International Journal of Systems Signal Control and Engineering Application

MEDWELL PUBLICATIONS SINCE 2000

Adaptive Controller Design for the Generalized Projective Synchronization of Hyperchaotic Lü and Hyperchaotic Cai Systems

[1]P. Sarasu and [2]V. Sundarapandian

[1]*Faculty of Computer Science and Engineering,*

[2]*Research and Development Centre, Vel Tech Dr. RR & Dr. SR Technical University Avadi, -600 062 Chennai, India*

Key words: Adaptive control, chaos synchronization, hyperchaos, hyperchaotic systems, generalized projective synchronization, hyperchaotic Lu system, hyperchaotic Cai system

Corresponding Author:
P. Sarasu
Faculty of Computer Science and Engineering Vel Tech Dr. RR & Dr. SR Technical University Avadi, -600 062 Chennai, India

Page No.: 79-94
Volume: 13, Issue 4, 2020
ISSN: 1997-5422
International Journal of Systems Signal Control and Engineering Application

Abstract: This study presents the adaptive controller design for the Generalized Projective Synchronization (GPS) of identical hyperchaotic Lu systems, i.dentical hyperchaotic Cai systems and non-identical hyperchaotic Lü and hyperchaotic Cai systems. The adaptive GPS synchronization results derived in this study have been proved using the adaptive control theory and Lyapunov stability theory. Since, the Lyapunov exponents are not required for these calculations, the proposed adaptive control method is very effective and convenient for achieving the Generalized Projective Synchronization (GPS) of the hyperchaotic systems addressed in this study. Numerical simulations are shown to demonstrate the effectiveness of the adaptive GPS synchronization results derived in this study for the hyperchaotic Lü and hyperchaotic Cai systems.

INTRODUCTION

Chaotic systems are systems are nonlinear dynamical systems which possess some special features, such as being extremely sensitive to small variations of initial conditions, having bounded trajectories in the phase space, and so on. The sensitive nature of chaotic systems is commonly called as the butterfly effect[1]. The chaos phenomenon was first observed in weather models by Lorenz[2].

Chaos is an interesting nonlinear phenomenon and has been studied well in the last three decades. Chaos theory has wide applications in several fields like physical systems[3], chemical systems[4], ecological systems[5], secure communications[6-8], etc. Hyperchaotic system is usually defined as a chaotic system having more than one positive Lyapunov exponent. Since hyperchaotic system has the characteristics of high capacity, high security and high efficiency, it has the potential of broad applications in nonlinear circuits, neural networks, lasers, secure communications, biological systems and so on. The hyperchaos phenomenon was first observed by Rossler[9].

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging research problem.

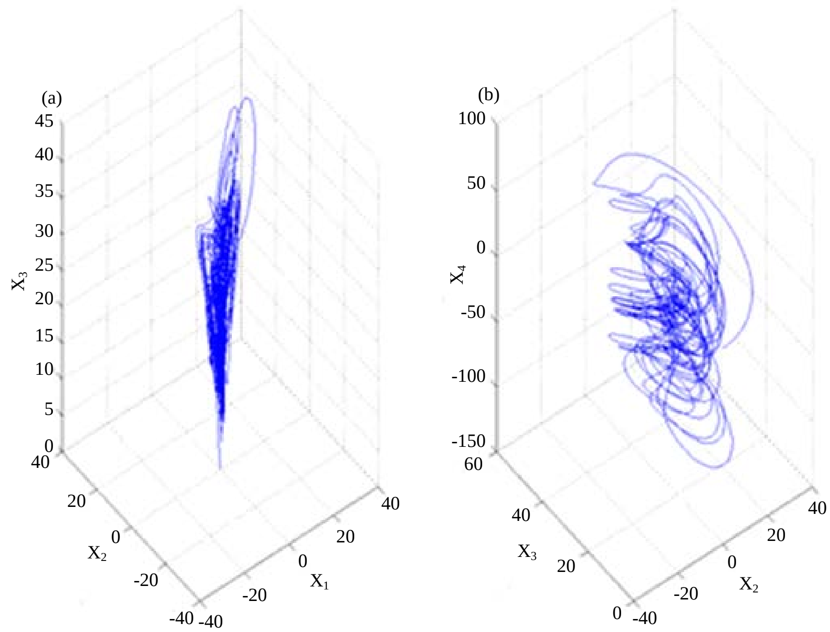

Fig. 1(a, b): The phase portrait of the hyperchaotic Lu system

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the idea of the chaos synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since, the seminal work by Pecora and Carroll[10], chaos synchronization problem has been studied intensively and extensively in the chaos literature. In the last two decades, various schemes have been successfully applied for chaos synchronization such as OGY method[11], active control method[12-15], adaptive control method[16-20], time-delay feedback method[21], backstepping design method[22-25], sampled-data feedback synchronization method[26], sliding mode control method[27-30], etc.

So, far many types of synchronization phenomenon have been presented such as complete synchronization[31], generalized synchronization[32], anti-synchronization[33-36], hybrid synchronization[37-39], projective synchronization[40], generalized projective synchronization[41, 42], etc.

Complete Synchronization (CS) is characterized by the equality of state variables evolving in time while Anti-Synchronization (AS) is characterized by the disappearance of the sum of relevant state variables evolving in time. In hybrid synchronization of two chaotic systems, one part of the systems is completely synchronized and the other part is anti-synchronized, so that, the Complete Synchronization (CS) and Anti-Synchronization (AS) co-exist in the systems.

Projective Synchronization (PS) is characterized by the fast that the master and slave systems could be synchronized up to a scaling factor. In Generalized Projective Synchronization (GPS), the responses of the synchronized dynamical states synchronize up to a constant scaling matrix α. It is easy to see that the complete synchronization and anti-synchronization are special cases of the generalized projective synchronization where the scaling matrix $\alpha = I$ and $\alpha = -I$, respectively.

This study describes the adaptive controller design for the GPS of the identical hyperchaotic Lü systems Chen *et al.*[43], the identical hyperchaotic Cai systems Wang and Cai[44] and the non-identical hyperchaotic Lu and hyperchaotic Cai systems. The adaptive GPS synchronization results for the hyperchaotic systems addressed in this paper have been established using the Lyapunov stability theory[45].

Systems description: In this study, we describe the hyperchaotic systems addressed in this study. The hyperchaotic Lu system[43] is described by the 4D Lu dynamics:

$$\begin{aligned} \dot{x}_1 &= a\left(x_2 - x_1\right) + x_4 \\ \dot{x}_2 &= cx_2 - x_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= dx_4 + x_1x_3 \end{aligned} \tag{1}$$

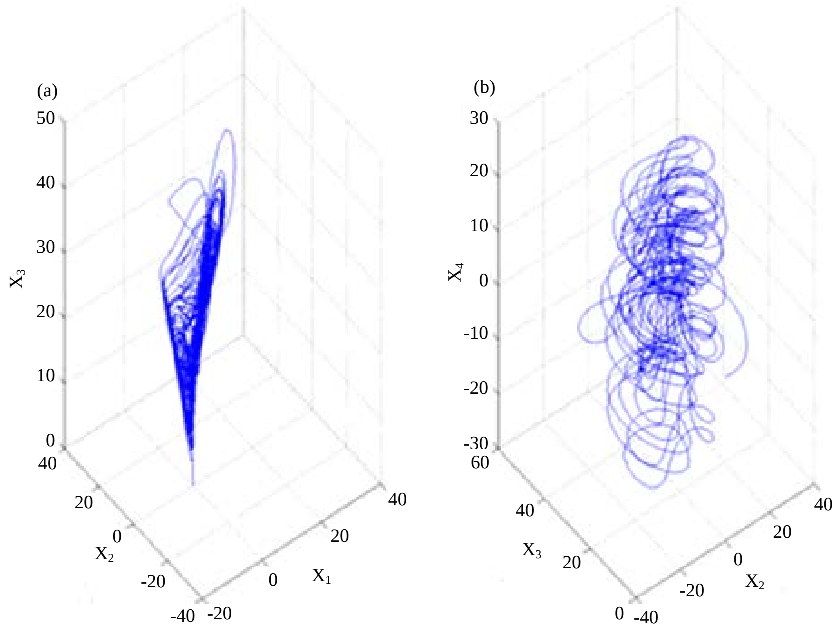

Fig. 2(a, b): The Phase Portrait of the Hyperchaotic Cai System

where x_1, x_2, x_3, x_4 are the states and a, b, c, d are constant, positive parameters of the system.The system Eq. 1 exhibits a hyperchaotic attractor when the system parameter values are chosen as:

$$a = 36,\ b = 3,\ c\ 20,\ d = 1.3$$

The phase portrait of the hyperchaotic Lu system Eq. 1 is depicted in Fig. 1. The hyperchaotic Cai system ([45], 2009) is described by the 4D Cai dynamics:

$$\begin{aligned} \dot{x}_1 &= p(x_2 - x_1) \\ \dot{x}_2 &= qx_1 + rx_2 - x_1x_3 + x_4 \\ \dot{x}_3 &= -sx_3 + x_2^2 \\ \dot{x}_4 &= -\varepsilon x_1 \end{aligned} \tag{2}$$

where, x_1, x_2, x_3, x_4 are the states and p, q, r, s, **ε** are constant, positive parameters of the system. The system Eq. 2 exhibits a hyperchaotic attractor when the system parameter values are chosen as:

$$p = 27.5,\ q = 3,\ r = 19.3,\ s = 2.9,\ \varepsilon = 3.3$$

The phase portrait of the hyperchaotic Cai system Eq. 2 is depicted in Fig. 2.

