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Pole Placement Based on Derivative of States

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Abstract: State feedback is one of the important concepts in control theory. There are well defined methods like Ackerman formula and Bass-Gura formula to find required gain matrix (K) which place close loop poles at the desired location. Here, instead of states, derivative of states are used in order to find the suitable control law. Effectiveness of method is shown by simulation.

INTRODUCTION

When input is applied to a system, output comes out. Sometimes output is not as good as desired. possible reasons may be slowness of response, stability problem or steady state errors (Chen, 2014; Ogata, 2010). In these cases, control is applied in order to make the response better. Figure 1, shows a general schematic of a system.

Here, there are m inputs and n outputs. H is a set of differential equations (ODE) which govern the output response. When system response is not desired, a control system must be added to the original system in order to make the overall response better. In order to do this, two control techniques can be used:

- Feed forward control
- Feed back control

It has been shown that feedback control has advantages over feed forward control [x]. Control problems can be divided into 2 groups (Chen, 2014; Ogata, 2010):

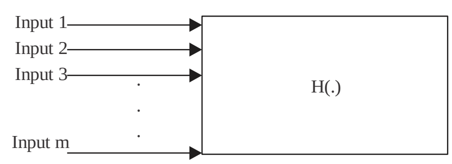

Fig. 1: General schematic of a Multi Input Multi Output system (MIMO)

- Tracking problems
- Regulation problems

In tracking problems, goal is to follow a reference with minimal error. In regulation problems, system output must be keep at the desired level despite of disturbance and input changes.

In order to design a controller, either Laplace transform based methods or state space based methods can be used.

A lot of work has been done on pole placement problem by using state feedback (Valasek and Olgac, 1999; Tuel, 1966). In this study, pole placement problem is solved by derivative of state feedback.

State derivative based feedback control law: Assume a Linear Time Invariant (LTI) system given by Eq. 1:

$$\dot{x} = Ax(t) + Bu(t) \tag{1}$$

$A \in IR^{n\times n}$, $x(t) \in IR^n$, $B \in IR^n$ and $u(t) \in IR$. x(t) is control input. Assume pair (A, B) is state vector and u(t) controllable, i.e., controllability matrix is full rank:

$$u(t) = -K\dot{x}(t) \tag{2}$$

Equation 2 uses derivative of states in order to produce control signal. After putting (Eq. 2) in (Eq. 1), following equation is obtained:

$$\dot{x} = (I + BK)^{-1} Ax(t) \tag{3}$$

Here, problem is to find gain matrix (K) such that close loop poles (Eigenvalues of $(I+BK)^{-1}A$) are desired set Δ_1, given by $\Delta_1 = \{\lambda_1, \lambda_2, ..., \lambda_n\}$. In order to solve the problem, following theorem is used.

Theorem 1: Assume A is an invertible matrix. Matrix A has eigen values $\{\lambda_1, \lambda_2, ..., \lambda_n\}$, if and only if eigen values of matrix A^{-1} are $\{\lambda^{-1}_1, \lambda^{-1}_2, ..., \lambda^{-1}_n\}$ (Strang, 2006). Inverse of $(I+BK)^{-1}A$ is found first:

$$\left((I+BK)^{-1} A\right)^{-1} = A^{-1}(I+BK) = A^{-1}+A^{-1}BK = A_{new}+B_{new}K \tag{4}$$

where, $A_{new} = A^{-1}$ and $B_{new} = A^{-1}B$. Assume a dynamical system as Eq. 5:

$$\dot{q} = A_{new}q + B_{new}W(t) \tag{5}$$

W(t) is the control input for this new system. Assume W(t) is chosen as:

$$W(t) = -Kq(t) \tag{6}$$

using this control law Eq. 5 can be write as:

$$\dot{q} = (A_{new} - B_{new}K)q(t) \tag{7}$$

If gain matrix K is choosen such that eigen values of $(A_{new}-B_{new}K)$ are $\Delta_2 = \Delta^{-1}_1 = \{\lambda^{-1}_1, \lambda^{-1}_2, ..., \lambda^{-1}_n\}$ then gain matrix K puts the eigenvalues of system in Eq. 1 at $\Delta_1 = \{\lambda_1, \lambda_2, ..., \lambda_n\}$.

So, problem of finding gain matrix K for derivative of state feedback control law in Eq. 2 can be solved by converting the problem to a new system of Eq. 5 and using a state feedback control law as Eq. 6.

MATERIALS AND METHODS

Method for determining gain matrix K: In order to solve the state feedback control problem for new system of Eq. 5, (pole placement) there are well known methods like [x].

- Ackermann method
- Bass-Gura method

These methods are studied with examples in (Chen, 2014). In this study, Ackerman method is used. Assume that:

$$\begin{aligned}\alpha(s) &= (s-\lambda_1^{-1})(s-\lambda 2-1,...,s-\lambda n-1 = \\ &sn+an-1sn-1+,...+\alpha_0\end{aligned} \tag{8}$$

Ackerman has shown [x] gain matrix K which places eigen values of close loop system in roots of α(s), i.e., $\{\lambda^{-1}_1, \lambda^{-1}_2, ..., \lambda^{-1}_n\}$ is given by:

$$K = q_n^T \alpha(A) \tag{9}$$

where, $q^T_n = [0, 0, ..., 0, 1]\varphi^{-1}_c$ and $\varphi^{-1}_c = [B_{new}, A_{new}, B_{new}, ..., A^{n-1}_{new}B_{new}]$ is called controllability matrix.

