International Journal of Systems Signal Control and Engineering Application

MEDWELL PUBLICATIONS SINCE 2000

LQR Controller and Optimal Estimation-Observer Design Applied for Pitch Aircraft B747

[1]Labane Chrif, [2]Zemalache Meguenni Kadda and [1]Laouer Mohammed
[1]LMMC at University, Dr Moulay Tahar SAIDA, 20000, Algeria
[2]Oran University of Science and Technology, USTOMB-ORAN, 31000, Algeria

Key words: Aircraft motion, LQR control, longitudinal stability, aircraft, flight control, observer design, control design using pole placement.

Corresponding Author:
Labane Chrif
LMMC at University, Dr Moulay Tahar SAIDA, 20000, Algeria

Page No.: 53-58
Volume: 13, Issue 3, 2020
ISSN: 1997-5422
International Journal of Systems Signal Control and Engineering Application

Abstract: Today's aircraft designs rely heavily on automatic control system to monitor and control many of aircraft's subsystem. The development of this research is to model a pitch, roll and sideslip controller based on design an autopilot that controls the pitch of an aircraft. The Linear Quadratic Controller is developed for controlling the pitch angle, roll angle and sideslip angle of an aircraft system. Simulation results for the response of pitch roll and sideslip controller are presented in time domain. Finally, the performances of pitch control systems are investigated and analyzed based on common criteria of impulse response in order to identify which control strategy delivers better performance with respect to the desired pitch angle. It is found from simulation, LQR controller give the best performance. In this article, we apply the linear observer without physical sensor to control the pitch angle of an aircraft B747 in which external reactions are done. A linear observer constructs to consolidate the angles of the movements of the aircraft in the mode of the longitudinal and lateral flight. This linear observer is based on the techniques of pole placement has been shown by the results of simulation.

INTRODUCTION

The rapid advancement of aircraft design from the very limited capabilities of the Wright brothers first successfully airplane today's high performance military, commercial and general aviation aircraft required the development of many technologies, those are aerodynamics, structures, materials, propulsion and flight controls. The development of automatic control system has played an important role in the growth of civil and military aviation. Modern aircraft include a variety of automatic control system that aids the flight crew in navigation, flight management and augmenting the stability characteristic of the airplane. For this situation an autopilot is designed that control the pitch of aircraft that can be used by the flight crew to lessen their workload during cruising and help them land their aircraft during adverse weather condition in the real situation[1]. The autopilot is an element within the flight control system. It is a pilot relief mechanism that assists in maintaining an attitude, heading, altitude or flying to navigation or landing references. Designing an autopilot requires control system theory background and knowledge of stability derivatives at different altitudes and Mach numbers for a given airplane[2]. Lot of works has been done in the past to control the pitch, roll and sideslip of an aircraft for the purpose of flight stability and yet this research still remains an open issue in the present and future works[3-6].

Aircraft control and movement: There are three primary ways for an aircraft to change its orientation relative to the passing air. Pitch (movement of the nose up or down), Roll (rotation around the longitudinal axis, that is, the axis which runs along the length of the aircraft) and Yaw (movement of the nose to left or right) Turning the aircraft (change of heading) requires the aircraft firstly to roll to achieve an angle of bank; when the desired change of heading has been accomplished the aircraft must again be rolled in the opposite direction to reduce the angle of bank to zero.

Flight dynamics: Flight dynamics is the science of air vehicle orientation and control in three dimensions. The three critical flight dynamics parameters are the angles of rotation in three dimensions about the vehicle's center of mass, known as pitch, roll and yaw (quite different from their use as Tait-Bryan angles).

Aerospace engineers develop control systems for a vehicle's orientation (attitude) about its center of mass. The control systems include actuators which exert forces in various directions and generate rotational forces or moments about the aerodynamic center of the aircraft and thus rotate the aircraft in pitch, roll or yaw. For example, a pitching moment is a vertical force applied at a distance[1, 4].

