Lactation Curve Models for Predicting Milk Yield and Different Factors Affecting Lactation Curve

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Abstract: Milk production during the entire lactation is a continuous physiological function which describes the rate of milk secretion with advancement in lactation. The biometrical properties of lactation are different in different genetic groups, even if the environmental and managerial factors are constant. There are various non-genetic factors which affects the shape of lactation curve. It can be observed that, despite of different quantitative levels of milk production and the trend of milk production over time shows the same tendency with an initial growing phase up to a maximum, followed by a more or less slow decline.

INTRODUCTION

The lactation curve can be defined as the graphical representation of milk yield against time (Brody *et al*., 1923). The typical shape of the lactation curve has two characteristic parts, i.e. a rapid increase from calving to a peak period in early stage of lactation and a gradual decline from peak yield to the end of lactation (Leon-Velarde *et al*., 1995). However, some workers suggested that there are three different stages of the lactation curve, namely ascending phase, persistent phase and descending phase. Various models have been tried by different workers to fit the lactation curve in dairy cattle. The basic trend of the lactation curve shape can be shown in Fig. 1 where there is a rapid increase in milk production until the maximum or peak production (y_m), the time at which peak occurs (y_t) and the rate of decrease of milk yield in the second phase of lactation, the second phase of the lactation curve is the longest phase of the lactation, some workers suggest it as declining phage of the

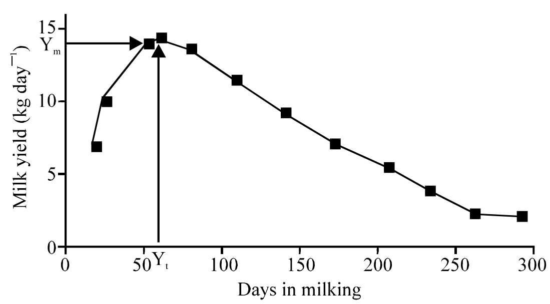

Fig. 1: The standard lactation curve of indigenous dairy cattle

lactation curve. The inverse of which represents the persistency of lactation. Therefore, the persistency is defined as the ability of the lactating animal to maintain a more or less constant milk yield in the declining phase of lactation and it represents the inherent capacity of the animal for sustainable milk production. Economically, the configuration of lactation curve is important since, the animals which produces milk yield at a moderate level steadily throughout the lactation is preferred than one which produces more at peak but little thereafter. The abrupt decline in the milk production after peak increases production cost because yield is distributed less equally over the complete lactation and also the cost of milk production depends to a large extent on the lactation yield and persistency. It has been reported that, cows with flat lactation curves are less susceptible to metabolic disorders, health and fertility problems (Solkner and Fuchs, 1987; Dekkers *et al*., 1998).

There are lots of advantages of evaluation or modeling the lactation curve such as to predict the milk yield of a cow in a lactation with minimum error and to use it in the process of cow/sire evaluation and thus enabling an anticipate choice of animals that have to be culled or that are affected by some disease but that do not show clinical signs (Vargas *et al*., 2000). It also helps for predicting expected missing values on field records and gives concise summary of biological efficiency of dairy cows and persistency of cow. Further, the knowledge of lactation curves in dairy cattle is important for decisions on herd management and selection strategies and is a key element in determining optimum strategies for insemination and replacement of dairy cows as well as for genetic evaluation of dairy cows for improvement of milk production traits (Macciota *et al*., 2005). Since, the shape of lactation curve is being influenced by several environmental as well as genetic factors, the individual cow are subjected to random variation and different lactation curve models very in their goodness of fit to individual lactation.

