for KLB approximation is  $T_K$  for K approximation and is  $T_W$  for W approximation.

For example, modeling the procedure with the G/G/1 K (Kingman approximation) is shown in the following equations:

$$N_q = \lambda \cdot T = \lambda \cdot T_K$$

$$T_K = \left[ \frac{\rho(\sqrt{2(m+1)}-1)}{M(1-\rho)} \right] \frac{(C_a^2 + C_s^2)}{2} \cdot \frac{1}{C \cdot SN} + \frac{1}{C \cdot SN}$$

$$L_{Qq} = \bar{d} \cdot N_q = \bar{d} \cdot \lambda \cdot T_K$$

$$V_{Qq} = C_1 \ln(L_{Qq}) + C_2 = C_1 \ln(\bar{d} \cdot \lambda \cdot T_K) + C_2$$

$$C_1 = 1.0052, C_2 = 0.6131$$

$$\lambda = E \cdot SN, \mu = C \cdot SN, SN = 40 \text{ km h}^{-1},$$

$$E = [0:C], C = 70 \text{ veh km}^{-1}$$

$$m = 1, \rho = \left( \frac{\lambda}{\mu} \right) = \frac{E}{C}, C_a = 0, C_s = 2$$

Figure 4 shows the graph of G/G/1 K Queuing Model assuming the sparameters  $C_1 = 1.0052$ ,  $C_2 = 0.6131$ ,  $C_a = 0$ ,  $C_s = 2$ ,  $C = 70 \text{ veh km}^{-1}$ ,  $SN = 40 \text{ km h}^{-1}$ . The shape of the curve in the figure shows the curve of similar shape at Fig. 1 or Eq. 8. The curve shows the following relationship if  $L_{q_q}$  increases;  $V_{Qq}$  also increases logarithmically. From this curve, it is difficult to determine how close the value  $V_{Qq}$  to the value  $V_Q$  (Fig. 1).

The physical phenomena of the normal traffic in front of the EV in condition green traffic signal, when detected pre-emptive signal are shown in Fig. 3. At Fig. 3,

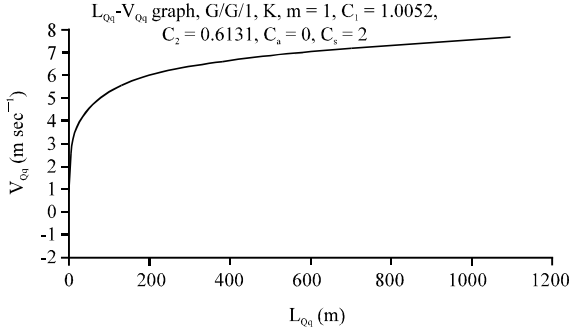


Fig. 4:  $L_{Qq}$ - $V_{Qq}$  graph during red traffic signal when pre-emptive signal is detected with G/G/1 K Queueing Model using  $C_1 = 1.0052$ ,  $C_2 = 0.6131$ ,  $C_a = 0$ ,  $C_s = 2$ ,  $C = 70 \text{ veh km}^{-1}$ ,  $SN = 40 \text{ km h}^{-1}$

it is shown that the speed  $V_Q$  is constant. Similar to the Fig. 3, for Queueing Model, the value of  $V_{Qq}$  is also constant.

**The validation of the model:** The reliability model of the Queueing Model depends on how close the results of the model to the real conditions. The process of determining whether the Queueing Model of the queue tail travel speed of vehicles in front of the EV is quite close to the queue tail travel speed in real conditions is achieved through the validation of the model (Van Woensel and Vandaele, 2006). That process is an iterative process that involves the calibration parameters model, comparing model output with real conditions and improved the models to achieve acceptable accuracy.

For the validation, it needs a tool. Some commonly used tools are: root mean square error, root mean square percentage error, residual variance and so on. For example, the root mean square error (rms) has a major drawback, namely squaring error means an emphasis on the magnitude of the error (Barcelo and Casas, 2005). It is very useful if the tool can measure the weight of two things: the amount of error and a basis for comparison between the model and the real conditions (Barcelo and Casas, 2005).