Roll, pitch and yaw refer to rotations about the respective axes starting from a defined equilibrium state. The equilibrium roll angle is known as wings level or zero bank angle, equivalent to a level heeling angle on a ship. Yaw is known as "heading". The equilibrium pitch angle in submarine and airship parlance is known as "trim" but in aircraft, this usually refers to angle of attack rather than.

Longitudinal modes: Oscillating motions can be described by two parameters, the period of time required for one complete oscillation and the time required to damp to half-amplitude or the time to double the amplitude for a dynamically unstable motion. The longitudinal motion consists of two distinct oscillations, a long-period oscillation called a phugoid mode and a short-period oscillation referred to as the short-period mode[5].

Phugoid (longer period) oscillations: The longer period mode, called the "phugoid mode" is the one in which there is a large-amplitude variation of air-speed, pitch angle and altitude but almost no angle-of-attack variation. The phugoid oscillation is really a slow interchange of kinetic energy (velocity) and potential energy (height) about some equilibrium energy level as the aircraft attempts to re-establish the equilibrium level-flight condition from which it had been disturbed. The motion is so slow that the effects of inertia forces and damping forces are very low. Although the damping is very weak, the period is so long that the pilot usually corrects for this motion without being aware that the oscillation even exists. Typically the period is 20-60 sec[2, 5].

Short period oscillations: With no special name, the shorter period mode is called simply the "short-period mode". The short-period mode is a usually heavily damped oscillation with a period of only a few seconds. The motion is a rapid pitching of the aircraft about the center of gravity. The period is so short that the speed does not have time to change, so, the oscillation is essentially an angle-of-attack variation. The time to damp the amplitude to one-half of its value is usually on the order of 1 second. Ability to quickly self damp when the stick is briefly displaced is one of the many criteria for general aircraft certification[5] (Fig. 1).

Aircraft dynamics longitudinal

Equations of movements: The general equations of the movement are governed by the equations of mechanics:

$$\begin{cases} m\dfrac{d\overrightarrow{u}}{dt} = \sum \overrightarrow{F_e} \\ \dfrac{d\overrightarrow{C}}{dt} = \sum \overrightarrow{M_e} \end{cases} \tag{1}$$

Equation of longitudinal motion:

$$\beta = p = r = \Phi = 0 \tag{2}$$

Longitudinal equations can be rewritten as:

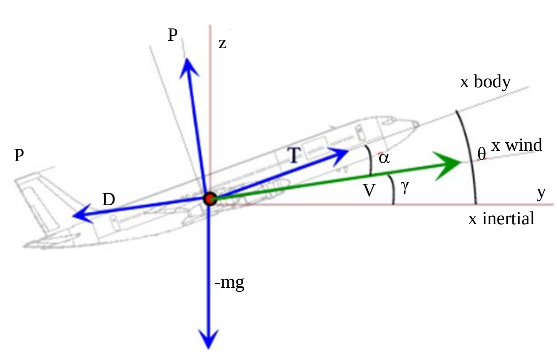

Fig. 1: Aerodynamic reference

$$\begin{cases} \dot{u} = \frac{X_u}{m}u + \frac{X_w}{m}w - \frac{g\cos\Theta_0}{m}\theta + \Delta X^c \\ \dot{w}\frac{Z_u}{m\text{-}Z_w}u + \frac{Z_w}{m\text{-}Z_w}w + \frac{Z_q + mU_0}{m\text{-}Z_w}q - \frac{mg\sin\Theta_0}{m\text{-}Z\dot{w}}\theta + \Delta Z^c \\ \dot{q} = \frac{\left[M_u + Z_u\Gamma\right]}{I_{yy}}u + \frac{\left[M_u + Z_u\Gamma\right]}{I_{yy}}w + \frac{\left[M_q + \left(Z_q + mU_0\right)\Gamma\right]}{I_{yy}} - \frac{mg\sin\Theta_0\Gamma}{Iyy}\theta + \Delta M^c \\ \dot{\theta} = q \end{cases} \tag{3}$$

