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## Fractional PI Stabilization of Delay Systems: A Thermal Application

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**Key words:** Fractional-order calcul, fractional order PI controller, first order delay system, thermal system

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**Abstract:** We propose in this paper an application of fractional-order  $PI^\lambda$  controller as an alternative to solve some of the control problems that can arise. It aims to apply the analytical tuning procedure to control the heat flow systems. The system modeled by first order system involving time delay, this kind of systems whose closed-loop characteristic equation are fractional order quasi-polynomials. Using the proposed method, the entire stability region of  $PI^\lambda$  controllers is obtained and visualized in the plane  $(K_p, K_i, \lambda)$ . Simulation and experimental results on thermal system are given to show the effectiveness of this type of controllers and this tuning rule.

### INTRODUCTION

Several engineering systems and industry processes are modeled using fractional calculus which can be defined as a natural extension of the classical mathematics (Tenreiro, 1997; Monje *et al.*, 2010). Usually, the famous tools to model dynamic systems at a macroscopic level are integrals and derivatives, hence, the earliest theoretical contributions related to the fractional derivatives and integrals were made by Euler, Liouville and Abel (Monje *et al.*, 2010). Also, it should be stated that the fractional order representation is more adequate to describe real world systems than those of integer order models (Tan *et al.*, 2009). Since, that time, many studies exhibit that differ-integral operators may be applied advantageously in diverse areas. Especially, in control theory and applications, it is extensively applied and researchers find many opportunities to develop new directions for research in fractional-order control.

Recently, with the development of fractional calculus theory, the implementation of fractional order controllers is become feasible. According to researchers by Xue and Chen (2002) are classified in four categories, Oustaloup's CRONE controller and its three generations, fractional PID controllers or Podlubny's  $PI^\lambda D^\mu$  (Podlubny, 1999), FO lead-lag compensator (Monje *et al.*, 2010) and FO phase this kind of controller. Some practical application studies are presented by Petras and Vinagre (2002), Bhambhani and Chen (2008). The water level control was studied by using  $PI^\lambda$  controller in Bhambhani and Chen (2008) where some numerical simulations and experimental results were given. In Petras and Vinagre (2002) the temperature control of heat solid modeled using fractional order calculus was made, respectively by using conventional PID and  $PI^\lambda D^\mu$  controllers. Traditional PID controller, proportional control and On/Off control are usually used for temperature control. In this study, as a new control scheme for temperature regulation, so, we proposed using

fractional-order PI<sup>λ</sup> controller for a more accurate temperature profile. Also, the fractional order controller could be advantageous for temperature control, since, there are frequently variations in parameters in heat flow systems and most of desired specifications which are not readily achieved simultaneously by traditional PID controller.

**MATERIALS AND METHODS**

**Mathematical background:** There are several definitions for fractional order integral and differential calculus, however the most frequently used are the Grunwald-Letnikov (GL), the Riemann-Liouville (RL) and the Caputo definitions (Monje *et al.*, 2010; Caponetto *et al.*, 2010a, b; Podlubny *et al.*, 1997). The GL definition is given by:

$${}_a D_t^r f(t) = \lim_{h \rightarrow 0} h^{-r} \sum_{j=0}^{\lfloor \frac{t-a}{h} \rfloor} (-1)^j \binom{r}{j} f(t-jh) \quad (1)$$

where, a and t are the limits of the operation, r ∈ ℝ and [.] means the integer part.

The Riemann-Liouville RL fractional integral of function f(t) is defined as:

$${}_a D_t^r f(t) = \frac{1}{\Gamma(r-n)} \frac{d_n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{r-n+1}} d\tau \quad (2)$$

Where, (n-1 < r < n). The Caputo fractional integral is defined as:

$${}_a D_t^r f(t) = \frac{1}{\Gamma(r-n)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{r-n+1}} d\tau \quad (3)$$

The continuous integro-differential operator is defined as:

$${}_a D_t^r = \begin{cases} \frac{d^r}{dt^r} & : r > 0 \\ 1 & : r = 0 \\ \int_a^t d\tau & : r < 0 \end{cases} \quad (4)$$

Fractional integrals and derivatives also appear in the theory of control of dynamical systems when its is described by a fractional differential equation.