Evaluation or validation methodology in this research is carried out by using the Theil coefficient (Van Woensel and Vandaele, 2006). The coefficients can show how similar the Queueing Model travel speed with the real travel speed or observed travel speed. Theil coefficient of Queueing Model speed  $V_{Qq}$  and observed speed  $V_Q$  is defined as:

$$\text{Theil}_{V_{Qq}, V_Q} = \frac{\sqrt{\sum(V_{Qq} - V_Q)^2}}{\sqrt{\frac{\sum V_{Qq}^2}{n} + \frac{\sum V_Q^2}{n}}} \quad (12)$$

where, n is the number of samples taken. Their coefficients are obtained by several experiments with setting the parameters of the Queueing Model.

**Modeling algorithm:** As described in the introduction section that the purpose of modeling in this research is to determine the appropriate model of the G/G/1 Model (KLB, K, W) to represent the physical phenomena of the normal vehicle traffic on a single lane intersection in the presence of the emergency vehicle. Modeling is approached by G/G/1 Queueing Model of the travel speed of the tail queue of the physical condition  $V_Q$ . There are two scenarios when pre-emption signal is detected by the control system. The first scenario is during red signal and the second scenario is during green signal.

The tail queue speed of the physical condition  $V_Q$  at the red traffic signal scenario is based on the Eq. 8 which is the simplification of the data field observations conducted by the research (Wang *et al.*, 2013). The model equation has a similar form to Eq. 8 which is a natural logarithmic equation. First, it computes  $N_q$  and then, it computes  $V_{Qq}$  (Fig. 5).

**Modeling algorithm:**

Input:  $T = T_{KLB}, T_K, T_W$

Output: determine the appropriate model of the G/G/1 Model to represent the physical phenomena of the normal vehicle traffic on a single lane intersection with the presence of the emergency vehicle.

1. Check special circumstance
2. If the traffic signal is red and the pre-emptive signal is detected then
3. Compute:  $N_q = \lambda \cdot T \cdot T = T_{KLB}$  (KLB approximation)  
 $T = T_K$  (Kingman approximation)  
 $T = T_W$  (Whitt approximation)
4. Compute:  $L_{Qq} = \bar{d} \cdot N_q$
5. Compute:  $V_{Qq} = C_1 \ln(L_{Qq}) + C_2$
6. else
7. If the traffic signal is green and the pre-emptive signal is detected then
8. Compute:  $N_q = \lambda \cdot T \cdot T = T_{KLB}$  (KLB approximation)  
 $T = T_K$  (Kingman approximation)  
 $T = T_W$  (Whitt approximation)
9. Compute:  $L_{Qq} = \bar{d} \cdot N_q$
10. Compute:  $V_{Qq} = \text{constant}$
11. end
12. Compute:

$$\text{Theil}_{V_{Qq}, V_Q} = \frac{\sqrt{\sum(V_{Qq} - V_Q)^2}}{\sqrt{\frac{\sum V_{Qq}^2}{n} + \frac{\sum V_Q^2}{n}}}$$

For green traffic signal scenario, the value of  $V_Q$  is constant. It is based on observations conducted by researchers (Wang *et al.*, 2013) shown in Fig. 3. In the same way with red signal scenarios, modeling is performed by computing  $N_q$ ,  $L_{Qq}$  and  $V_{Qq}$ .