With:

$$\Delta X^c = \frac{X_{\delta_e}}{m}\delta_e + \frac{X_p}{m}\delta_p$$

$$\Delta Z^c = \frac{Z_{\delta_e}}{m - Z_{\dot{\omega}}}\delta_e + \frac{Z_{\delta_p}}{m - Z_{\dot{\omega}}}\delta_p$$

$$\Delta M^c = \frac{M_{\delta_e} + Z_{\delta_e\Gamma}}{I_{yy}}\delta_e + \frac{M_{\delta_p} + Z_{\delta_p\Gamma}}{I_{yy}}\delta_p$$

Rewrite in state space form as:

$$A = \begin{bmatrix} \frac{X_u}{m} & \frac{X_{\dot{\omega}}}{m} & 0 & -g\cos\Theta_0 \\ \frac{Z_u}{m\text{-}Z_{\dot{\omega}}} & \frac{Z_{\omega}}{m\text{-}Z_{\dot{\omega}}} & \frac{Z_g + mU_0}{m\text{-}Z_{\dot{\omega}}} & \frac{-mg\sin\Theta_0}{m\text{-}Z_{\dot{\omega}}} \\ I_{yy}^{1}\left[M_u + Z_u\Gamma\right] & I_{yy}^{1}\left[M_u + Z_u\Gamma\right] & I_{yy}^{1}\left[M_g + \left(Z_g + mU_0\right)\Gamma\right] & \text{-}I_{yy}^{1}mg\sin\Theta_0\Gamma \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{X_{\delta_e}}{m} & \frac{X_{\delta_p}}{m} \\ \frac{Z_{\delta_e}}{m\text{-}Z_{\dot{\omega}}} & \frac{Z_{\delta_p}}{m\text{-}Z_{\dot{\omega}}} \\ I_{yy}^{-1}\left[M_{\delta_E} + Z_u\Gamma\right] & I_{yy}^{-1}\left[M_{\delta_p} + Z_u\Gamma\right] \\ 0 & 0 \end{bmatrix} C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{4}$$

$$\square X^c = \frac{X_{\delta_e}}{m}\delta_e + \frac{X_p}{m}\delta_p$$

Since $u \approx 0$ in this mode, then $\dot{u} \approx 0$ and can eliminate the X force equation:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{Z_w}{m\text{-}Z_{\dot{w}}} & \frac{Z_q + mU_0}{m\text{-}Z_{\dot{w}}} & \frac{\text{-}mg\sin\Theta_0}{m\text{-}Z_{\dot{w}}} \\ \frac{\left[M_w + Z_w\Gamma\right]}{I_{yy}} & \frac{\left[M_q + \left(Z_q + mU_0\right)\Gamma\right]}{I_{yy}} & \frac{\text{-}mg\sin\Theta_0}{I_{yy}} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta Z^C \\ \Delta M^C \\ 0 \end{bmatrix} \tag{5}$$

Typically find that $Z_{\dot{w}} \leq m \text{ et } Z_q \leq mU_0$. Check for 747:

$$\text{-}Z_{\dot{w}} = 1909 \leq m = 2.8866 * 10e5$$

$$\text{-}Z_q = 4.5 * 10e5 \leq mU_0 = 6.8 * 10e7$$

$$\Gamma = \frac{M_{\dot{w}}}{m\text{-}Z_{\dot{w}}} \Rightarrow \Gamma \approx \frac{M_{\dot{w}}}{m}$$

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\dfrac{Z_w}{m-Z_{\dot{w}}} & U_0 & -g\sin\Theta_0\\ \dfrac{\left[M_w+Z_w\dfrac{M_{\dot{w}}}{m}\right]}{I_{yy}} & \dfrac{\left[M_q+(mU_0)\dfrac{M_{\dot{w}}}{m}\right]}{I_{yy}} & \dfrac{-mg\sin\Theta_0}{I_{yy}}\dfrac{M_{\dot{w}}}{m}\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix}+\begin{bmatrix}\Delta Z^C\\ \Delta M^C\\ 0\end{bmatrix} \tag{6}$$