**Fractional order integrals:**

$$I_c^n f(t) \triangleq \frac{1}{\Gamma(\alpha)} \int_c^t (t-\tau)^{\alpha-1} f(\tau) d\tau, t > c, n \in \mathbb{R}^+ \quad (5)$$

where, t > c, n ∈ ℤ<sup>+</sup>

$$I_c^n f(t) \triangleq \frac{1}{\Gamma(\alpha)} \int_c^t (t-\tau)^{\alpha-1} f(\tau) d\tau, t > c, n \in \mathbb{Z}^+ \quad (6)$$

$$I_c^n f(t) \triangleq \frac{1}{\Gamma(\alpha)} \int_c^t (t-\tau)^{\alpha-1} f(\tau) d\tau, t > c, n \in \mathbb{R}^+ \quad (7)$$

$$I_c^n f(t) = \phi_\alpha(t) * f(t) \quad (8)$$

where the factorial function is define as:

$$\Gamma(\alpha) = \int_0^\infty e^{-u} u^{\alpha-1} du \quad (9)$$

and

$$\phi_\alpha(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}, \alpha \in \mathbb{R}^+ \quad (10)$$

using the Laplace transform, we have:

$$L\{\phi_\alpha(t) * f(t)\} = \phi_\alpha(s) \times f(s) \quad (11)$$

$$L\left\{\frac{t^{\alpha-1}}{\Gamma(\alpha)}\right\} = \frac{1}{s^\alpha} \quad (12)$$

**Fractional-order derivatives:** The fractional-order derivative is not a fractional order integral by a direct substitution α by -α. It provide an excellent instrument for the description of memory and hereditary properties of various processes (Caponetto *et al.*, 2010a). The fractional derivative of order of function f(t) can be defined as:

$$\frac{d^\alpha}{dt^\alpha} f(t) \begin{cases} f^{(n)}(t) & \text{if } \alpha = n \in \mathbb{N} \\ \frac{t^{n-\alpha-1}}{\Gamma(n-\alpha)} * f^{(n)}(t) & \text{if } n-1 < \alpha < n \end{cases} \quad (13)$$

Fractional quasi-polynomial: As a preliminary step, we define the fractional quasi-polynomial as follows:

$$\delta(s) = q_0(s) + \sum_{i=1}^n q_i(s) e^{-L_i s}$$

is a fractional quasi-polynomial of degree m. where:

$$q_i(s) = \sum_{j=0}^m a_{ij} s^{\alpha_j}$$

α<sub>0</sub> < α<sub>1</sub> < ..., < α<sub>n</sub> are generally fractional power, a<sub>ij</sub> are real numbers and L<sub>1</sub> < L<sub>2</sub> < ..., < L<sub>n</sub> represent time delays. There are two classes of fractional quasi-polynomials:

- Retarded-type fractional quasi-polynomials if  $\deg q_0(s) > \deg q_i(s)$  for all  $i = 1, 2, \dots, n$
- Neutral-type fractional quasi-polynomials if  $\deg q_0(s) = \deg q_i(s)$  for at least one  $i = 1, 2, \dots, n$

**Controller design concept:** The aim of this section is to present the system which will be controlled by a fractional order PI controller and to present the design of the fractional controller. The first order time delay systems can be described by:

$$G(s) = \frac{K}{1+Ts} e^{-Ls} \quad (14)$$

Our tuning strategy, is based on Hermite-Biehler theorem and the Pontryagin condition to determine the  $K_p$  and  $K_i$  parameters. The fractional PI <sup>$\lambda$</sup>  controller transfer function  $C(s)$  is given by the following equation:

$$C(s) = K_p + \frac{K_i}{s^\lambda} \quad (15)$$

In Fig. 1,  $r(t)$  is the reference input or the setpoint signal,  $e(t)$  is the error,  $u(t)$  is the control and  $y(t)$  is the output signal. Consequently, the control input of the PI <sup>$\lambda$</sup>  controller is:

$$u(t) = K_p (r(t) - y(t)) + K_i D_t^{-\alpha} (r(t) - y(t)) \quad (16)$$

where,  $D_t^{-\alpha}$  is the fractional differential/integral operators. The control design method proposed in this study is based on a Hermite-Biehler and Pontryagin theorem which consist on interlacement property of the real roots of the polynomial characteristic.