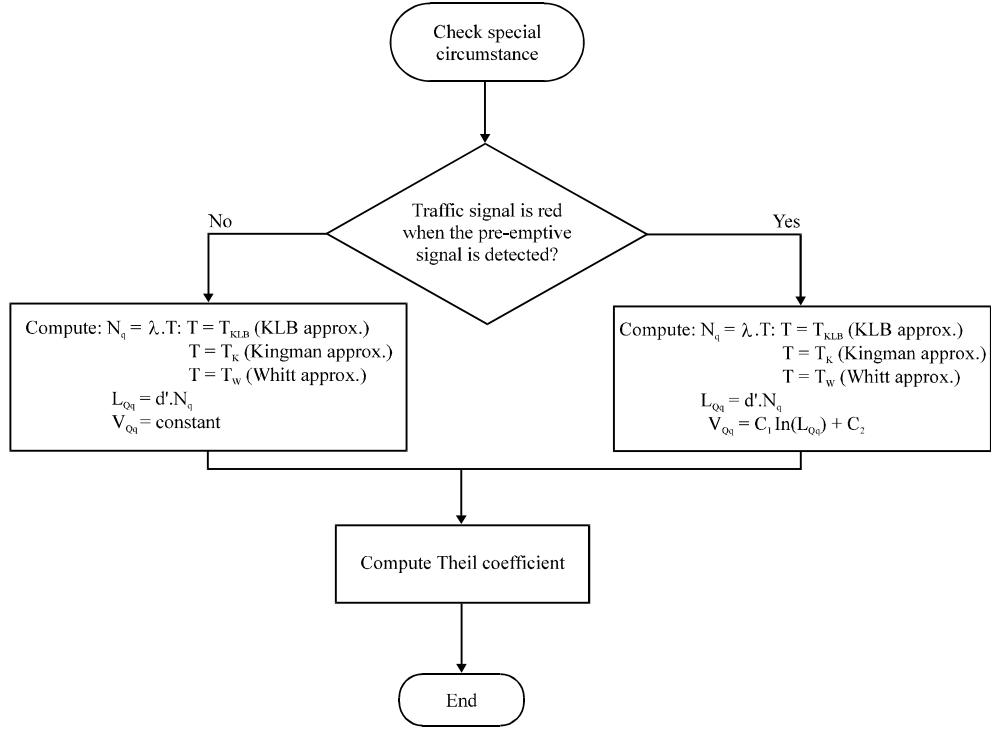


Fig. 5: Flow chart of modeling algorithm

Criteria of the G/G/1 Model approximations (with the approximations: KLB, K, W) is to have the value of queue tail speed value  $V_{Qq}$  closest to the value of  $V_Q$ . A criterion is obtained by validating or testing Theil coefficient. In the tests are carried out by changing the parameters in the G/G/1 Queue Model.

## RESULTS AND DISCUSSION

If the Queueing Model speed approaches the observed speed, Theil coefficient value becomes lower or goes to 0. Otherwise, the Theil coefficient value goes to 1 (Van Woensel and Vandaele, 2006). If the result of the model indicates the opposite direction to the value of the real condition, then the Theil coefficient leads to value  $>1$  (Barcelo and Casas, 2005).

The G/G/1 Model used in this experiment is characterized by the parameters: the arrival process, service times and First In First Out (FIFO) queueing schemes. The arrival process is characterized by the arrival rate  $\lambda$  and variance (or with squared coefficient of variation of inter-arrival times  $C_a^2$ ). The service time is characterized by two parameters which are: mean service time ( $\mu^{-1}$ ) and variance (or with squared coefficient of variation of service time  $C_s^2$ ).

Table 2 shows the Theil coefficient values from some experiments in the scenario of red traffic signal. This experiments conducted by using the maximum traffic density  $C$  is  $70 \text{ veh km}^{-1}$ . The nominal speed or free flow speed  $SN$  is  $40 \text{ km h}^{-1}$ . The value of  $SN$  is referred to the research by Wang *et al.* (2013) while the value of  $C$  is referred to the research by Vandaele *et al.* (2000). The model is tested by changing the parameters:  $\lambda = E \cdot SN$ ,  $E = [0:C]$ ,  $\mu = C \cdot SN$ ,  $C_a^2 = 0$  and  $C_s^2 = 4$ ,  $C_a^2 = 0$  and  $C_s^2 = 1$ ,  $C_a^2 = 0.25$  and  $C_s^2 = 0.25$ ,  $C_a^2 = 1$  and  $C_s^2 = 1$ ,  $C_a^2 = 1$  and  $C_s^2 = 0$ ,  $C_a^2 = 4$  and  $C_s^2 = 0$ . The number of samples  $n$  is 69.

As there is no specific method for determining the parameters  $C_a$ ,  $C_s$  so that the steps are by “trial and error” some of the parameter values and then by testing the Theil coefficient. From the experiment results (with  $C_1 = 1.0052$ ,  $C_2 = 0.6131$ ), the Theil coefficient value for Kingman’s approximation is lower than the value of KLB approximation and Whitt approximation. Table 2 shows the influenced of the combination values of  $C_a$  and  $C_s$  to the Theil coefficient.