ADAPTIVE GENERALIZED PROJECTIVE SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC LÜ SYSTEMS

Theoretical results: In this study, we deploy adaptive control to derive results for the Generalized Projective Synchronization (GPS) of the identical hyperchaotic Lu systems (2006) when the system parameters are unknown. Thus, the master system is described by the hyperchaotic Lu dynamics:

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_4 \\ \dot{x}_2 &= cx_2 - x_1x_3 \\ \dot{x}_3 &= -bx_3 + x_1x_2 \\ \dot{x}_4 &= dx_4 + x_1x_3 \end{aligned} \tag{3}$$

where, x_1, x_2, x_3, x_4 are the states and a, b, c, d are unknown parameters of the system. Also, the slave system is described by the controlled hyperchaotic Lu dynamics We define the parameter estimation errors as:

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_4 + u_1 \\ \dot{y}_2 &= cy_2 - y_1y_3 + u_2 \\ \dot{y}_3 &= -by_3 + y_1y_2 + u_3 \\ \dot{y}_4 &= dy_4 + y_1y_3 + u_4 \end{aligned} \tag{4}$$

where y_1, y_2, y_3, y_4 are the states and u_1, u_2, u_3, u_4 are the adaptive controls to be desigend. The GPS synchronization errors are defined as:

$$e_i = y_i - \alpha_1 x_i, (i = 1, 2, 3, 4) \tag{5}$$

where, the scales α_1, α_2, α_3, α_4 are real numbers. The error dynamics is obtained as:

$$\begin{aligned} \dot{e}_1 &= a(y_2-\alpha_1x_2-e_1)+y_4-\alpha_1x_4+u_1 \\ \dot{e}_2 &= ce_2-y_1y_3+\alpha_2x_1x_3+u_2 \\ \dot{e}_3 &= -by_3+y_1y_2-\alpha_3x_1x_2+u_3 \\ \dot{e}_4 &= de_4+y_1y_3-\alpha_4x_1x_3+u_4 \end{aligned} \tag{6}$$

We consider the adaptive controller defined by:

$$\begin{aligned} u_1 &= -\hat{a}(y_2-\alpha_1x_2-e_1)-y_4+\alpha_1x_4-k_1e_1 \\ u_2 &= \hat{c}e_2-y_1y_3-\alpha_2x_1x_3-k_2e_2 \\ u_3 &= -\hat{b}e_3-y_1y_2+\alpha_3x_1x_2-k_3e_3 \\ u_4 &= -\hat{d}e_4-y_1y_3+\alpha_4x_1x_3-k_4e_4 \end{aligned} \tag{7}$$

where, $\hat{a}$, $\hat{b}$, $\hat{c}$, $\hat{d}$ are estimates of the parameters a, b, c, d, respectively. Substituting Eq. 6 into Eq. 6, we obtain the closed-loop error dynamics:

$$\begin{aligned} \dot{e}_1 &= (\alpha-\hat{\alpha})(y_2-\alpha_1x_2-e_1)-k_1e_1 \\ \dot{e}_2 &= (c-\hat{c})e_2-k_2e_2 \\ \dot{e}_3 &= -(b-\hat{b})e_3-k_3e_3 \\ \dot{e}_4 &= (d-\hat{d})e_4k_4e_4 \end{aligned} \tag{8}$$

We define the parameter estimation errors as:

$$\begin{aligned} e_a &= a-\hat{a} \\ e_b &= b-\hat{b} \\ e_c &= c-\hat{c} \\ e_d &= d-\hat{d} \end{aligned} \tag{9}$$

Using Eq. 9, the error dynamics Eq. 8 is simplified as:

$$\begin{aligned} \dot{e}_1 &= e_a(y_2-\alpha_1x_2-e_1)-k_1e_1 \\ \dot{e}_2 &= e_ce_2-k_2e_2 \\ \dot{e}_3 &= -e_be_3-k_3e_3 \\ \dot{e}_4 &= e_de_4-k_4e_4 \end{aligned} \tag{10}$$

For the derivation of the update law for adjusting the estimates of parameters, the Lyapunov method is used. We consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_4^2+e_a^2+e_b^2+e_c^2+e_d^2\right) \tag{11}$$

which is positive definite on R^8. We note that:

$$\begin{aligned} \dot{e}_a &= -\dot{\hat{a}} \\ \dot{e}_b &= -\dot{\hat{b}} \\ \dot{e}_c &= -\dot{\hat{c}} \\ \dot{e}_d &= -\dot{\hat{d}} \end{aligned} \tag{12}$$

Differentiating Eq. 11 along the trajectories of the system Eq. 12 and using Eq. 12, we find that:

$$\begin{aligned} \dot{V} &= -k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2+ \\ & e_a\left[e_1(y_2-\alpha_1x_2-e_1)-\dot{\hat{a}}\right]+ \\ & e_b\left[-e_3^2-\dot{\hat{b}}\right]+e_c\left[-e_2^2-\dot{\hat{c}}\right]+e_d\left[-e_4^2-\dot{\hat{d}}\right] \end{aligned} \tag{13}$$

In view of Eq. 13, the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= e_1(y_2-\alpha_1x_2-e_1)+k_5e_a \\ \dot{\hat{b}} &= -e_3^2+k_6e_b \\ \dot{\hat{c}} &= e_2^2+k_7e_c \\ \dot{\hat{d}} &= e_4^2+k_8e_d \end{aligned} \tag{14}$$

where the gains $k_{5\text{-}8}$ are positive constants.

Theorem 1: The adaptive control law (Eq. 7) achieves General Projective Synchronization (GPS) between the identical hyperchaotic Lu systems Eq. 3 and 4 where the parameter update law is given by Eq. 14 and the gains k_i, (i = 1, 2, ..., 8) are positive constants. The GPS errors e_i, (i = 1, 2, 3, 4) and the parameter estimation errors e_a, e_b, e_c, e_d converge exponentially to zero as $t\to\infty$ for all initial conditions.

Proof: Upon substituting the parameter update law Eq. 13 into the Eq. 14, we obtain the derivative of the quadratic Lyapunov function V as:

$$\dot{V} = -k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_1^2-k_5e_a^2-k_6e_b^2-k_7e_c^2-k_8e_d^2 \tag{15}$$

which is a negative definite function on R^8. Hence, by Lyapunov stability theory[45], it follows that the GPS errors $e_1\to 0$, $e_2\to 0$, $e_3\to 0$, $e_4\to 0$, exponentially as $t\to\infty$ and the parameter estimator errors $e_a\to 0$, $e_b\to 0$, $e_c\to 0$, $e_d\to 0$, exponentially as $t\to\infty$ for all initial conditions.

Numerical results: For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential Eq. 3 and 4 with the adaptive controller Eq. 7. The parameter estimates of the identical systems Eq. 3 and 4 are taken, so that, the hyperchaotic Lu systems exhibit hyperchaotic strange attractor, i.e.,

$$a = 36,\ b = 3,\ c = 20,\ d = 1.3$$

We take the state feedback gains as:

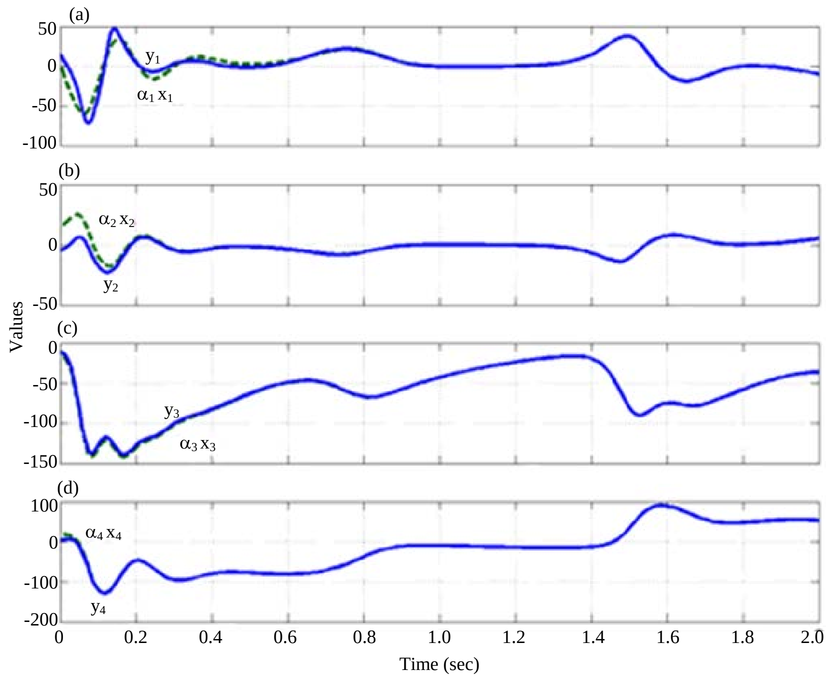

Fig. 3(a-d): GPS of the identical hyperchaotic Lu systems

$$k_i = 4 \text{ for } i = 1, 2, ..., 8$$

The initial values of the parameter estimates are chosen as:

$$\hat{a}(0) = 7, \hat{b}(0) = 8, \hat{c}(0) = 5, \hat{d}(0) = 17$$

The initial values of the master system Eq. 3 are chosen as:

$$x_1(0) = 2, x_2(0) = -23, x_3(0) = 7, x_4(0) = 12$$

The initial values of the slave system Eq. 4 are chosen as:

$$y_1(0) = 15, \ y_2(0) = -5, y_3(0) = -12, y_4(0) = 2$$

The GPS scales α_i are chosen as:

$$\alpha_1(0) = 1.8, \ \alpha_2 = -0.6, \alpha_3 = -2.4, \alpha_4 = 1.5$$

Figure 3 shows the GPS between the identical hyperchaotic Lu systems Eq. 3 and 4. Figure 4 shows the time-history of the GPS errors e_1, e_2, e_3, e_4. Figure 5 shows that the parameter estimates $\hat{a}, \hat{b}, \hat{c}, \hat{d}$ converge to the chosen values of the system parameters a, b, c, d, respectively as $t \to \infty$. Figure 6 shows the time-history of the parameter estimation errors e_a, e_b, e_c, e_d.