The closed-loop characteristic polynomial of a first order time delay system is given by:

$$\delta(s) = (KK_i + KK_p s^\lambda) e^{-Ls} + (1+Ts) s^\lambda \quad (17)$$

$$\delta(s) = e^{Ls} \delta(s) = (KK_i + KK_p s^\lambda) + (1+Ts) s^\lambda e^{-Ls} \quad (18)$$

Replacing the term  $Ls$  in the previous expression by  $z$  we obtain:

$$\delta^*(z) = KK_i + KK_p \left(\frac{z}{L}\right)^\lambda + \left(1+T\left(\frac{z}{L}\right)\right) \left(\frac{z}{L}\right)^\lambda e^z \quad (19)$$

We consider:  $z = j\omega$ .

$$\delta^*(j\omega) = KK_i + KK_p \left(\frac{j\omega}{L}\right)^\lambda + \left(1+T\left(\frac{j\omega}{L}\right)\right) \left(\frac{z}{L}\right)^\lambda (\cos(\omega) + j \sin(\omega)) \quad (20)$$

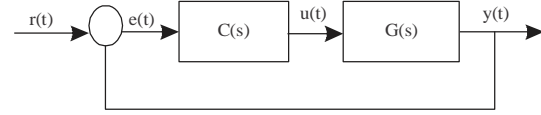


Fig. 1: Closed-loop control of a time-delay system

$$\delta^*(j\omega) = \left( KK_i + KK_p \left(\frac{j\omega}{L}\right)^\lambda \right) + \left(\frac{j\omega}{L}\right)^\lambda \left( \cos(\omega) - \frac{T}{L} \omega \sin(\omega) + j \left( \frac{T}{L} \omega \cos(\omega) + \sin(\omega) \right) \right) \quad (21)$$

The expression (Eq. 21) can be rewritten:

$$\delta^*(j\omega) = \delta_r^*(\omega) + j\delta_i^*(\omega) \quad (22)$$

$$\delta_r^*(\omega) = KK_i + \left( KK_p + \cos(\omega) - \frac{T}{L} \omega \sin(\omega) \right) \frac{|\omega|^\lambda}{(L)^\lambda} - \left( \frac{T}{L} \omega \cos(\omega) + \sin(\omega) \right) \left| \text{Im}\{(j)^\lambda\} \right| \frac{|\omega|^\lambda}{(L)^\lambda} \text{sign}(\omega) \quad (23)$$

$$\delta_i^*(\omega) = \left( KK_p + \cos(\omega) - \frac{T}{L} \omega \sin(\omega) \right) \left| \text{Im}\{(j)^\lambda\} \right| \frac{|\omega|^\lambda}{(L)^\lambda} + \left( \frac{T}{L} \omega \cos(\omega) + \sin(\omega) \right) \left| \text{Re}\{(j)^\lambda\} \right| \frac{|\omega|^\lambda}{(L)^\lambda} \quad (24)$$

Clearly, the controller parameter  $K_p$  only affects the imaginary part of  $\delta^i(\omega)$  whereas both parameters  $K_p$  and  $K_i$  appear in the real part  $\delta^r(\omega)$ . In order to solve our stabilization problem, we need first to determine the range of  $K_p$  for which a solution to the PI <sup>$\lambda$</sup>  stabilization problem of a closed-loop stable plant is given. According to Pontryagin Theorem,  $\delta^i(\omega)$  has only real roots for every:

$$K_p \in [K_{u-}, K_{u+}]$$

where,  $K_{u-}$  and  $K_{u+}$  are respectively the lower and the upper bound of  $K_p$  range. The successive step is to establish the ranges of the values of  $K_p$  and  $K_i$  that fulfil the interlacing condition between the roots of  $\delta_r^*(\omega)$  and  $\delta_i^*(\omega)$ .

However, we present our theorem Hafsi *et al.* (2013) useful to compute the stability region of first order first order system with time delay. Based on the first property of Hermite-Biehler (Silva *et al.*, 2002; Guillermo *et al.*, 2001; Hafsi *et al.*, 2015) which consist that all the roots of the polynomial characteristic of the closed loop equation are real.