Figure 6 and 7 show the  $L_Q - V_Q$  graph in the scenario of red traffic signal when pre-emptive signal is detected. Both figures show quite clearly that the G/G/1 W is not appropriate to represent the physical phenomena (Eq. 8) compared to the G/G/1 K and G/G/1 KLB. Figure 6 and 7

Table 2: The Theil coefficient with  $C = 70 \text{ veh km}^{-1}$ ,  $SN = 40 \text{ km h}^{-1}$

		$C_1 = 1.0052, C_2 = 0.6131$					
Coefficients	$C_a^2 = 0, C_s^2 = 4$	$C_a^2 = 0, C_s^2 = 1$	$C_a^2 = 0.25, C_s^2 = 0.25$	$C_a^2 = 1, C_s^2 = 1$	$C_a^2 = 1, C_s^2 = 0$	$C_a^2 = 4, C_s^2 = 0$	
KLB	0.1919	0.2665	0.2944	0.2096	0.2457	0.2145	
K	0.1777	0.2457	0.2796	0.2096	0.2457	0.1777	
W	0.3650	0.4252	0.4253	0.4340	0.3654	0.3649	

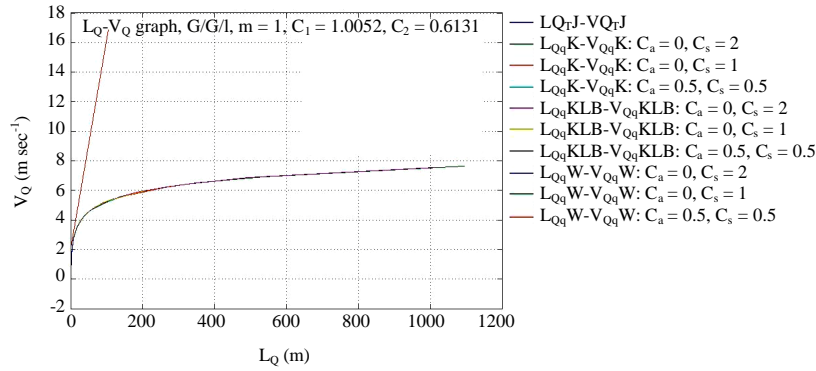


Fig. 6:  $L_Q$ - $V_Q$  graph during red traffic signal when pre-emptive signal is detected with G/G/1 Queueing Model using  $C_1 = 1.0052$ ,  $C_2 = 0.6131$ ,  $C_a = 0$  and  $C_s = 2$ ,  $C_a = 0$  and  $C_s = 1$ ,  $C_a = 0.5$  and  $C_s = 0.5$ ,  $C = 70 \text{ veh km}^{-1}$ ,  $SN = 40 \text{ km h}^{-1}$

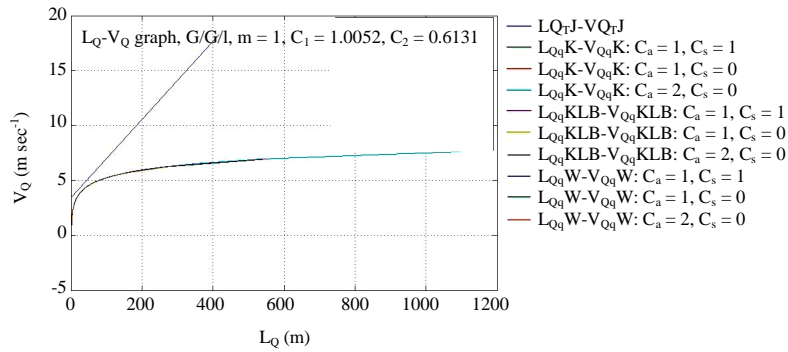


Fig. 7:  $L_Q$ - $V_Q$  graph during red traffic signal when pre-emptive signal is detected with G/G/1 Queueing Model using  $C_1 = 1.0052$ ,  $C_2 = 0.6131$ ,  $C_a = 1$  and  $C_s = 1$ ,  $C_a = 1$  and  $C_s = 0$ ,  $C_a = 2$  and  $C_s = 0$ ,  $C = 70 \text{ veh km}^{-1}$ ,  $SN = 40 \text{ km h}^{-1}$