ADAPTIVE GENERALIZED PROJECTIVE SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC CAI SYSTEMS

Theoretical results: In this study, we deploy adaptive control to derive results for the Generalized Projective Synchronization (GPS) of the identical hyperchaotic Cai systems (2009) when the system parameters are unknown. Thus, the master system is described by the hyperchaotic Cai dynamics:

$$\begin{aligned} \dot{x}_1 &= p(x_2 - x_1) \\ \dot{x}_2 &= qx_1 + rx_2 - x_1x_3 + x_4 \\ \dot{x}_3 &= -sx_3 + x_2^2 \\ \dot{x}_4 &= -\varepsilon x_1 \end{aligned} \tag{16}$$

where, x_1, x_2, x_3, x_4 are the states and p, q, r, s, ε are unknown parameters of the system. Also, the slave system is described by the controlled hyperchaotic Cai dynamics:

$$\begin{aligned} \dot{y}_1 &= p(y_2 - y_1) \\ \dot{y}_2 &= qy_1 + ry_2 - y_1y_3 + y_4 + u_2 \\ \dot{y}_3 &= -sy_3 + y_2^2 + u_3 \\ \dot{y}_4 &= -\varepsilon y_1 + u_4 \end{aligned} \tag{17}$$

where, y_1, y_2, y_3, y_4 are the states and u_1, u_2, u_3, u_4 are the adaptive controls to be designed. The GPS synchronization errors are defined as:

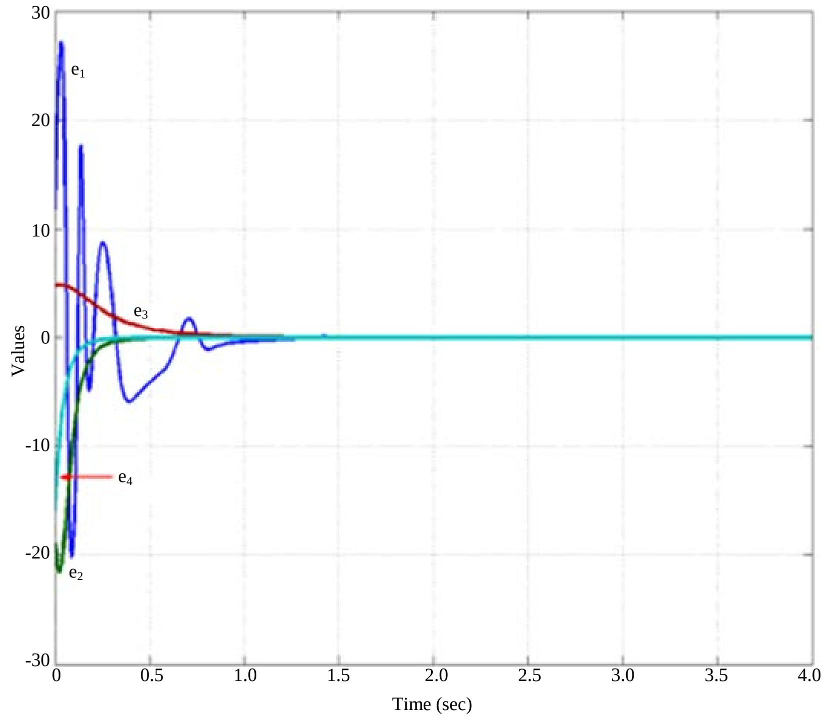

Fig. 4: Time history of the GPS errors

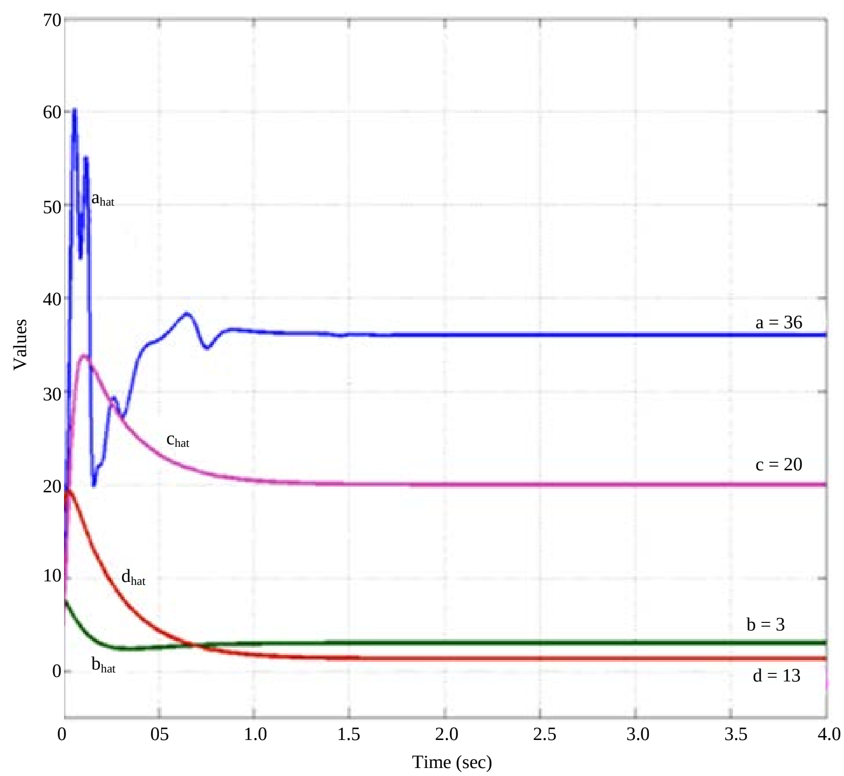

Fig. 5: Time history of the parameter estimates

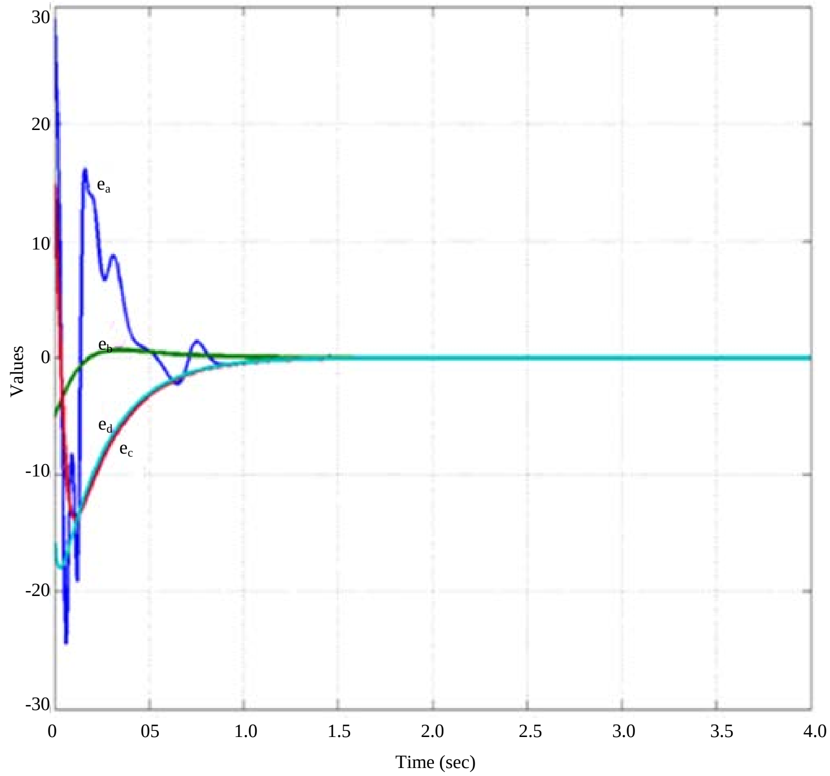

Fig. 6: Time history of the parameter estimation errors

$$e_i = y_i - \alpha_i x_i, \ (i = 1, 2, 3, 4) \tag{18}$$

where the scales α_1, α_2, α_3, α_4 are real numbers. The error dynamics is obtained as:

$$\begin{aligned} \dot{e}_1 &= p(y_2-\alpha_1 x_2-e_1)+u_1 \\ \dot{e}_2 &= q(y_1-\alpha_2 x_1)+re_2-y_1y_3+ \\ &\quad \alpha_2 x_1 x_3+y_4-\alpha_2 x_4+u_2 \\ \dot{e}_3 &= -se_3+y_2^2-\alpha_3 x_2^2 u_3 \\ \dot{e}_4 &= -\varepsilon(y_1-\alpha_4 x_1)+u_4 \end{aligned} \tag{19}$$