**Theorem 1:** Hafsi *et al.* (2013), we consider a first order plant given by the following transfer function:

$$G(s) = \frac{K}{1+Ts} e^{-Ls}$$

where the parameters T, L and K are positive. We can determine the set of all stabilizing ( $K_p, K_i$ ) values for the given plant using the fractional order PI controller ( $PI^\lambda$ ):

$$C(s) = K_p + \frac{K_i}{s^\lambda}$$

The stabilizing set of parameters  $K_p$  values for a closed-loop stable plant is given by:

$$\bullet \max\left(-\frac{1}{K}, K_{u-}\right) < K_p < K_{u+}$$

Where:

$$K_{u-} = \frac{1}{K} \left( \left( \frac{T}{L} \alpha_1 \cos(\alpha_1) + \sin(\alpha_1) \right) \frac{\text{real}\{(j)^\lambda\}}{\text{Im}\{(j)^\lambda\}} \right.$$

$$\left. - \left( \cos(\alpha_1) - \frac{T}{L} \alpha_1 \sin(\alpha_1) \right) \right)$$

$$K_{u+} = -\frac{1}{K} \left( \left( \frac{T}{L} \alpha_1 \cos(\alpha_1) + \sin(\alpha_1) \right) \frac{\text{real}\{(j)^\lambda\}}{\text{Im}\{(j)^\lambda\}} \right.$$

$$\left. + \left( \cos(\alpha_1) - \frac{T}{L} \alpha_1 \sin(\alpha_1) \right) \right)$$

$\alpha_1 \in [-\pi; 0]$  and  $\alpha_2 \in [0; \pi]$  are respectively the solutions of the two previous equations:

$$\tan(\alpha_1) = -\frac{\frac{T}{L} \alpha_1 \left| \text{Im}\{(j)^\lambda\} \right| + \left(1 + \frac{T}{L}\right) \left| \text{real}\{(j)^\lambda\} \right|}{\left(1 + \frac{T}{L}\right) \left| \text{Im}\{(j)^\lambda\} \right| - \frac{T}{L} \alpha_2 \left| \text{real}\{(j)^\lambda\} \right|}$$

$$\tan(\alpha_2) = -\frac{\frac{T}{L} \alpha_2 \left| \text{Im}\{(j)^\lambda\} \right| + \left(1 + \frac{T}{L}\right) \left| \text{real}\{(j)^\lambda\} \right|}{\left(1 + \frac{T}{L}\right) \left| \text{Im}\{(j)^\lambda\} \right| - \frac{T}{L} \alpha_2 \left| \text{real}\{(j)^\lambda\} \right|}$$

Once the  $K_p$  range established, we determine  $K_i$  as follows:

$$\bullet \max\{-m_j K_p - b_j\} < K_i < \min\{-m_j K_p - b_j\}_{j=1,3,5,\dots}$$

Where:

$$m_j = m(\omega_j) = -\left| \text{real}\{(j)^\lambda\} \right| \frac{|\omega|^\lambda}{(L)^\lambda}$$

$$b_j = b(\omega_j) = -\left( \cos(\omega_j) - \frac{T}{L} \omega_j \sin(\omega_j) \right) \left| \text{real}\{(j)^\lambda\} \right| \frac{|\omega|^\lambda}{(L)^\lambda} + \left( \frac{T}{L} \omega_j \cos(\omega_j) + \sin(\omega_j) \right) \left| \text{Im}\{(j)^\lambda\} \right| \frac{|\omega|^\lambda}{(L)^\lambda} \text{sign}(\omega_j)$$

$\omega_j, j = 1, 2, 3, \dots$  are the roots, arranged in ascending order of magnitude of  $\delta^*_i(\omega)$ .

**Application to a thermal system:** Practical experience play an important role in demonstrating the accuracy of the new controller  $PI^\lambda$  for different for values of  $\lambda$ .

**Modelling and identification:** Temperature control arises in diverse engineering fields, but note that it is very difficult to find a precise model of transfer function of the output temperature change from the power input, thus, we use an approximated integer-order transfer function for heat flow while a fractional PI controller is used to regulate the temperature profile.

