

## Speed Control of Ward Leonard Layout System using $H^\infty$ Optimal Control

<sup>1</sup>Mustefa Jibril, <sup>1</sup>Messay Tadese and <sup>2</sup>Eliyas Alemayehu Tadese

<sup>1</sup>*School of Electrical and Computer Engineering, Dire Dawa Institute of Technology, Dire Dawa, Ethiopia*

<sup>2</sup>*Faculty of Electrical and Computer Engineering, Jimma Institute of Technology, Jimma, Ethiopia*

**Key words:** Ward Leonard layout,  $H^\infty$  optimal control synthesis controller,  $H^\infty$  optimal control synthesis via.  $\gamma$ -iteration controller

### Corresponding Author:

Mustefa Jibril

*School of Electrical and Computer Engineering, Dire Dawa Institute of Technology, Dire Dawa, Ethiopia*

Page No.: 26-29

Volume: 14, Issue 1, 2021

ISSN: 1997-5422

International Journal of Systems Signal Control and Engineering Application

Copy Right: Medwell Publications

**Abstract:** In this study, modelling designing and simulation of a Ward Leonard layout system is done using robust control theory. In order to increase the performance of the Ward Leonard layout system with  $H^\infty$  optimal control synthesis and  $H^\infty$  optimal control synthesis via.  $\gamma$ -iteration controllers are used. The open loop response of the Ward Leonard layout system shows that the system needs to be improved. Comparison of the Ward Leonard layout system with  $H^\infty$  optimal control synthesis and  $H^\infty$  optimal control synthesis via.  $\gamma$ -iteration controllers to track a desired step speed input have been done. Finally, the comparative simulation results prove the effectiveness of the proposed Ward Leonard layout system with  $H^\infty$  optimal control synthesis controller in improving the percentage overshoot and the settling time.

## INTRODUCTION

Ward Leonard layout, additionally referred to as the Ward Leonard Drive system become a widely used DC motor speed manipulate system added by way of Harry Ward Leonard in 1891. It was applied to railway locomotives utilized in World War I and become utilized in anti-aircraft radars in World War II<sup>[1]</sup>. Connected to automated anti-aircraft gun administrators, the monitoring motion in two dimensions needed to be extraordinarily smooth and particular. The MIT Radiation Laboratory decided on Ward-Leonard to equip the well-known radar SCR-584 in 1942. The Ward Leonard layout become widely used for elevators till thyristor drives have become available inside the Nineteen Eighties because it supplied easy velocity control and steady torque. Many Ward Leonard control structures and versions on them stay in use<sup>[2]</sup>.

## MATERIALS AND METHODS

### Mathematical modelling of the Ward Leonard layout:

The Ward Leonard layout system is shown in Fig. 1. The equations of the Ward Leonard layout are as follows. The Kirchhoff's law of voltages of the excitation field of the generator G is:

$$V_f = R_f i_f + L_f \frac{di_f}{dt} \quad (1)$$

The voltage  $v_g$  of the generator G is proportional to the current  $i_f$ , i.e.,

$$V_g = K_f i_f$$

The voltage  $v_m$  of the motor M is proportional to the angular velocity  $\omega_m$ , i.e.,

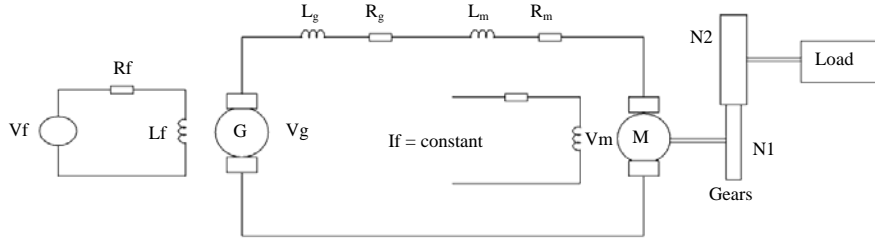


Fig. 1: Ward Leonard layout

$$V_m = K_2 \omega_m$$

The differential equation for the current  $i_a$  is

$$(R_g + R_m)i_a + (L_g + L_m)\frac{di_a}{dt} = V_g - V_m = K_1 i_f - K_2 \omega_m \quad (2)$$