We consider the adaptive controller defined by:

$$\begin{aligned} u_1 &= -\hat{p}(y_2-\alpha_1 x_2-e_1)+k_1 e_1 \\ u_2 &= -\hat{q}(y_1-\alpha_2 x_1)+\hat{r}e_2+y_1y_3- \\ &\quad \alpha_2 x_1 x_3-y_4-\alpha_2 x_4-k_2 e_2 \\ u_3 &= \hat{s}e_3-y_2^2+\alpha_3 x_2^2-k_3 e_3 \\ u_4 &= \hat{\varepsilon}(y_1-\alpha_4 x_1)-k_4 e_4 \end{aligned} \tag{20}$$

where, $\hat{p}$, $\hat{q}$, $\hat{r}$, $\hat{s}$, $\hat{\varepsilon}$ pare estimates of the parameters p, q, r, s, ε, respectively. Substituting Eq. 19 into Eq. 20, we obtain the closed-loop error dynamics:

$$\begin{aligned} \dot{e}_1 &= (p-\hat{p})(y_2-\alpha_1 x_2-e_1)-k_1 e_1 \\ \dot{e}_2 &= (q-\hat{q})(y_1-\alpha_2 x_1)+(r-\hat{r})e_2-k_2 e_2 \\ \dot{e}_3 &= -(s-\hat{s})e_3-k_3 e_3 \\ \dot{e}_4 &= -(\varepsilon-\hat{\varepsilon})(y_1-\alpha_4 x_1)-k_4 e_4 \end{aligned} \tag{21}$$

We define the parameter estimation errors as:

$$\begin{aligned} e_p &= p-\hat{p} \\ e_q &= q-\hat{q} \\ e_r &= r-\hat{r} \\ e_s &= s-\hat{s} \\ e_\varepsilon &= \varepsilon-\hat{\varepsilon} \end{aligned} \tag{22}$$

Using Eq. 22, the error dynamics Eq. 21 is simplified as

$$\begin{aligned} \dot{e}_1 &= e_p(y_2-\alpha_1 x_2-e_1)-k_1 e_1 \\ \dot{e}_2 &= e_q(y_1-\alpha_2 x_1)+e_r e_2-k_2 e_2 \\ \dot{e}_3 &= -e_s e_3-k_3 e_3 \\ \dot{e}_4 &= -e_\varepsilon(y_1-\alpha_4 x_1)-k_4 e_4 \end{aligned} \tag{23}$$

For the derivation of the update law for adjusting the estimates of parameters, the Lyapunov method is used. We consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\left[e_1^2+e_2^2+e_3^2+e_4^2+e_p^2+e_q^2+e_r^2+e_s^2+e_\varepsilon^2\right] \quad (24)$$

which is positive definite on R^9. We note that:

$$\begin{aligned} \dot{e}_p &= -p \\ \dot{e}_q &= -\dot{\hat{q}} \\ \dot{e}_r &= -\dot{\hat{r}} \\ \dot{e}_s &= -\dot{\hat{s}} \\ \dot{e}_\varepsilon &= -\dot{\hat{\varepsilon}} \end{aligned} \quad (25)$$

Differentiating Eq. 23 along the trajectories of the system Eq. 23 and using Eq. 25, we find that:

$$\begin{aligned} &\dot{V} = k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2+ \\ &e_p\left[e_1(y_2-\alpha_1x_2-e_1)-\dot{\hat{p}}\right]+ \\ &e_q\left[e_2(y_1-\alpha_2x_1)-\dot{\hat{p}}\right]+e_r\left[e_2^2-\dot{\hat{r}}\right]+ \\ &e_s\left[-e_3^2-\dot{\hat{s}}\right]+e_\varepsilon\left[-e_4(y_1-\alpha_4x_1)-\dot{\hat{\varepsilon}}\right] \end{aligned} \quad (26)$$

In view of Eq. 26, the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{p}} &= e_1(y_1-\alpha_2x_1-e_1)+k_5e_p \\ \dot{\hat{q}} &= e_2(y_1-\alpha_2x_1)+k_6e_q \\ \dot{\hat{r}} &= e_2^2+k_7e_r \\ \dot{\hat{s}} &= -e_3^2+k_8e_s \\ \dot{\hat{\varepsilon}} &= -e_4(y_1-\alpha_4x_1)+k_9e_\varepsilon \end{aligned} \quad (27)$$

Theorem 2: The adaptive control law Eq. 20 achieves General Projective Synchronization (GPS) between the identical hyperchaotic Cai systems Eq. 16 and 17 where the parameter update law is given by Eq. 27 and the gains k_i, (i = 1, 2, ..., 9) are positive constants. The GPS errors e_i, (i = 1, 2, 3, 4) and the parameter estimation errors e_p, e_q, e_r, e_s, e_ε converge exponentially to zero as $t\to\infty$ for all initial conditions.

Proof: Upon substituting the parameter update law Eq. 26 into the Eq. 27, we obtain the derivative of the quadratic Lyapunov function V as:

$$\dot{V} = -k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2-k_5e_p^2-k_6e_q^2-k_7e_r^2-k_8e_s^2-k_9e_\varepsilon^2 \quad (28)$$

which is a negative definite function on R^8. Hence, by Lyapunov stability theory[45], it follows that the GPS errors $e_1\to0$, $e_2\to0$, $e_3\to0$, $e_4\to0$ exponentially as $t\to\infty$ and the parameter estimator errors $e_p\to0$, $e_q\to0$, $e_r\to0$, $e_s\to0$, $e_\varepsilon\to0$, exponentially as $t\to\infty$ for all initial conditions.

Numerical results: For the numerical simulations, the fourth order runge-kutta method is used to solve the two systems of differential Eq. 16 and 17 with the adaptive controller Eq. 20. The parameter estimates of the identical hyperchaotic Cai systems Eq. 16 and 17 are taken so that the systems exhibit hyperchaotic strange attractors, i.e:

$$p = 27.55,\ q = 3,\ r = 19.3,\ s = 2.9,\ \varepsilon = 3.3$$

We take the state feedback gains as:

$$k_i = 4 \text{ for } i = 1, 2, ..., 9$$

The initial values of the parameter estimates are chosen as:

$$\hat{p}(0) = 2,\ \hat{q}(0) = 8,\ \hat{r}(0) = 5,\ \hat{s}(0) = 13,\ \hat{\varepsilon}(0) = 24$$

The initial values of the master system Eq. 16 are chosen as:

$$x_1(0) = 4,\ x_2(0) = 11,\ x_3(0) = -3,\ x_4(0) = -10$$

The initial values of the slave system Eq. 17 are chosen as:

$$y_1(0) = -6,\ y_2(0) = -22,\ y_3(0) = 24,\ y_4(0) = 14$$

The GPS scales ai are chosen as

$$\alpha_1 = -1.9,\ \alpha_2 = -1.5,\ \alpha_3 = 2.3,\ \alpha_4 = -17$$

Figure 7 shows the GPS between the identical hyperchaotic Cai systems Eq. 16 and 17. Figure 8 shows the time-history of the GPS errors e_1, e_2, e_3, e_4. Figure 9 shows that the parameter estimates $\hat{p}, \hat{q}, \hat{r}, \hat{s}, \hat{\varepsilon}$ converge to the chosen values of the system parameters p, q, r, s, ε, respectively as $t\to\infty$. Figure 10 shows the time-history of the parameter estimation errors e_p, e_q, e_r, e_s, e_ε.

ADAPTIVE GENERALIZED PROJECTIVE SYNCHRONIZATION OF NON-IDENTICAL HYPERCHAOTIC LU AND HYPERCHAOTIC CAI SYSTEMS

Theoretical results: In this study, we deploy adaptive control to derive results for the Generalized