Figure 2 shows the set of test equipment: computer, AD/DA converter (NI-USB-6009 data acquisition module) and thermal system. The thermal system includes aluminium rod, of 41 cm length and 2 cm section, heating resistor, PWM converter and a sensor to measure the temperature. The system block diagram is presented in Fig. 3.

The input signal of the thermal system is a thermal flux generated by a heating resistor which is controlled by a computer with AD/DA converter. The controller and the heating resistor are separated by a dimmer phase angle (PWM converter). The output signal of this system is the aluminium rod temperature measured by an LM35DZ sensor. Then, this signal is amplified to obtain an output voltage varying from 0-5 V.

In order to determine the thermal system model, we have applied to the heating resistor a pseudo random binary sequence given by Fig. 4. This last depicts also the corresponding temperature at distance  $d = 6$  cm of the aluminum rod extremity. We note that the sample time is



Fig. 2: Real schema of the thermal system

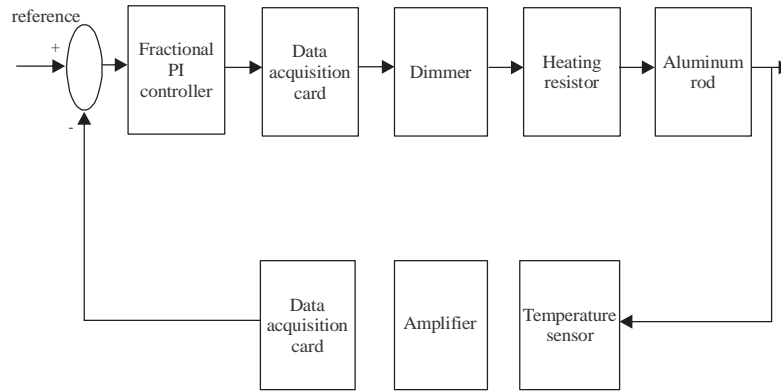


Fig. 3: Synoptic schema of the closed loop of thermal system

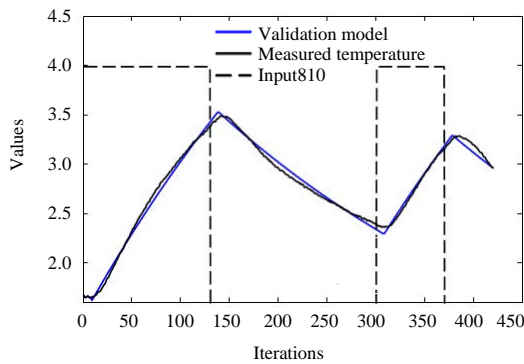


Fig. 4: Identification data

equal to 10 sec. Through a simple system identification process, the thermal system can be approximately modeled by the following transfer function:

$$G(s) = \frac{K}{Ts+1} e^{-Ls}$$

where: the delay time  $L = 12.2$ , response time  $T = 164.22$  and the static gain  $K$  is equal to  $0.8667$ . If the system model is determined, it is easy to design a controller to control the system.

**Simulation results:** Extensive simulation results are included to illustrate the simple yet practical nature of these type of controller. This study presents some simulation results to validate the design method.

In view of Theorem 1, an algorithm to determine the stabilizing set of the fractional PI controller parameter for the plant  $G(s) = \frac{0.8667}{164.22s+1} e^{-12.2s}$  is presented as follows:

**Algorithm:**

- Step 1. Choose fractional integral of order  $\lambda$
- Step 2. Compute the  $K_p$  range using first part of the theorem 1
- Step 3. Calculate the  $K_i$  range for each  $K_p$  computed in step 2
- Step 4. Return to step 1