LACTATION CURVE MODELS

Different workers has suggested different models for modeling the lactation curve and also modified it timely. However, following are some of the lactation curve models given by different workers:

- Exponential decline function (Brody *et al*., 1923)
- Parabolic exponential model (Sikka, 1950)
- Inverse polynomial model (Nelder, 1966)
- Gamma type function (Wood, 1967)
- Quadratic model (Dave, 1971)
- Quadratic-cum-log model (Malhotra *et al*., 1980)
- Exponential function (Wilmink, 1987)
- Polynomial Regression function (Ali and Scaeffer, 1987)
- Simple linear regression (Madalena *et al*., 1979)
- Cobby and Le Du model (Cobby and Le Du, 1978)
- Linear modal model (Molina and Boschini, 1979)
- Linear cum log model (Singh and Gopal, 1982)
- Reparametrized wood's model (Dhanoa, 1981)
- Papajcsik and Bodero models (Papajcsik and Bodero, 1988)
- Multiphasic logistic function (Grossman and Koops, 1988)
- Mixed Log model (Guo and Swalve, 1995)

Most of these models have been used on the buffalo and on exotic cattle. Very few models however have been used in the indigenous cattle. Out of all these models used, the present review focused on some of them used extensively in the dairy cattle especially in the indigenous cattle.

Exponential decline function (Brody *et al*., 1923): According to Gahlot *et al*. (1988) the exponential function could not describe the initial increase in milk yield in Rathi cattle. Yadav and Sharma (1985) also concluded that this model explains only the declining phase of the lactation curve in Hariana halfbreds. However, Kolte *et al*. (1986) in Sahiwal cattle reported that exponential decline function explained lower R^2-value (26%) and slightly higher standard error of estimate (0.391) and suggested that this function will be less reliable. Similar results were reported by Singh and Bhat (1978) in Hariana cows and Yadav *et al*. (1977b) in Hariana cows and its Friesian crosses.

Parabolic exponential model (Sikka, 1950): Sikka (1950) reported that parabolic exponential function was superior to exponential decline function in describing the lactation curve in Ayrshire cattle. Singh and Bhat (1978) observed that lactation of varying duration (<44 weeks) was better explained by parabolic exponential function in Hariana cattle. However, contrary results were reported by Yadav and Sharma (1985) that this function described only the declining trend and could not define the shape of lactation curve efficiently in Hariana halfbreds. According to Singh and Bhat (1978) and Gahlot *et al*. (1988) the model gave good fit (R^2 = 74%) for milk yield during first lactation but did not fit at all before the peak was attained in Hariana and Rathi cattle, respectively. It was so because the function was symmetric around the peak yield.

Inverse polynomial model (Nelder, 1966): Inverse polynomial model was developed by Nelder (1966). Singh and Bhat (1978) reported that inverse polynomial explained 99.9% of the variation in Hariana cattle. Gahlot *et al*. (1988) stated that inverse polynomial function gives best fit in Rathi cattle. However, this function was superior to exponential decline function for explaining lactation curve using weekly milk data of Hariana cows (Singh and Bhat, 1978). Singh and Bhat (1978) also reported 99.9% variability in lactation milk yield can be explained by this function. Kolte *et al*. (1986)

reported that this function provided the best fit with the highest R^2-value (57.5%) which was followed by gamma-type function in Sahiwal cattle. Batra (1986) observed that this function provided a better fit than gamma function based on comparison of R^2-value for weekly milk yields. Yadav *et al*. (1997a) found inverse polynomial function explained 99% of the variation in average lactation curve in Hariana cattle as compared to 95% of variation explained by both gamma and parabolic exponential functions. However, Olori *et al*. (1999) showed that this function under-predicted the milk yield around peak production and then over-predicted immediately afterwards in Holstein Friesian cattle.