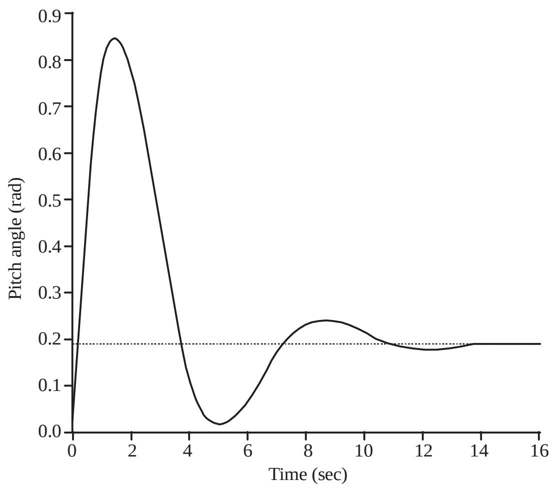

Fig. 2: Open loop impulse response (pitch angle)

The transfer function can be represented in state-space form and output equation as state by Eq. 7 and 8:

$$\begin{bmatrix}\dot{w}\\ \dot{q}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}-0.3149 & 235.8928 & 0\\ -0.0034 & -0.4282 & 0\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix}+\begin{bmatrix}-5.5079\\ 0.0021\\ 0\end{bmatrix}\delta_e \tag{7}$$

$$y=\begin{bmatrix}0 & 0 & 1\end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix}+[0] \tag{8}$$

This research presents investigation into the development of pitch control schemes for pitch angle and pitch rate of an aircraft systems. Pitch control systems with full state feedback controller are investigated. A modern controller (LQR) controls the pitch of an aircraft system. Performance of one control strategy with respect to the pitch. Simulation results are shown in Fig. 2.

$X = [u, \omega, q, \gamma]^T$ and $\gamma = \theta-\alpha$ represent flight path angle, with $\alpha = \omega$, $u = [\delta_e/\delta_p]$. The input (elevator deflection angle δ_e) will be 0.2 rad (11°) and the output is the pitch angle (θ) (Fig. 3).

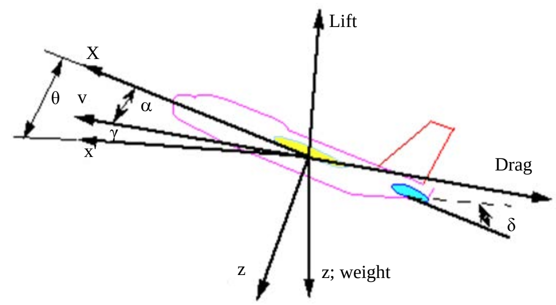

Fig. 3: Aerodynamics angles

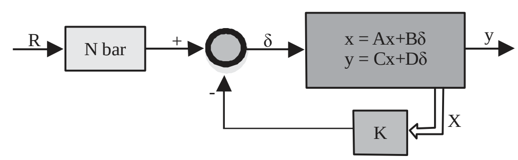

Fig. 4: Full-state feedback controller with reference input

LINEAR QUADRATIC REGULATOR CONTROLLER

Modern control theory has made a significant impact on the aircraft industry in recent years[3]. LQR is a method in modern control theory that used state-space approach to analyze such a system. Using state space methods it is relatively simple to work with a multi-output system. The system can be stabilized using full-state feedback system. The configuration of this control system is shown in Fig. 4.