The torque  $T_m$  of the motor is proportional to the current  $i_a$ , i.e.,

$$T_m = K_3 i_a$$

The rotational motion of the rotor is described by

$$\left( J_m + \left( \frac{N_1}{N_2} \right)^2 J_L \right) \frac{d\omega_m}{dt} + \left( B_m + \left( \frac{N_1}{N_2} \right)^2 B_L \right) \omega_m = K_3 i_a \quad (3)$$

Here,  $J_m$  is the moment of inertia and  $B_m$  the viscosity coefficient of the motor; likewise, for  $J_L$  and  $B_L$  of the load. From the above relations, we can determine the transfer function of the Ward Leonard (WL) layout (including the load):

$$G_{WL}(s) = \frac{\omega_L}{V_f(s)} = \frac{K_1 K_2 \frac{N_1}{N_2}}{\left( (L_g + L_m)s + (R_g + R_m) \right) \left( (L_f s + R_f) \left[ \left( J_m + \left( \frac{N_1}{N_2} \right)^2 J_L \right) s + \left( B_m + \left( \frac{N_1}{N_2} \right)^2 B_L \right) \right] + K_2 K_3 \right)}$$

Where:

$$\omega_y = \frac{N_1}{N_2} \omega_m$$

The parameters of the system is shown in Table 1. The transfer function of the system becomes:

$$G(s) = \frac{1}{209.8s^3 + 806.7s^2 + 971s + 357.8}$$

And the state space representation becomes:

Table 1: System parameter

Parameter	Symbol	Values
Motor coil inductance	$L_m$	18 H
Motor coil resistance	$R_m$	20 $\Omega$
Moment of inertia of the motor	$J_m$	66
Damping coefficient of the motor	$B_m$	28
Moment of inertia of the Load	$J_L$	23
Damping coefficient of the Load	$B_L$	18
Generator Coil inductance	$L_g$	16 H
Generator coil resistance	$R_g$	28 $\Omega$
Generator field inductance	$L_f$	10 H
Generator field resistance	$R_f$	18 $\Omega$
Generator voltage constant	$K_1$	8
Motor voltage constant	$K_2$	16
Motor torque constant	$K_3$	18
Gear one	$N_1$	64
Gear two	$N_2$	32

$$\dot{x} = \begin{pmatrix} -9.9242 & -16.1380 & -5.9529 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} u$$

$$y = (0 \ 0 \ 41.6667) x$$

### The proposed controllers design

**$H_\infty$  optimal control synthesis controller design:**  $H_\infty$  optimal control synthesis solve the small-gain infinity-norm robust control problem; i.e., find a stabilizing controller  $F(s)$  for a system  $P(s)$  such that the closed-loop transfer function satisfies the infinity-norm inequality:

$$\|T_{y,u}\|_\infty \triangleq \sup \sigma_{\max}(T_{y,u}(j\omega)) < 1$$

The block diagram of the system with  $H_\infty$  optimal control synthesis controller is shown in Fig. 2. An important use of the infinity-norm control theory is for direct shaping of closed-loop singular value Bode plots of control systems. In such cases, the system  $P(s)$  will typically be the plant augmented with suitable loop-shaping filters. The  $H_\infty$  optimal control synthesis controller transfer function is:

$$F(s) = \frac{0.269s^3 + 1.034s^2 + 1.245s + 0.4587}{s^4 + 3.856s^3 + 4.669s^2 + 1.755s + 0.01709}$$

**H<sup>∞</sup> optimal control synthesis via  $\gamma$ -iteration controller design:**

H<sup>∞</sup> optimal control synthesis via  $\gamma$ -iteration compute the optimal H<sup>∞</sup> controller using the loop-shifting two-Riccati formulae<sup>[3, 4]</sup>. The output is the optimal “ $\gamma$ ” for which the cost function can achieve under a preset tolerance:

$$\left\| \begin{bmatrix} \gamma T_{y,u_1} (\text{gamin d}) \\ T_{y,u_1} (\text{otherind}) \end{bmatrix} \right\|_{\infty} \leq 1$$

The search of optimal g stops whenever the g relative error between two adjacent stable solutions is less than the tolerance specified. For most practical purposes, the tolerance can be set at 0.01 or 0.001<sup>[5]</sup>. The block diagram of the system with H<sup>∞</sup> optimal control synthesis via  $\gamma$ -iteration controller is shown in Fig. 3.