An example: In order to show effectiveness of proposed method, an example is given. Assume the circuit shown in Fig. 2. State space model is given by:

$$\begin{pmatrix} \dot{i} \\ \dot{V}_1 \\ \dot{V}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-1}{L} \\ 0 & \frac{-1}{R_1C_1} & \frac{1}{R_1C_1} \\ \frac{1}{c_2} & \frac{1}{R_1C_2} & \frac{-1}{R_1C_2} \end{pmatrix} \begin{pmatrix} i \\ V_1 \\ V_2 \end{pmatrix} + \begin{pmatrix} \frac{1}{L} \\ 0 \\ 0 \end{pmatrix} V_{in} \tag{10}$$

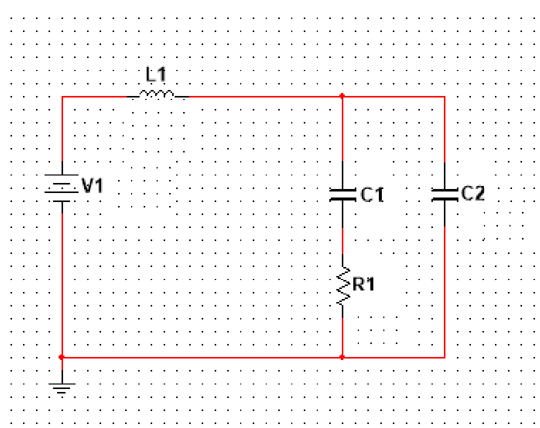

Fig. 2: Circuit for illustrative example

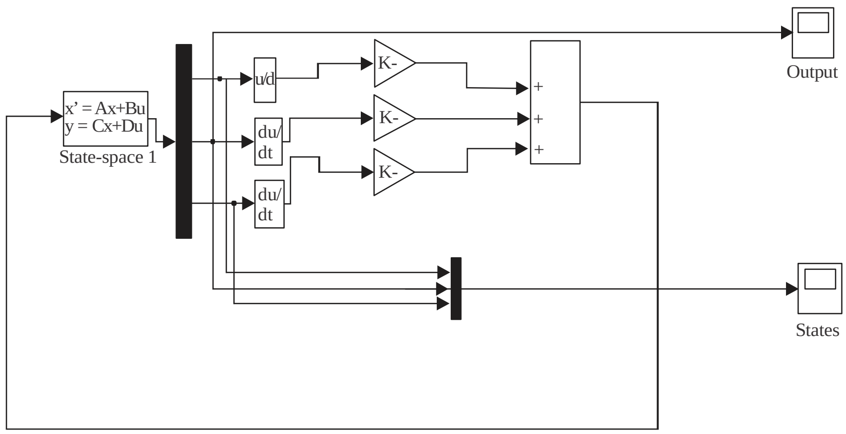

Fig. 3: Simulink diagram of studied example

Where:

$[i, V_1, V_2]^T$ = State vector
I = Inductor current
V_1 = Capacitor
C_1 = Voltage
V_2 = Capacitor
C_2 = Voltage
V_{in} = Assumed as control input

For simplicity, we take L_1 = 1H, $C_1 = 1_F$, C_2 = 1F and R_1 = 1Ω. With this values Eq. 10 can be rewritten as:

$$\begin{pmatrix} \dot{i} \\ \dot{V}_1 \\ \dot{V}_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} i \\ V_1 \\ V_2 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} V_{in} \quad (11)$$

Equation 11 is controllable and has poles at -1.755, -0.123±0.745j. Assume Δ_1, i.e., set of desired eigen values, is given by:

$$\Delta_1 = \{-3, -4, -5\} \quad (12)$$

after applying the aforementioned procedure K = [10 33 26]. So:

$$V_{in} = -[10\ 33\ 26] \begin{pmatrix} \dot{i} \\ \dot{V}_1 \\ \dot{V}_2 \end{pmatrix} = -10\dot{i} - 33\dot{V}_1 - 26\dot{V}_2$$

is the desired control law.

RESULTS AND DISCUSSION

Simulation: In order to simulate the system MATLAB®/ Simulink® has used. MATLAB® has great variety of tools in order to simulate dynamical systems. Figure 3 shows simulink diagram of the example studied before.

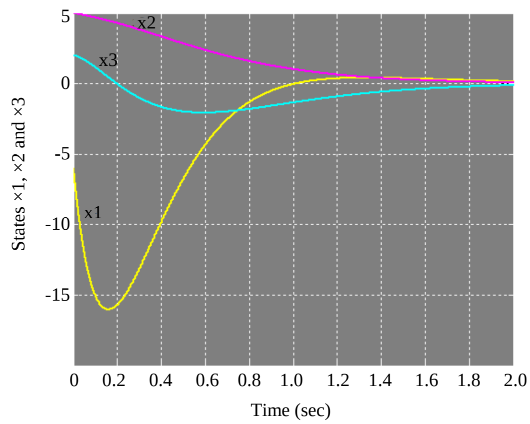

Fig. 4: Simulation results initial condition is $[-5, 5\ 2]^T$

Results (state variable of circuit: i, V_1, V_2) are shown in Fig. 4. Figure 4 shows that applying the control law given by $V_{in} = -10i\text{-}33\dot{V}_1 - 26V_2$ can force the system to return to equilibrium point $[0, 0, 0]^T$, also close loop system has a more fast response than to original system.

CONCLUSION

Pole placement problem is studied in recent decades by many researchers. In this study, instead of states, derivative of states is used for placing poles at the desired location.

A method is described for finding required gain matrix. Effectiveness of studied method is shown with an example. Next step is to apply the studied method to an industrial application.

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