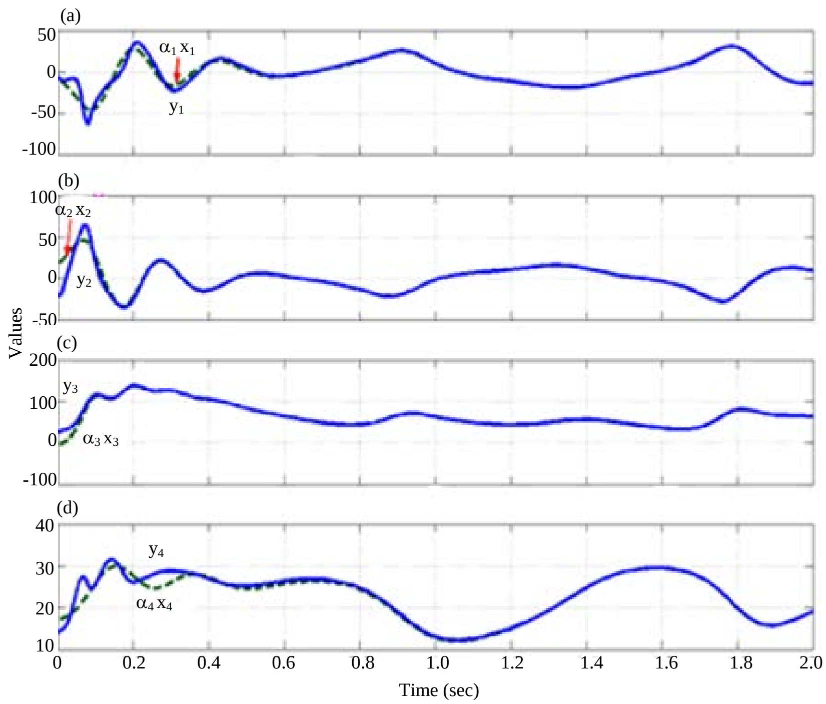

Fig. 7(a-d): GPS of the identical hyperchaotic Cai systems

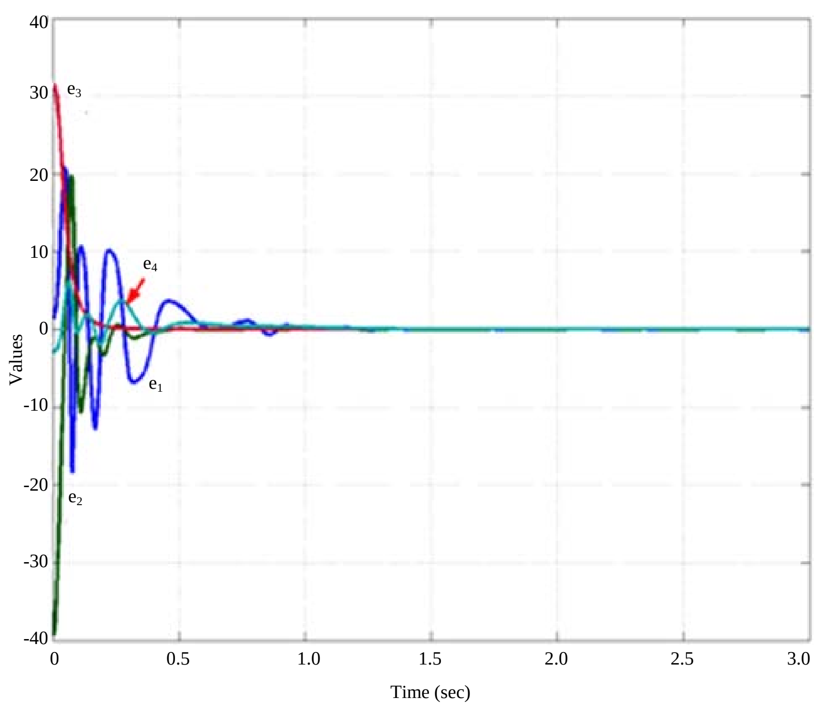

Fig. 8: Time history of the GPS errors

Projective Lu system (2006) and hyperchaotic Cai system (2009), when the system parameters are unknown. Thus, the master system is described by the hyperchaotic Lu dynamics: Synchronization (GPS) of the non-identical hyperchaotic.

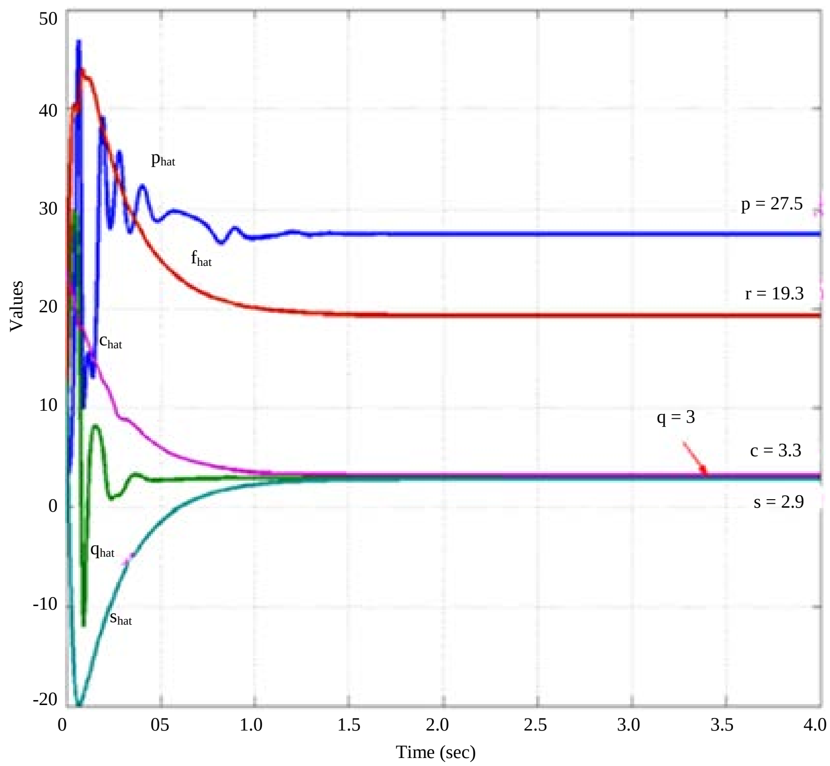

Fig. 9: Time history of the parameter estimates

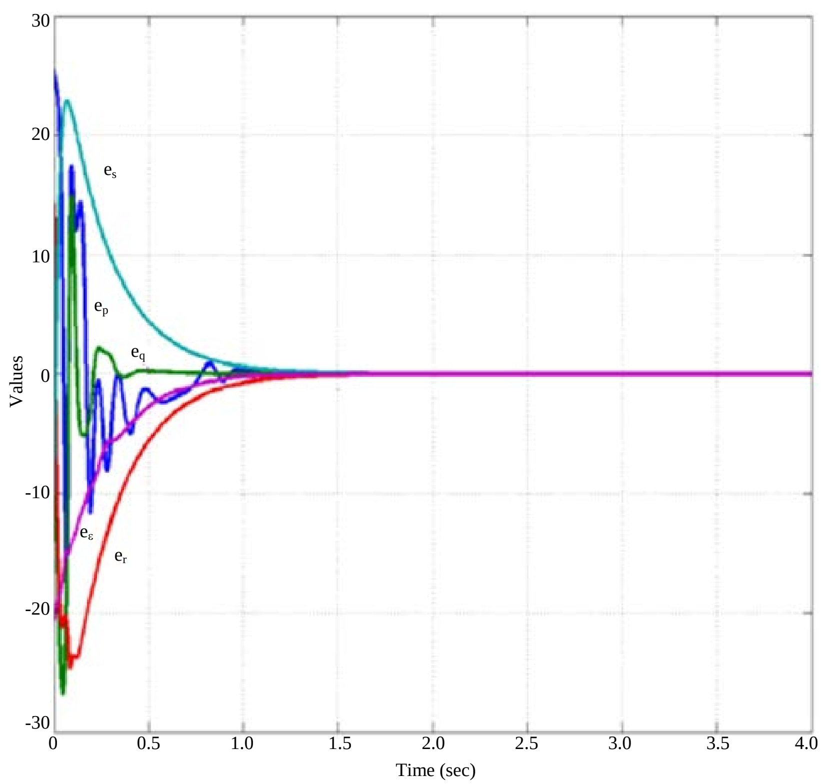

Fig. 10: Time history of the parameter estimation errors

$$\begin{aligned} \dot{x}_1 &= a(x_2-x_1)+x_4 \\ \dot{x}_2 &= cx_2-x_1x_3 \\ \dot{x}_3 &= -bx_3+x_1x_2 \\ \dot{x}_4 &= dx_4+x_1x_3 \end{aligned} \tag{29}$$

where, x_1, x_2, x_3, x_4 are the states and a, b, c, d are unknown parameters of the system. Also, the slave system is described by the controlled hyperchaotic Cai dynamics:

$$\begin{aligned} \dot{y}_1 &= p(y_2-y_1)+u_1 \\ \dot{y}_2 &= qy_1+ry_2-y_1y_3+y_4+u_2 \\ \dot{y}_3 &= -sy_3+y_2^2+u_3 \\ \dot{y}_4 &= -\varepsilon y_1+u_4 \end{aligned} \tag{30}$$

where y_1, y_2, y_3, y_4 are the states, p, q, r, s, ε are unknown parameters of the system and u_1, u_2, u_3, u_4 are the adaptive controls to be designed. The GPS synchronization errors are defined as:

$$e_i = y_i-\alpha_i x_i,\ (i = 1, 2, 3, 4) \tag{31}$$

where the scales α_1, α_2, α_3, α_4 are real numbers. The error dynamics is obtained as:

$$\begin{aligned} \dot{e}_1 &= p(y_2-y_1)-\alpha_1\left[a(x_2-x_1)+x_4\right]+u_1 \\ \dot{e}_2 &= qy_1+ry_2-y_1y_3+y_4-\alpha_2\left[cx_2-x_1x_3\right]+u_2 \\ \dot{e}_3 &= -sy_3+y_2^2-\alpha_3\left[-bx_3+x_1x_2\right]+u_3 \\ \dot{e}_4 &= -\varepsilon y_1-\alpha_4\left[dx_4+x_1x_3\right]+u_4 \end{aligned} \tag{32}$$

We consider the adaptive controller defined by:

$$\begin{aligned} \dot{u}_1 &= -\hat{p}(y_2-y_1)-\alpha_1\left[\hat{a}(x_2-x_1)+x_4\right]-k_1e_1 \\ u_2 &= -\hat{q}y_1-\hat{r}y_2+y_1y_3-y_4+\alpha_2\left[\hat{c}x_2-x_1x_3\right]k_2e_2 \\ u_3 &= \hat{s}y_3-y_2^2+\alpha_3\left[-\hat{b}x_3+x_1x_2\right]-k_3e_3 \\ u_4 &= \hat{\varepsilon}y_1+\alpha_4\left[\hat{d}x_4+x_1x_3\right]-k_4e_4 \end{aligned} \tag{33}$$

where $\hat{a}, \hat{b}, \hat{c}, \hat{d}, \hat{p}, \hat{q}, \hat{r}, \hat{s}, \hat{\varepsilon}$ are estimates of the parameters a, b, c, d, p, r, s, ε, respectively. Substituting Eq. 32 into Eq. 33 we obtain the closed-loop error dynamics:

$$\begin{aligned} \dot{e}_1 &= (p-\hat{p})(y_2-y_1)-\alpha_1(a-\hat{a})(x_2-x_1)-k_1e_1 \\ \dot{e}_2 &= (q-\hat{q})y_1+(r-\hat{r})y_2-\alpha_2+(c-\hat{c})x_2-k_2e_2 \\ \dot{e}_3 &= -(s-\hat{s})y_3+\alpha_3\left(b-\hat{b}\right)x_3-k_3e_3 \\ \dot{e}_4 &= -(\varepsilon-\hat{\varepsilon})y_1-\alpha_4\left(d-\hat{d}\right)x_4-k_4e_4 \end{aligned} \tag{34}$$