Gamma type function (Wood, 1967): Various studies have been done for fitting gamma type function to test day milk yield records in indigenous, crossbred as well as exotic dairy cattle. The gamma type function have been used in most lactation curve model studies because it includes the basic features of lactation curve with three parameters which allow the calculation of average production, maximum production and day of maximum production. Yadav *et al*. (1977a, b) fitted Gamma type function to average weekly yields in Hariana cows and its Friesian crosses with ½ Friesian (1/2 F) and ¾ Friesian (3/4 F) inheritance and reported goodness of fit of the function as 95, 98 and 93%, respectively. Rao and Sundaresan (1979) reported that gamma function to be best fit in describing individual lactation in Sahiwal cows. Madalena *et al*. (1979) reported 74.1% R^2-value for gamma function when fitted on the records of Holstein Friesian and Gir crosses.

Kellogg *et al*. (1977) fitted gamma function in Holstein Friesian cattle and explained as high variation as 98.8-99.8%. Singh and Bhat (1978) reported 97.3% of R^2-value for gamma function in describing the average lactation curve in Hariana cattle. Rao and Sundaresan (1979) fitted this function to daily yield in different weeks of different lactations of Sahiwal cows and found an overall goodness of fit (R^2-value) of 75.9%. Gahlot *et al*. (1988) fitted gamma function to average monthly yields in Rathi cows and reported a goodness of fit of 94.68%. Morant and Gnanasakthy (1989) reported a high and positive association among the lactation curve parameters in Holstein cattle. Whereas Tekerli *et al*. (2000) found a negative correlation between initial milk yield and the ascending phase of milk yield but a positive association was obtained between ascending and descending phase of milk yield in Holstein cattle. Silvestre etc used this function for modelling the lactation curves of dairy cattle based on test day milk yield records and concluded that the accuracy of the function decreased when the interval between test days increased. Rose (2008) reported the goodness of fit (R^2-value) of gamma type function for first lactation as 82.2% and third lactation as 88.8% in the Karan Fries cattle. However, Rashia (2010) explained the goodness of fit as 87.90% for weekly test day milk yields and 95.9% for monthly test day yields in the same breed.

Mixed Log Function (Guo and Swalve, 1995): Mixed Log Function was first constructed by Guo and Swalve (1995). This function has been used extensively in crossbred and in exotic dairy cattle. Olori *et al*. (1999) reported the R^2-value with mixed log function as 96.4% in Holstein Friesian cattle he also shown that mixed log function under predicted the milk yield around peak production and then over predicted immediately afterwards. Cilek and Keskin (2008) reported 92.7% of R^2-value of mixed log function in Simmental cows. Till date, mixed log function has not been explored to describe the lactation curve in the indigenous cattle. However, work is in progress to fit the mixed log function in Sahiwal cattle using weekly test day records in Dairy Cattle Breeding Division, National Dairy Research Institute, Karnal.

Lactation curve models reported by different workers along with their coefficient of determination (R^2-value) have been shown in Table 1. The literature in the table has been reviewed on the models which have been extensively used in dairy cattle.

Table 1: Coefficient of determination (R^2) of various lactation curve models

Models	Location	Breed	R^2- value (%)	References
Parabolic exponential function	IVRI, Izatnagar	HF X Haryana	95	Yadav *et al.* (1977a, b)
Parabolic exponential function	IVRI, Izatnagar	Hariana	93.6	Singh and Bhat (1978)
Parabolic exponential function	Hissar	Crossbred	53.6	Yadav and Sharma (1985)
Parabolic exponential function	Nagpur	Sahiwal	26.4	Kolte *et al.* (1986)
Parabolic exponential function	IVRI, Izatnagar	HF X Haryana	80.2	Singh *et al*. (1987)
Parabolic exponential function	Bikaner	Rathi	74	Gahlot *et al*. (1988)
Parabolic exponential function	Krushinagar	Kankrej	99.2	Prajapati *et al*. (1992)
Parabolic exponential function	GCBF, Morvi	Gir	68.1	Pundir and Kaushik (1993)
Parabolic exponential function	Nagpur	Sahiwal	84	Gore *et al*. (1996)
Parabolic exponential function	Pantnagar	HF X Sahiwal	90.9	Kumar *et al*. (1997)
Parabolic exponential function	Pantnagar	Jersey-Sahiwal	86.1-99	Singh *et al*. (1997)
Gamma type function	IVRI, Izatnagar	HF X Haryana	99	Yadav *et al*. (1977a)
Gamma type function	IVRI, Izatnagar	Hariana	95	Yadav *et al*. (1977b)
Gamma type function	IVRI, Izatnagar	Hariana	97.3	Singh and Bhat (1978)
Gamma type function	IVRI, Izatnagar	HF	95.9	Bhat *et al*. (1978)
Gamma type function	Northern India	Sahiwal	75.9	Rao and Sundaresan (1979)