The H<sup>∞</sup> optimal control synthesis via  $\gamma$ -iteration controller transfer function is:

$$G(s) = \frac{0.2585s^3 + 0.9939s^2 + 1.196s + 0.4408}{s^4 + 3.856s^3 + 4.669s^2 + 1.755s + 0.01709}$$

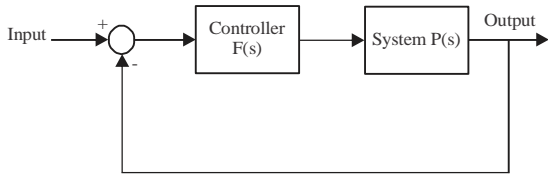


Fig. 2: Block diagram of the system with H<sup>∞</sup> optimal control synthesis controller

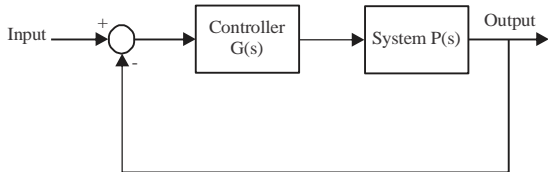


Fig. 3: Block diagram of the system with H<sup>∞</sup> optimal control synthesis via  $\gamma$ -iteration controller

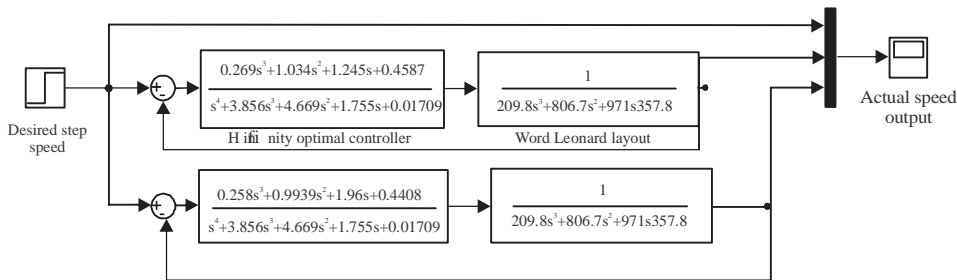


Fig. 6: Simulink model of the Ward Leonard layout system with H<sup>∞</sup> optimal control synthesis and H<sup>∞</sup> optimal control synthesis via  $\gamma$ -iteration controllers

**RESULTS AND DISCUSSION**

**Ward Leonard layout system open loop response:** The Simulink model of the open loop Ward Leonard layoutsystem and the simulation result of the system for a constant field voltage input of 100 volt is shown in Fig. 4 and 5, respectively<sup>[6-8]</sup>.

The simulation result shows that the Ward Leonard layout output speed is 0.75 rad/sec which needs a performance improvement.

**Comparison of the proposed controllers for tracking a desired step speed:** The Simulink model of the Ward Leonard layout system with H<sup>∞</sup> optimal control synthesis and H<sup>∞</sup> optimal control synthesis via  $\gamma$ -iteration controllers are shown in Fig. 6.

The simulation result of the Ward Leonard layout system with H optimal control synthesis and H<sup>∞</sup> optimal