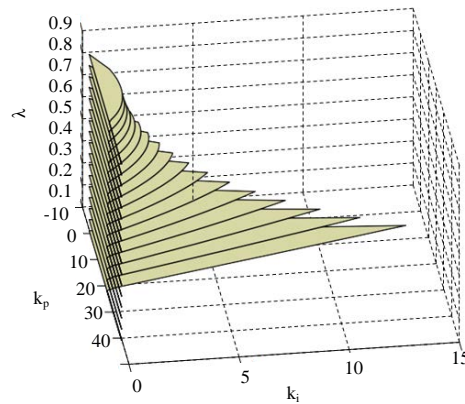


Fig. 5: Stability region of the thermal system for  $0 < \lambda < 1$

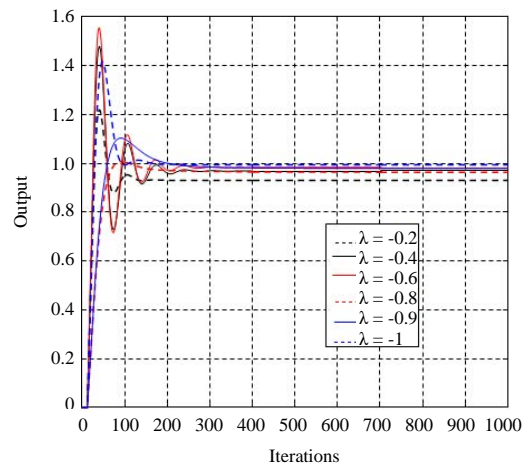


Fig. 6: Step responses of  $PI^\lambda$  controller where  $0 < \lambda < 1$

By sweeping over the fractional order  $\lambda$  range and using the algorithm presented earlier, we obtained the admissible set of  $(K_p, K_i)$  for each  $\lambda$  value. By varying the value of the non-integer order  $\lambda$  in the range of  $[0.1: 0.1: 1.9]$ , the stability regions of each  $PI^\lambda$  controllers are plotted in the  $(K_p, K_i, \lambda)$  plane. Figure 5 and 6 shows the

global stability regions obtained using  $\lambda \in [0.1: 0.1: 1]$  and  $\lambda \in [1.1: 0.1: 1.9]$  values for the fractional order  $PI^\lambda$  controllers.

Shown in 6 the step responses of the closed-loop systems for different values of  $\lambda$ . In fact, for  $\lambda \in [0.1: 0.1: 0.9]$  and for a fixed point of  $(K_p, K_i)$  in the stability region (5) correspondingly to each  $\lambda$  values. So, we constate that the effects of the integral action is to eliminate the steady-state error. In fact, for a small values of  $\lambda$  such as  $\lambda = 0.1, \lambda = 0.2$  the output does not converge to the reference value and when this fractional order integral increase the steady-state error decrease. It is observed that the response of  $PI^\lambda$  controlled system show a fairly better response in comparison with the case of the  $PI^{0.3}$  controller. Also, it is interesting to note that the fractional order integrator  $\lambda$  which is an extra degree of freedom contribute to the performance improvement of the

closed-loop system by making the step responses reveal that the steady-state error decreases as the order  $\lambda$  increases. In Fig. 7 and 8 the step responses of the model:

$$G(s) = \frac{0.8667}{164.22s + 1} e^{-12.2}$$

corresponding to the thermal system controlled by the fractional controller  $PI^\lambda$  where  $\lambda = 1.1, 1.2, 1.3, 1.4, 1.5$ . are represented. We observe from this figure that the output of the model converge to reference input which confirm the effectively of this kinds of regulators.

### RESULTS AND DISCUSSION

**Experimental results:** In order to illustrate the effectiveness the approche given by the theorem (1), we consider the thermal system given by Fig. 2-4.

In fact, the thermal system is defined as a first order system with time delay. The evolutions of the reference signal, the measured temperature (output signal), control signal obtained with a fractional order controllers  $PI^{-0.9}$  and  $PI^{-1.2}$  are represented respectively in Fig. 9, 10.

From Fig. 9, we observe a small steady-state error which cost on the weak value of the integrator order ( $\lambda = 0.9$ ) which confirm the simulation results given in Fig. 6. Whereas for the  $PI^{-1.2}$  controller, we remark that measured temperature meets the desired requirements and provide a small variation which gives a null steady-state error.