Table 1: Continue

Models	Location	Breed	R^2- value (%)	References
Gamma type function	Brazil	HF and HF X Gir	0.71-0.74	Madalena *et al*. (1979)
Gamma type function	USA	HF	86	Shanks *et al*. (1980)
Gamma type function	USA	HF	59-88	Shanks (1981)
Gamma type function	Nigeria.	HF X Bunaji	71	Abubakar and Buvanendran (1981)
Gamma type function	Akola	Crossbreed	68.6-83.8	Pande (1983)
Gamma type function	Hissar	Crossbred	52.4-57.3	Yadav and Sharma (1985)
Gamma type function	Nagpur	Sahiwal	29	Kolte *et al*. (1986)
Gamma type function	Canada	HF	74.7	Ali and Schaeffer (1987)
Gamma type function	IVRI, Izatnagar	HF X Haryana	64.6	Singh *et al*. (1987)
Gamma type function	Udaipur	Rathi	94.6	Gahlot *et al*. (1988)
Gamma type function	China	Simmental cattle	92	Xu *et al*. (1988)
Gamma type function	Costa Rica	Jersey	64	Aguirre and Boschini (1992)
Gamma type function	GCBF, Morvi	Gir	62.7	Pundir and Kaushik (1993)
Gamma type function	Akola	Gaolao X Exotic	83.7	Pande (1983)
Gamma type function	USA	HF	89	Palmer *et al*. (1994)
Gamma type function	Ghana	HF	85	Ahunu and Kabuga (1994)
Gamma type function	Pantnagar	Jersey- Sahiwal	87.2-99.5	Singh *et al*. (1997)
Gamma type function	Pantnagar	HF X Sahiwal	92.7	Kumar *et al*. (1997)
Gamma type function	UK	HF	94.4	Olori *et al*. (1999)
Gamma type function	Turkey	HF	71	Tekerli *et al*. (2000)
Gamma type function	Turkey	S. Anatolian Red	64.6-69.7	Orman *et al*. (2000)
Gamma type function	Brazil	Caracu cows	98.6	Faro *et al*. (2001)
Gamma type function	Brazil	Caracu cows	31.1	Faro and Albuquerque (2002)
Gamma type function	Hisar	Sahiwal crosses	94.5	Singh *et al*. (2002)
Gamma type function	Turkey	HF	59.5	Kocak and Ekiz (2008)
Gamma type function	NDRI, Karnal	Karan Fries	82.2-88.8	Rose (2008)
Gamma type function	Serbia	Simmental	92.7	Cilek and Keskin (2008)
Gamma type function	Iran	HF	70	Atashi *et al*. (2009)
Gamma type function	NDRI, Karnal	Karan Fries	87.9	Rashia (2010)
Inverse polynomial function	IVRI, Izatnagar	HF X Haryana	95	Yadav *et al*. (1977a)
Inverse polynomial function	IVRI, Izatnagar	Hariana	99	Yadav *et al*. (1977b)
Inverse polynomial function	IVRI, Izatnagar	Hariana	99.9	Singh and Bhat (1978)