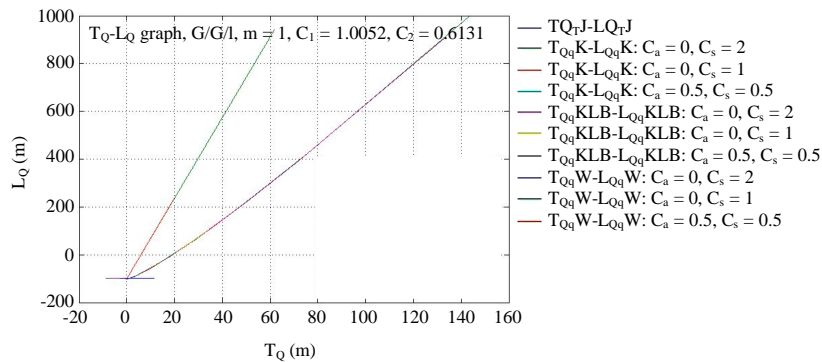


Fig. 8:  $T_Q$ - $L_Q$  (trajectory of queue tail) graph during a red signal when pre-emptive signal is detected with G/G/1 Queueing Model using  $C_1 = 1.0052$ ,  $C_2 = 0.6131$ ,  $C_a = 0$  and  $C_s = 2$ ,  $C_a = 0$  and  $C_s = 1$ ,  $C_a = 0.5$  and  $C_s = 0.5$ ,  $C = 70 \text{ veh km}^{-1}$ ,  $SN = 40 \text{ km h}^{-1}$



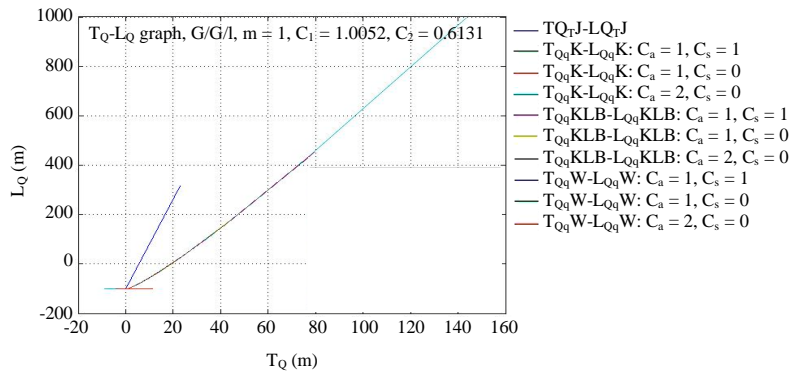


Fig. 9:  $T_Q-L_Q$  (trajectory of queue tail) graph during a red signal when pre-emptive signal is detected with G/G/1 Queuing Model using  $C_1 = 1.0052$ ,  $C_2 = 0.6131$ ,  $C_a = 1$  and  $C_s = 1$ ,  $C_a = 1$  and  $C_s = 0$ ,  $C_a = 2$  and  $C_s = 0$ ,  $C = 70 \text{ veh km}^{-1}$ ,  $SN = 40 \text{ km h}^{-1}$

show that it is quite difficult to determine how close the G/G/1 K and G/G/1 KLB approximations to the physical phenomena (Eq. 8).

Figure 8 and 9 show the  $T_Q-L_Q$  graph (trajectory of the queue tail) with a red signal scenario. Assume that the tail queue is 100 m from the stop line. Both figures also show quite clearly that the G/G/1 W is not appropriate to represent the physical phenomena (Fig. 2) compared to the G/G/1 K and G/G/1 KLB. Figure 8 and 9 show that it is quite difficult to determine how close the G/G/1 K and G/G/1 KLB approximations to the physical phenomena (Fig. 2 and TQTJ-LQTJ graph according to Eq. 6).

During green signal scenario (based on the research), it is obtained that the speed of normal vehicles in front of the EV has a constant value  $V_Q$  (Fig. 3). As discussed previously, if the speed of the queue model  $V_{qq}$  is made constant then the speed is independent of the G/G/1 Approximation Model. Therefore, when it is assumed  $V_{qq} = V_Q$ , the Theil coefficient is equal to 0. Conversely, if it is assumed differently, it generates a certain Theil coefficient.

### CONCLUSION

This study aimed to determine the appropriate model approximations for the G/G/1 Model in order to represent the physical phenomena of the normal vehicle traffic on a single lane intersection in the presence of emergency vehicle. The approach was performed by modeling the travel speed of the queue tail with the G/G/1 approximations: Kraemer-Lagenbach-Belz, Kingman and Whitt Approximation. A criterion of the G/G/1 Model approximations was to have the smallest Theil coefficient

value of the queue tail travel speed. From the experiment, it can be concluded that G/G/1 Kingman approximation provided the appropriate model to represent the physical phenomena of the normal vehicle traffic on a single lane intersection with the presence of the emergency vehicle.

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