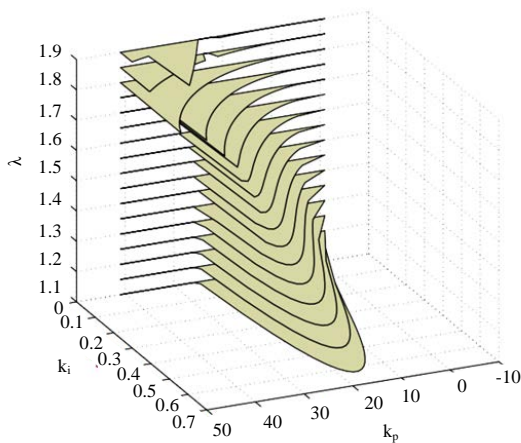


Fig. 7: Stability region of the thermal system for  $1.1 < \lambda < 1.9$

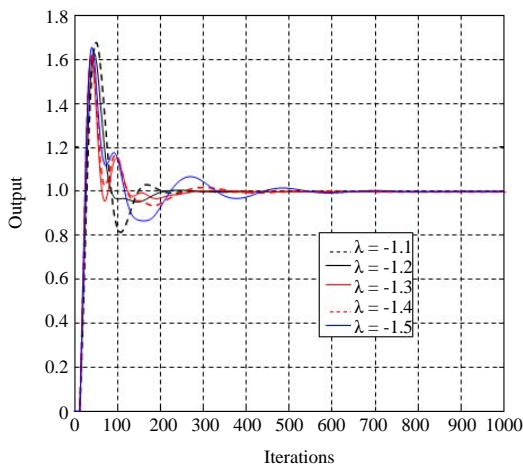


Fig. 8: Step responses of  $PI^\lambda$  where  $1.1 < \lambda < 1.9$

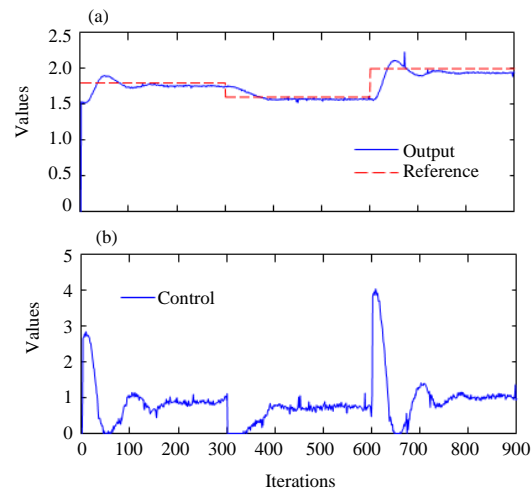


Fig. 9(a, b): Closed-loop results obtained with  $PI^{-0.9}$  controller where  $K_p = 10$  and  $K_i = 0.7$

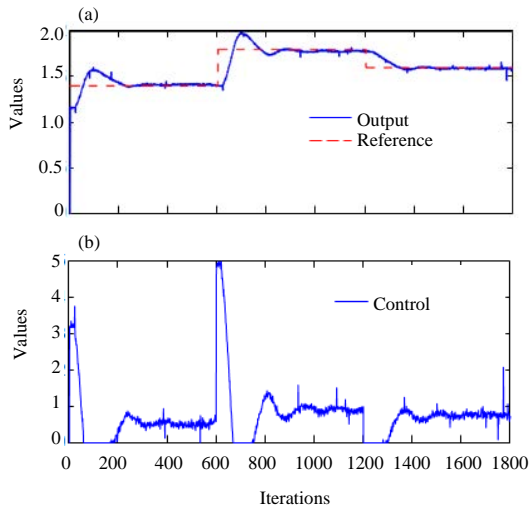


Fig. 10(a, b): Closed-loop results obtained with  $PI^{-1.2}$  controller where  $K_p = 15$  and  $K_i = 0.5$

### CONCLUSION

In conclusion, simulation and experimental results show that the closed-loop system can achieve favorable dynamic performance.

In this study, fractional order proportional integral controller is designed for temperature profile control of a metallic rod. The FO-PI controller tuned by our analytical method based on Hermite-Biehler theorem gives a strong performance. Also, from experimental and simulation results, it is observed that the designed fractional order controller works efficiently.

### RECOMMENDATION

In our future research efforts, we would like to consider a fractional model uncertainty problem which will be controlled by a robust fractional order  $PI^\lambda$  and  $PI^\lambda D^\mu$  controllers.

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