Inverse polynomial function	Hissar	Crossbreed	90	Yadav and Sharma (1985)
Inverse polynomial function	Nagpur	Sahiwal	57.5	Kolte *et al*. (1986)
Inverse polynomial function	Canada	Crossbreed	67.9	Batra (1986)
Inverse polynomial function	IVRI, Izatnagar	HF X Haryana	68.38-91.7	Singh *et al*. (1987)
Inverse polynomial function	Hisar	Crossbred	>90	Yadav and Sharma (1988)
Inverse polynomial function	Udaipur	Rathi	99.7	Gahlot *et al*. (1988)
Inverse polynomial function	Mhow	Jersey	81.5	Shobha and Khan (1990)
Inverse polynomial function	Jabalpur	Jersey	66.4-75.8	Roy and Katpatal (1993)
Inverse polynomial function	GCBF, Morvi	Gir	90.3	Pundir and Kaushik (1993)
Inverse polynomial function	Nagpur	Sahiwal	99	Gore *et al*. (1996)
Inverse polynomial function	Pantnagar	HF X Sahiwal	99.5	Kumar *et al*. (1997)
Inverse polynomial function	Pantnagar	Jersey X Sahiwal	93.1-99.6	Singh *et al*. (1997)
Inverse polynomial function	UK	HF	97.9	Olori *et al*. (1999)
Inverse polynomial function	Turkey.	HF	96	Tekerli (2000)
Inverse polynomial function	Hissar	Sahiwal	98	Singh *et al*. (2002)
Inverse polynomial function	Brazil	Caracu cows	97.6	Faro *et al*. (2001)
Inverse polynomial function	Brazil	Caracu cows	28.4	Faro and Albuquerque (2002)
Inverse polynomial function	USA	HF	97.9	Sharifi *et al*. (2009)
Exponential decline function	IVRI, Izatnagar	HF X Hariana	95	Yadav *et al*. (1977a)
Exponential decline function	Costa Rica	Guernsey	>0.90	Boschini and Sanchez (1980)
Exponential decline function	Hissar	Crossbred	28.8-61.1	Yadav and Sharma (1985)
Exponential decline function	Nagpur	Sahiwal	26	Kolte *et al*. (1986)
Exponential decline function	Costa Rica	Jersey	55	Aguirre and Boschini (1992)
Exponential decline function	GCBF, Morvi	Gir	40.1	Pundir and Kaushik (1993)
Exponential decline function	Nagpur	Sahiwal	83	Gore *et al*. (1996)
Exponential decline function	Pantnagar	Jersey X Sahiwal	98.3	Singh *et al*. (1997)
Exponential decline function	Pantnagar	HF X Sahiwal	90	Kumar *et al*. (1997)
Mixed log function	UK	HF	96.4	Olori *et al*. (1999)
Mixed log function	USA	HF	59.1	Kocak and Ekiz (2008)
Mixed log function	Serbia	Simmental cow	92.7	Cilek and Keskin (2008)