In designing LQR controller, lqr function in MATLAB can be used to determine the value of the vector K which determined the feedback control law. This is done by choosing two parameter values, input R = 1 and $Q = C^T*C$ where C^T is the matrix transpose of C from state Eq. 6 and 11. The controller can be tuned by changing the nonzero elements in q matrix which is done in m-file code as obtained:

$$\begin{aligned}&R = 1\\ &Q = [000;\ 000;\ 00x]\\ &K = \text{lqr}[A,\ B,\ Q,\ R]\end{aligned} \tag{9}$$

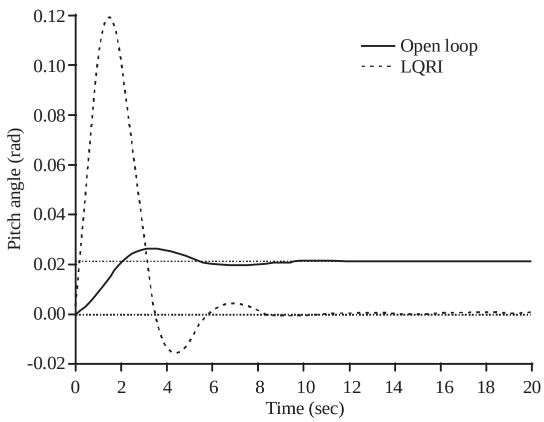

Fig. 5: Comparison of open-and closed-loop impulse response for the LQR example

Consequently by tuning the value of, the following values of matrix K are obtained. If is increased even higher, improvement to the response should be obtained even more. But for this case, the values of is chosen because it satisfied the design requirements while keep as small as possible.

In order to reduce steady state error of the system output, a value of constant gain Nbar should be added after the reference. With a full-state feedback controller all the states are feedback. The steady-state value of the states should be computed, multiply that by the chosen gain and used a new value as the reference for computing the input. Nbar can be found using the user-defined function which can be used in m-file code. The method used in simulation work is done by exported both value of matrix and constant gain. For this controller design, the value of constant gain, Nbar are found to be, Nbar = 100. The response for pitch angle control of an aircraft system using LQR controller are shown in Fig. 5.

OBSERVER DESIGN

When we can't measure all the states (as is commonly the case), we can build an observer to estimate them, while measuring only the output. For the magnetic ball example, we will add three new, estimated states to the system. The schematic is as follows (Fig. 6):

The observer is basically a copy of the plant; it has the same input and almost the same differential equation. An extra term compares the actual measured output y to the estimated output $\hat{y}$ this will cause the estimated states to approach the values of the actual states. The error dynamics of the observer are given by the poles of (A-L*C)

First we need to choose the observer gain L. Since, we want the dynamics of the observer to be much faster than the system itself, we need to place the poles at least five times farther to the left than the dominant poles of the system. If we want to use place, we need to put the three observer or for poles at different locations.

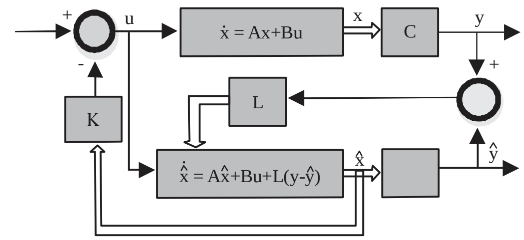

Fig. 6: Block diagram of the observer design

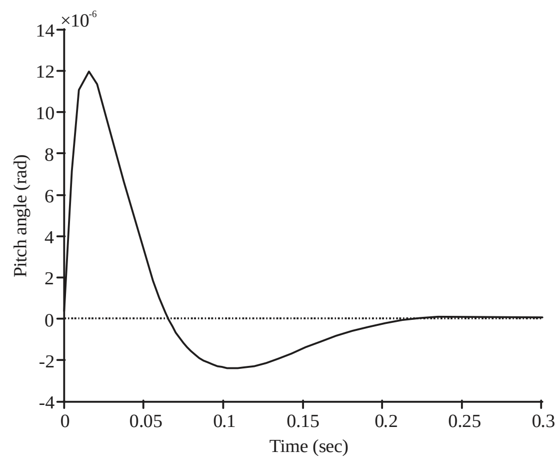

Fig. 7: Observer design response for the pitch angle θ

Because of the duality between controllability and observability, we can use the same technique used to find the control matrix but replacing the matrix B by the matrix C and taking the transposes of each matrix (consult your text book for the derivation):