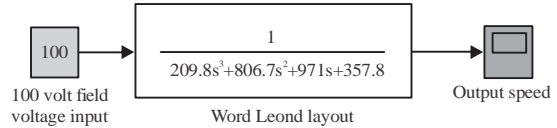


Fig. 4: Simulink model of the open loop of Ward Leonard layout system

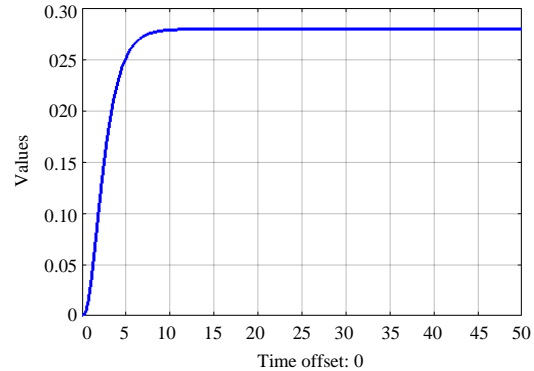


Fig. 5: Simulation result; Open loop Ward Leonard layout output speed

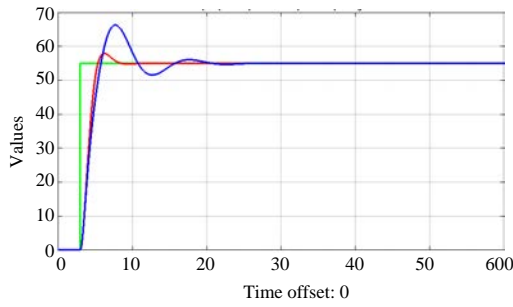


Fig. 7: Simulation result; Output speed response for a step desired speed signal

Table 2: Step response data

Performance data	H <sup>∞</sup> optimal	H <sup>∞</sup> optimal via. $\gamma$ -iteration
Rise time	3.8 sec	3.8 sec
Per. overshoot	3.63%	21.8%
Settling time	8 sec	26 sec
Peak value	57 rad/sec	67 ad/sec

control synthesis via.  $\gamma$ -iteration controllers for tracking a desired step speed (from 0-55 rad/sec) input is shown in Fig. 7. The performance data of the rise time, percentage overshoot, settling time and peak value is shown in Table 2.

### CONCLUSION

In this study, a Ward Leonard layoutsystem is designed using a DC motor generator combination. In order to improve the performance of the system, a robust control technique with H<sup>∞</sup> optimal control synthesis and H<sup>∞</sup> optimal control synthesis via.  $\gamma$ -iteration controllers are used. The open loop response of the system shows that the system needs improvement. The comparison of the proposed controllers is done to track a desired step speed and the results proves that the system with H<sup>∞</sup> optimal

control synthesis controller improves the settling time and the percentage overshoot than the system with H<sup>∞</sup> optimal control synthesis via.  $\gamma$ -iteration controller.

### REFERENCES

01. Telford, C.L. and R.H. Todd, 2013. An investigation of the ward leonard system for use in a hybrid or electric passenger vehicle. Proceedings of the International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, August 4-7, 2013, American Society of Mechanical Engineers, Portland, Oregon, USA., pp: 1-10.
02. Mustefa, J., M. Tadesse and E.A. Tades, 2020. DC motor speed control with the presence of input disturbance using neural network based model reference and predictive controllers. Int. Res. J. Modernization Eng. Technol. Sci., 2: 103-110.
03. Kofinas, P. and A.I. Dounis, 2018. Fuzzy Q-learning agent for online tuning of PID controller for DC motor speed control. Algorithms, Vol. 11, No. 10. 10.3390/a11100148
04. Mikova, L., I. Virgala and M. Kelemen, 2016. Speed control of DC motor. Am. J. Mech. Eng., 4: 380-384.
05. Priyanka, K. and A. Mariyammal, 2018. DC motor control using PWM. Int. J. Innovative Sci. Res. Technol., 3: 584-587.
06. Ismail, N.L., K.A. Zakaria, N.M. Nazar, M. Syaripuddin, A.S.N. Mokhtar and S. Thanakodi, 2018. DC motor speed control using fuzzy logic controller. AIP Conf. Proc. Vol. 1930, No. 1.
07. Tadesse, M., M. Jibril and E. Alemayehu, 2020. Performance investigation of DC motor angular velocity using optimal and robust control method. Int. J. Adv. Res. Innovative Ideas Edu., 6: 1016-1022.
08. Hassan, A.A., N.K. Al-Shamaa and K.K. Abdalla, 2017. Comparative study for DC motor speed control using PID controller. Int. J. Eng. Technol., 9: 4181-4192.