We define the parameter estimation errors as:

$$\begin{aligned} &e_a = a-\hat{a},\ e_b = b-\hat{b},\ e_c = c-\hat{c} \\ &e_d = d-\hat{d},\ e_p = p-\hat{p},\ e_q = q-\hat{q} \\ &e_r = r-\hat{r},\ e_s = s-\hat{s},\ e_\varepsilon = \varepsilon-\hat{\varepsilon} \end{aligned} \tag{35}$$

Using Eq. 35, the error dynamics Eq. 34 is simplified as:

$$\begin{aligned} \dot{e}_1 &= e_p(y_2-y_1)-\alpha_1e_a(x_2-x_1)-k_1e_1 \\ \dot{e}_2 &= e_qy_1+e_ry_2-\alpha_1e_cx_2-k_2e_2 \\ \dot{e}_3 &= -e_sy_3+\alpha_3e_bx_3-k_3e_3 \\ \dot{e}_4 &= -e_\varepsilon y_1-\alpha_4e_dx_4-k_4e_4 \end{aligned} \tag{36}$$

For the derivation of the update law for adjusting the estimates of parameters, the Lyapunov method is used. We consider the quadratic Lyapunov function defined by:

$$V = \frac{1}{2}\left[e_1^2+e_2^2+e_3^2+e_4^2+e_a^2+e_b^2+e_c^2+e_d^2+e_p^2+e_q^2+e_r^2+e_s^2+e_\varepsilon^2\right] \tag{37}$$

which is positive definite on R^{13}. We note that:

$$\begin{aligned} &\dot{e}_a = -\dot{\hat{a}},\ \dot{e}_b = -\dot{\hat{b}},\ \dot{e}_c = -\dot{\hat{c}} \\ &\dot{e}_d = -\dot{\hat{d}},\ \dot{e}_p = -\dot{\hat{p}},\ \dot{e}_q = -\dot{\hat{q}} \\ &\dot{e}_r = -\dot{\hat{r}},\ \dot{e}_s = -\dot{\hat{s}},\ \dot{e}_\varepsilon = -\dot{\hat{\varepsilon}} \end{aligned} \tag{38}$$

Differentiating Eq. 36 along the trajectories of the system Eq. 37 and using Eq. 38, we find that:

$$\begin{aligned} &e_a\left[-\alpha_1e_1(x_2-x_1)-\dot{\hat{a}}\right]+e_b\left[-\alpha_3e_3x_3-\dot{\hat{b}}\right]+ \\ &e_c\left[-\alpha_2e_2x_2-\dot{\hat{c}}\right]+e_d\left[-\alpha_4e_4x_4-\dot{\hat{d}}\right]+e_p\left[e_1(y_2-y_1)-\dot{\hat{p}}\right]+ \\ &e_q\left[e_2y_1-\dot{\hat{q}}\right]+e_r\left[e_2y_2-\dot{\hat{r}}\right]+e_s\left[-e_3y_3-\dot{\hat{s}}\right]+e_\varepsilon\left[-e_4y_1-\dot{\hat{\varepsilon}}\right] \end{aligned} \tag{39}$$

In view of Eq. 39, the estimated parameters are updated by the following law:

$$\begin{aligned} \dot{\hat{a}} &= -\alpha_1e_1(x_2-x_1)+k_5e_a \\ \dot{\hat{b}} &= \alpha_3e_3x_3+k_6e_b \\ \dot{\hat{c}} &= \alpha_2e_2x_2+k_7e_c \\ \dot{\hat{d}} &= \alpha_4e_4x_4+k_8e_d \\ \dot{\hat{p}} &= e_1(y_2-y_1)+k_9e_p \\ \dot{\hat{q}} &= e_2y_1+k_{10}e_q \\ \dot{\hat{r}} &= e_2y_2+k_{11}e_r \\ \dot{\hat{s}} &= -e_3y_3+k_{12}e_s \\ \dot{\hat{\varepsilon}} &= -e_4y_1+k_{13}e_\varepsilon \end{aligned} \tag{40}$$

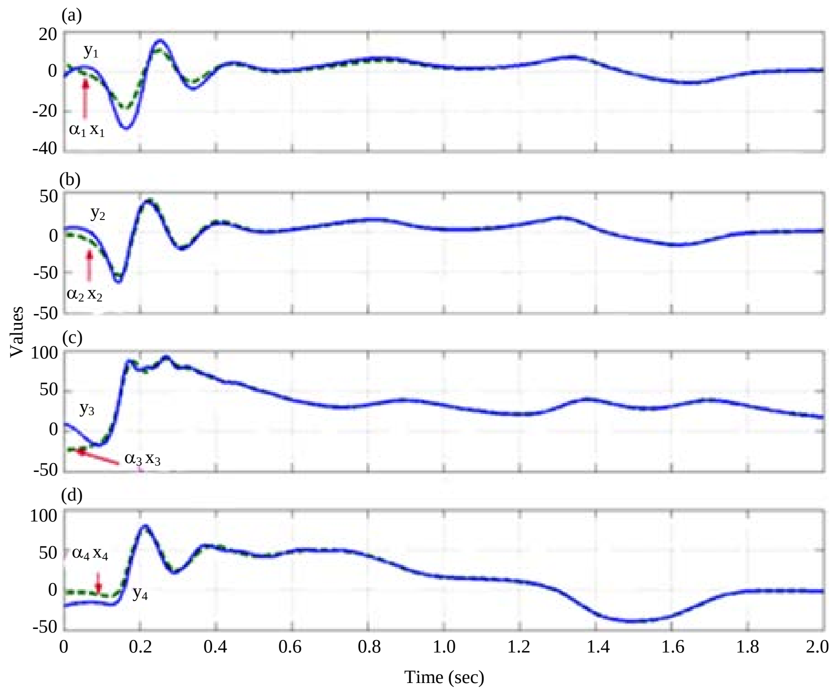

Fig. 11(a-d): GPS of the hyperchaotic Lu and hyperchaotic Cai systems

where the gains k_i, (i = 5, ..., 13) are positive constants.

Theorem 3: The adaptive control law Eq. 33 achieves General Projective Synchronization (GPS) between the non-identical hyperchaotic Lü system Eq. 29 and the hyperchaotic Cai system Eq. 30 where the parameter update law is given by Eq. 40) and the gains k_i, (i = 1, 2, ..., 13) are positive constants. The GPS errors e_i, (i = 1, 2, 3, 4) and the parameter estimation errors e_a, e_b, e_c, e_d, e_p, e_q, e_r, e_s, e_ε converge exponentially to zero as $t\to\infty$ for all initial conditions.

Proof: Upon substituting the parameter update law Eq. 40 into the Eq. 39 we obtain the derivative of the quadratic Lyapunov function V as:

$$\dot{V} = -k_1e_1^2-k_2e_2^2-k_3e_3^2-k_4e_4^2-k_5e_a^2-k_6e_b^2-k_7e_c^2-k_8e_d^2-k_9e_p^2-k_{10}e_q^2-k_{11}e_r^2-k_{12}e_s^2-k_{13}e_\varepsilon^2 \quad (41)$$

which is negative definite function on R^{13}. Hence, by Lyapunov stability theory Eq. 40 it follows that the GPS errors $e_1\to 0$, $e_2\to 0$, $e_3\to 0$, $e_4\to 0$ exponentially as $t\to\infty$ and the parameter estimator errors $e_a\to 0$, $e_b\to 0$, $e_c\to 0$, $e_d\to 0$, $e_p\to 0$, $e_q\to 0$, $e_r\to 0$, $e_s\to 0$, $e_\varepsilon\to 0$, exponentially as $t\to$ for all initial conditions (Fig. 11).