Table 2: Least square means for group effects on lactation curve traits (Atashi *et al*. 2009)

Lactation curve parameters	Season				Parity		
	Spring	Summer	Fall	Winter	1	2	≥3
a	2.720[a]	2.700[a]	2.670[b]	2.680[b]	2.5300[c]	2.8000[a]	2.7600[b]
b	0.260[c]	0.250[c]	0.270[b]	0.280[a]	0.2400[c]	0.2500[b]	0.2900[a]
c	0.005[b]	0.005[d]	0.005[c]	0.005[a]	0.0033[c]	0.0050[b]	0.0057[a]

a = Parameter to represent yield at the beginning of lactation, b and c are factors associated with the inclining and declining slope of the lactation curves, respectively; [a-c]Means with different superscripts within variable and lactation curve trait differ significantly

DIFFERENT FACTORS AFFECTING LACTATION CURVE

There are several factors which influences the lactation curve in dairy cattle such as the genetic background, period of calving, feeding, environmental conditions, herd, parity, season, age at calving, health status of the animal, etc. (Macciotta *et al*., 2005). However, present review focused on the five main factors, i.e., period of calving, season, age at calving, herd and parity.

Effect of parity: Rao and Sundaresan (1979) reported that the least-squares analysis of traits associated with lactation curve shape indicated that parity had a significant influence on the lactation curve. The effects of season of calving and parity reported by Atashi *et al*. (2009) have been shown in Table 2.

They also reported the R^2-value for parity groups were 65, 79 and 82% for the first, second and later lactation, respectively. Tekerli *et al*. (2000) reported that parity has significant effect on the shape of the lactation curve in Holstein Friesian cattle. Similarly, Horan *et al*. (2005) also reported that parity had a significant effect on all three lactation curve parameters in Holstein Friesian cows.

Effect of season of calving: Yadav *et al*. (1977a, b) reported that season did not affect all the component of inverse polynomial function however, the average slope of the curve get affected in Hariana and its Friesian crosses. Further, they fitted gamma function on average yields and reported a significant influence of season of calving on all the parameters of Wood function. Similarly, Rao and Sundaresan (1979) found a significant influence of season of calving on shape of the lactation curve in Sahiwal cattle. However, Mehto *et al*. (1980) reported a significant effect of season of calving on all the three parameters of gamma function in Hariana crossbreds. They further added that the initial milk production was highest in summer calvers, the milk yield in ascending phase was reportedly maximum in spring calvers and minimum in cows calving during the rainy season. However, Tekerli *et al*. (2000) reported that season of calving has significant effect on the shape of the lactation curve in Holstein Friesian cattle. Similar effect was found by Atashi *et al*. (2009) in Holstein Cattle.

Effect of period of calving: Wood (1967) observed significant effect of period calving on the inclining slope of the lactation curve in Holstein Friesian cattle. Similarly, Rao and Sundaresan (1979) found a significant influence of period on the shape of lactation curve in Sahiwal cattle. Mehto *et al*. (1980) found significant effect of year/period of calving on milk yield and declining phase of milk yield using Wood function in Hariana cattle. Dedkova and Nemcova (2003) found that period of calving had highly significant influence on the shape of lactation curve in Holstein cows using Wilmink function. Singh and Bhat (1978) reported that all the component of parabolic exponential except b (linear constant which measure the average slope of the curve) was significantly affected by period and month of calving.

Effect of age at calving: Yadav *et al*. (1977b) reported a significant effect of age at first calving on the inclining and declining parameters of Wood function and fitted Wood function to average weekly yield in Hariana and its Friesian crosses. Rao and Sundaresan (1979) reported a significant influence of age at calving on the shape of lactation curve of Sahiwal cows fitted using the Wood function. Mehto *et al*. (1980) in Hariana crossbreds observed that initial milk yield linearly increased with advancement in age of the cows in terms of her lactation sequence. However, Tekerli *et al*. (2000) reported the effect of Days In Milk (DIM) at monthly test day was highly significant ($p<0.01$) for coefficients of the lactation curve in Holstein Cows. The effect of calving age was significant ($p<0.05$) only for peak and lactation yields. Dedkova and Nemcova (2003) found that cows with lower age at calving showed best persistency while a slowing growing slop up to production peak was found with higher age at calving in Holstein cattle using Wood function.

CONCLUSION

It observed from different types of lactation curve that the trend of production over time follows the same tendency with an initial growing phase up to a maximum milk yield and then followed by a more or less slow decline. The important factors (i.e., calving year, calving season, age groups and parity) affect not only total milk yield but also the rate of milk production throughout the length of lactation (i.e., the shape of the lactation curve). Therefore, there is vast scope for the improvement in the herd by studying of lactation curve. Study of lactation curve is helpful for prediction of milk yield at any point of lactation and predicting incomplete records (missing values) with minimum error under field records. It also gives concise summary of biological efficiency of dairy cows and it also helps for designing suitable breeding and management strategies for breed improvement program in dairy cattle as well as for genetic evaluation of dairy cows.

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