$$L = \text{place}\left(A', C', [\text{op1 op2 op3 op4}]\right) \quad (10)$$

The equations in the block diagram above are given for $\hat{x}$. It is conventional to write the combined equations for the system plus observer using the original state plus the error state $e = x\text{-}\hat{x}$. We use as state feedback u = -K $\hat{x}$. We arrive at the combined state and error equations with the full-state feedback and an observer (Fig. 7):

$$\begin{aligned} A_t &= \left[\text{A-B}*\text{k} \;\; \text{B}*\text{k zeros}\left(\text{size}(A)\right)\text{A-L}*\text{C}\right] \\ B_t &= \left[\text{B}*\text{Nber zeros}\left(\text{size}(B)\right)\right] \\ C_t &= \left[\text{C zeros}\left(\text{size}(C)\right)\right] \end{aligned} \quad (11)$$

To see how the response looks to a nonzero initial condition with no reference input, add the following lines

Table 1: Parameters of aircraft

Parameters	Values
X_u	-1.982e3
X_w	4.025e3
Z_u	-2.595e4
Z_w	-030e4
Z_q	-4.524e5
δ_e	1.909e3
μ	1.593e4
g	9.81
M_ω	1.563e5
M_q	-1.521e7
M_{wd}	-1.702e4
S	511
Θ	0
U_0	235
$\bar{C}$	8.324

into your m-file. We typically assume that the observer begins with zero initial condition $\hat{x} = 0$. This gives us that the initial condition for the error is equal to the initial condition of the state[7]. Simulation results are shown in Fig. 8. The Observer design response for the pitch angle θ.

The results observer design provides good performance in term of steady state error. As depicted from Fig. 7, it can be observed that the pitch angle follows the reference value, respectively. This observer design is able to give a good response without produce any problem (Table 1).

CONCLUSION

The validated model of pitch, roll and sideslip control of an aircraft is very helpful in developing the control strategy for actual system. Pitch, roll and sideslip control of an aircraft is a system which requires a pitch, roll and sideslip controller to maintain the angle at it desired value. This can be achieved by reducing the error signal which is the difference between the output angle the desired angle. The control approach of LQR is capable on controlling the pitch angle, roll angle and sideslip angle of the aircraft system for value of 0.2 radian (11.5°). Simulation and analysis results show that, LQR controller relatively give the better performance. For advanced work, effort can be devoted in developing more robustness control techniques, following by implement the proposed control algorithm to real plant for validating of the theoretical result.

Finally, the LQR gives a very good following to the outputs of aircraft with a steady shift error limited and the observer design is an optimal observatory.

REFERENCES

01. Nelson, R.C., 1989. Flight Stability and Automatic Control. McGraw-Hill Co., New York, USA., ISBN-13: 978-0070462182, pp: 286.
02. Stojiljkovic, B., L. Vasov, C. Mitrovic and D. Cvetkovic, 2009. The application of the root locus method for the design of pitch controller of an F-104A aircraft. Strojniski Vestnik: J. Mech. Eng., 55: 555-560.
03. Chrif, L. and Z.M. Kadda, 2014. Aircraft control system using LQG and LQR controller with optimal estimation-Kalman filter design. Procedia Eng., 80: 245-257.
04. Eick, R.S., 2003. A reconfiguration scheme for flight control adaptation to fixed position actuator failures. Ph.D. Theses, University of Florida, Gainesville, Florida.
05. Bates, D.G., R. Kureemun and I. Postlethwaite, 2002. Quantifying the robustness of flight control systems using Nichols exclusion regions and the structured singular value. Proc. Inst. Mech. Eng. Part I. J. Syst. Control Eng., 215: 625-638.
06. Berenji, H.R., S. Saraf, P.W. Chang and S.R. Swanson, 2002. Pitch control of the space shuttle training aircraft. IEEE. Trans. Control Syst. Technol., 9: 542-551.
07. Zhu, J.H., 2006. A survey of advanced flight control theory and application. Proceedings of the Multi Conference on Computational Engineering in Systems Applications Vol. 1, October 4-6, 2006, IEEE, Beijing, China, pp: 655-658.