Numerical results: For the numerical simulations, the fourth order Runge- Kutta method is used to solve the two systems of differential Eq. 29 and 30 with the adaptive controller Eq. 33. The parameter estimates of the non-identical hyperchaotic Lu system Eq. 29 and the hyperchaotic Cai system Eq. 30 are chosen, so that, the systems exhibit 3-scroll chaotic attractors, i.e:

$$a = 36,\ b = 3,\ c = 20,\ d = 1.3,\ p = 27.5$$
$$q = 3,\ r = 19.3,\ s = 2.9,\ \varepsilon = 3.3$$

We take the state feedback gains as:

$$k_i = 4 \text{ for } i = 1, 2, ..., 13$$

The initial values of the parameter estimates are chosen as:

$$\hat{a}(0) = 2,\ \hat{b}(0) = 9,\ \hat{c}(0) = 12,\ \hat{d}(0) = 6$$
$$\hat{p}(0) = 4,\ \hat{q}(0) = 1,\ \hat{r}(0) = 7,\ \hat{s}(0) = 3,\ \hat{\varepsilon}(0) = 18$$

The initial values of the master system Eq. 16 are chosen as (Fig. 12 and 13):

$$x_1(0) = 10,\ x_2(0) = -4,\ x_3(0) = -17,\ x_4(0) = 6$$

The initial values of the slave system Eq. 17 are chosen as:

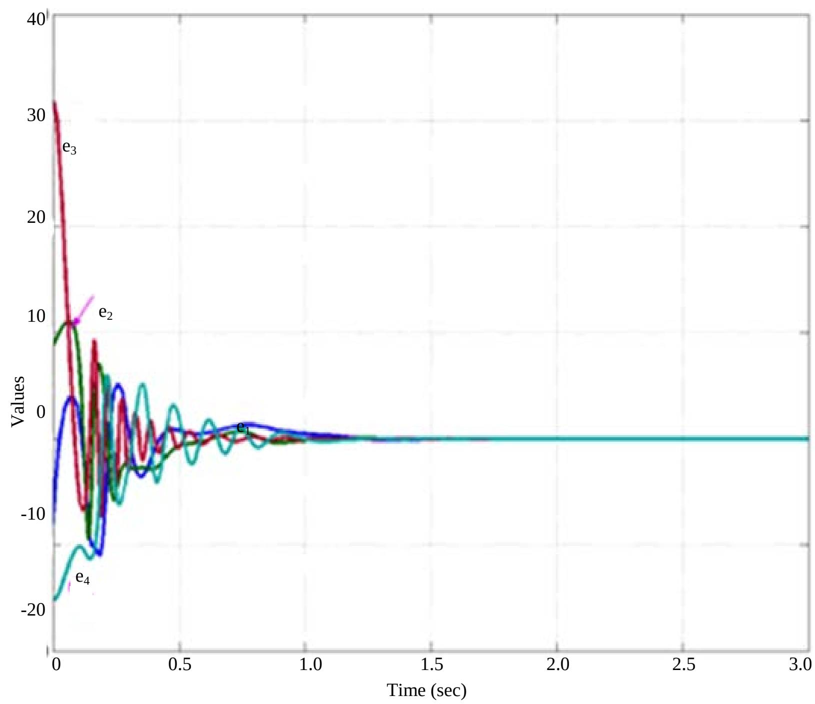

Fig. 12: Time history of the GPS errors

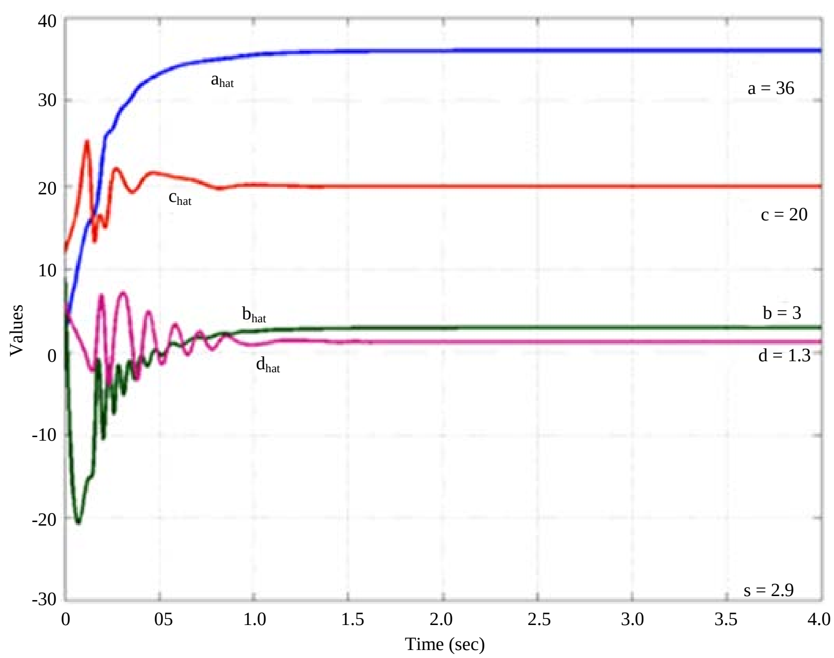

Fig. 13: Time history of the parameter estimates

$$y_1(0) = -3,\ y_2(0) = 4,\ y_3(0) = 8,\ y_4(0) = -20$$

The GPS scales α_i are chosen as:

$$\alpha_1 = 0.5,\ \alpha_2 = 1.2,\ \alpha_3 = 1.4,\ \alpha_4 = -0.8$$

Figure 11 shows the GPS between the non-identical hyperchaotic Lu and hyperchaotic Cai systems. Figure 12 shows the time-history of the GPS errors e_1, e_2, e_3, e_4. Figure 13 shows that the parameter estimates $\hat{a}, \hat{b}, \hat{c}, \hat{d}$, converge to the chosen values of the

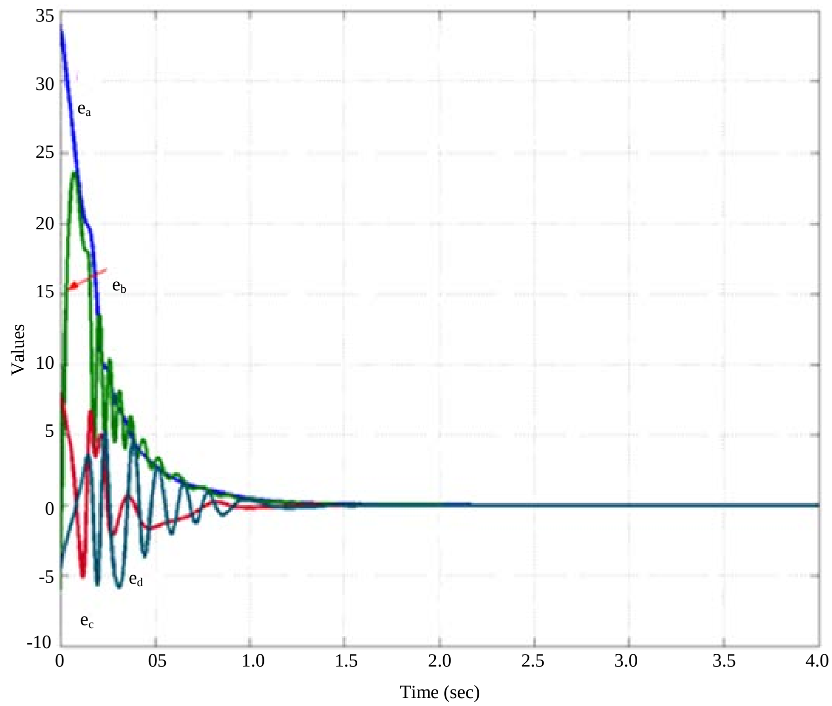

Fig. 14: Time history of the parameter estimation errors e_a, e_b, e_c, e_d

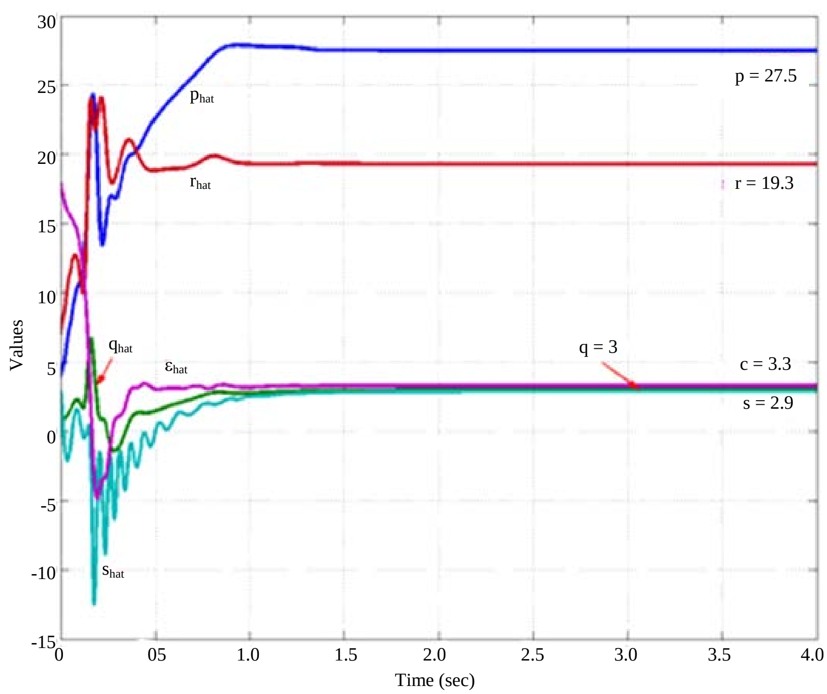

Fig. 15: Time history of the parameter estimates $\hat{p}, \hat{q}, \hat{r}, \hat{s}, \hat{\varepsilon}$

system parameters a, b, c, d, respectively as $t\to\infty$. Figure 14 shows the time-history of the parameter estimation errors e_a, e_b, e_c, e_d. Figure 15 shows that the parameter estimates $\hat{p}, \hat{q}, \hat{r}, \hat{s}, \hat{\varepsilon}$ converge to the chosen values of the system parametersp, q, r, s, ε respectively as $t\to\infty$. Figure 16 shows the time-history of the parameter estimation errors e_p, e_q, e_r, e_s, e_ε.

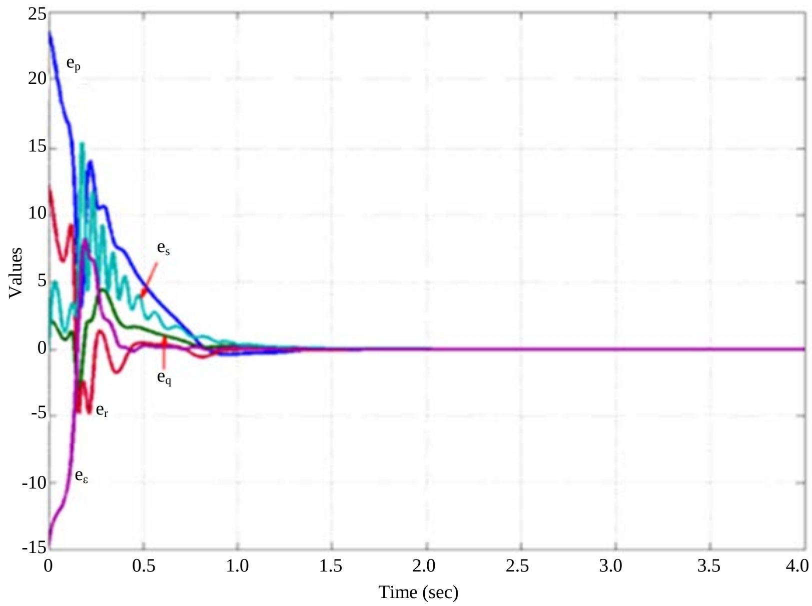

Fig. 16: Time history of the parameter estimation errors e_p, e_q, e_r, e_s, e_ε

CONCLUSION

In this study, we have designed adaptive controllers for achieving Generalized Projective Synchronization (GPS) of hyperchaotic systems, viz. the identical hyperchaotic Lu systems (2006), the identical hyperchaotic Cai systems (2009) and the non-identical hyperchaotic Lu and Cai systems when the system parameters are unknown. The adaptive GPS synchronization results for the hyperchaotic systems addressed in this paper have been proved using the Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed adaptive control method is very effective and convenient for the GPS of hyperchaotic systems. Numerical simulations have been presented to validate and demonstrate the effectiveness of the GPS synchronization results derived in this paper for the hyperchaotic systems.

REFERENCES

01. Alligood, K.T., T. Sauer and J.A. Yorke, 1997. Chaos: An Introduction to Dynamical Systems. Springer-Verlag, New York..
02. Lorenz, E.N., 1963. Deterministic nonperiodic flow. J. Atmos. Sci., 20: 130-141.
03. Lakshmanan, M. and K. Murali, 1996. Chaos in Nonlinear Oscillators: Controlling and Synchronization. World Scientific, Singapore, ISBN-13: 9789810221430, Pages: 325.
04. Han, S.K., C. Kerrer and Y. Kuramoto, 1995. Dephasing and bursting in coupled neural oscillators. Phys. Rev. Lett., 75: 3190-3193.
05. Blasius, B., A. Huppert and L. Stone, 1999. Complex dynamics and phase synchronization in spatially extended ecological system. Nature, 399: 354-359.
06. Cuomo, K.M. and A.V. Oppenheim, 1993. Circuit implementation of synchronized chaos with applications to communications. Phys. Rev. Lett., 71: 65-68.
07. Kocarev, L. and U. Parlitz, 1995. General approach for chaotic synchronization with application to communication. Phys. Rev. Lett., 74: 5028-5031.
08. Murali, K. and M. Lakshmanan, 2003. Secure communication using a compound signal using sampled-data feedback. Applied Math. Mech., 11: 1309-1315.
09. Rossler, O.E., 1979. An equation for hyperchaos. Phys. Lett. A, 71: 155-157.
10. Pecora, L.M. and T.L. Carroll, 1990. Synchronization in chaotic systems. Phys. Rev. Lett., 64: 821-824.
11. Ott, E., C. Grebogi and J.A. Yorke, 1990. Controlling chaos. Phys. Rev. Lett., 64: 1196-1199.
12. Ho, M.C. and Y.C. Hung, 2002. Synchronization of two different chaotic systems using generalized active control. Phys. Lett. A, 301: 424-428.
13. Huang, L., R. Feng and M. Wang, 2004. Synchronization of chaotic systems via nonlinear control. Phys. Lett. A, 320: 271-275.
14. Chen, H.K., 2005. Global chaos synchronization of new chaotic systems via nonlinear control. Chaos Solitons Fractals, 23: 1245-1251.
15. Sundarapandian, V., 2011. Global chaos synchronization of arneodo and shimizu-morioka chaotic systems by active nonlinear control. Int. J. Adv. Sci. Technol., 2: 32-42.

16. Lu, J., X. Wu, X. Han and J. Lu, 2004. Adaptive feedback synchronization of a unified chaotic system. Phys. Lett. A, 329: 327-333.
17. Chen, S. and J. Lu, 2002. Synchronization of an uncertain unified system via adaptive control. Chaos Solitons Fractals, 14: 643-647.
18. Sundarapandian, V., 2011. Adaptive control and synchronization of a highly chaotic attractor. Int. J. Inform. Syst. Tech., 1: 1-11.
19. Sundarapandian, V., 2011. Adaptive control and synchronization of uncertain Liu-Chen-Liu system. Int. J. Comput. Inform. Syst., 3: 1-6.
20. Sundarapandian, V., 2011. Adaptive control and synchronization of hyperchaotic Newton-Leipnik system. Int. J. Adv. Inform. Technol., 1: 22-33.
21. Park, J.H. and O.M. Kwon, 2003. A novel criterion for delayed feedback control of time-delay chaotic systems. Chaos Solitons Fractals, 17: 709-716.
22. Wang, C. and S.S. Ge, 2001. Adaptive synchronization of uncertain chaotic systems via backstepping design. Chaos Solitons Fractals, 12: 1199-1206.
23. Xiau-Qun, W. and L. Jun-An, 2003. Parameter identification and backstepping control of uncertain system. Chaos, Solitons Fractals, 18: 721-729.
24. Park, J.H., 2006. Synchronization of Genesio chaotic system via backstepping approach. Chaos Solitons Fractals, 27: 1369-1375.
25. Vincent, U.E., 2007. Controlling directed transport in inertia ratchets via adaptive backstepping control. Acta Phys. Pol. B, 38: 2459-2469.
26. Lee, S.H., V. Kapila, M. Porfiri and A. Panda, 2010. Master-slave synchronization of continuously and intermittently coupled sampled-data chaotic oscillators. Commun. Nonlinear Sci. Numer. Simul., 15: 4100-4113.
27. Slotine, J.J. and S.S. Sastry, 1983. Tracking control of non-linear systems using sliding surface with application to robotic manipulators. Int. J. Control, 38: 465-492.
28. Utkin, V.I., 1993. Sliding mode control design principles and applications to electric drives. IEEE Trans. Ind. Electron., 40: 23-36.
29. Sundarapandian, V., 2011. Global chaos synchronization of Lu-Chen-Cheng four-scroll chaotic attractors by sliding mode control. Comput. Sci. Eng. Int. J., 1: 26-35.
30. Vaidyanathan, S. and S. Sampath, 2011. Sliding mode controller design for global chaos synchronization of hyperchaotic Lorenz systems. Comput. Sci. Eng.: Int. J., 1: 61-70.
31. Lu, J., X. Yu and G. Chen, 2004. Chaos synchronization of general complex dynamical networks. Phys. A. Stat. Mech. Appl., 334: 281-302.
32. Wang, Y.W. and Z.H. Guan, 2006. Generalized synchronization of continuous chaotic systems. Chaos Solitons Fractals, 27: 97-101.
33. Zhang, X. and H. Zhu, 2008. Anti-synchronization of two different hyperchaotic systems via active and adaptive control. Int. J. Nonlinear Sci., 6: 216-223.
34. Chiang, T., J. Lin, T. Liao and J. Yan, 2008. Anti-synchronization of uncertain unified chaotic systems with dead-zone nonlinearity. Nonlinear Anal., 68: 2629-2637.
35. Sundarapandian, V. and R. Karthikeyan, 2012. Adaptive anti-synchronization of uncertain Tigan and Li systems. J. Eng. Applied Sci., 7: 45-52.
36. Vaidyanathan, S. and R. Karthikeyan, 2011. Anti-synchronization of Lu and Pan chaotic systems by adaptive nonlinear control. Int. J. Soft Comput., 6: 111-118.
37. Sundarapandian, V., 2011. Hybrid synchronization of hyperchaotic rossler and hyperchaotic lorenz systems by active control. Int. J. Adv. Sci. Technol., 2: 1-10.
38. Sundarapandian, V., 2011. Hybrid synchronization of Lorenz and Pehlivan systems by active nonlinear control. Int. J. Adv. Sci. Technol., 2: 10-20.
39. Sundarapandian, V., 2011. Hybrid synchronization of Harb and Wang chaotic systems by nonlinear control. Int. J. Comput. Inform. Syst., 3: 7-12.
40. Jia, Q., 2007. Projective synchronization of a new hyperchaotic lorenz system. Phys. Lett. A, 370: 40-45.
41. Li, X., 2009. Generalized projective synchronization using nonlinear control method. Int. J. Nonlinear Sci., 8: 79-85.
42. Sarasu, P. and V. Sundarapandian, 2011. Active controller design for generalized projective synchronization of four-scroll chaotic systems. Int. J. Syst. Signal Control Engin. Appli., 4: 26-33.
43. Chen, A., J. Lu, J. Lu and S. Yu, 2006. Generating hyperchaotic Lu attractor via state feedback control. Phys. A. Stat. Mech. Appl., 364: 103-110.
44. Wang, H. and G. Cai, 2009. Controlling hyperchaos in a novel hyperchaotic system. J. Inf. Comput. Sci., 4: 251-258.
45. Hahn, W., 1967. The Stability of Motion. Springer-Verlag, Berlin.