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Mathematical Modeling Between Population Growth Rate and Increase Food Rate of Malthus's Theorem

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Abstract: In this study an effort has been given attention to establish a mathematical relationship between rate of increase of food and rate of population growth of Malthus's Theorem. It is seen that Malthus's Theorem of rate of increase of food and rate of population growth follows positive exponential model. To verify the stability of the model, Cross Validity-Prediction Power (CVPP) is applied in this study. It is found that the coefficient of determination of the fitted model is 100% and the fitted model is also100% stable. And its all parameters are also highly significant.

Key words: Malthus's theorem, positive exponential model, coefficient of determination (R^2) Cross-Validity Prediction Power (CVPP), t-test, F-test

INTRODUCTION

Robert[1] told that population was growing by geometrical ratio, that is, 1, 2, 4, 8, 16, 32, 64, 128 and so on. On the other hand, food was increasing by arithmetic ratio, that is, 1, 2, 3, 4, 5, 6, 7, 8 and so on. These are in fact his assumptions. He actually tried to say that population were increasing faster than food. Though, he was severely criticized for his theory during his time and even if still now. But his theory is still now applicable to the developing countries like Bangladesh. But Malthus did not give any mathematical relation between them. In the era of globalizations, mathematical modeling is very realistic and sophisticated tools to represent data in mathematics. But, modeling has rare been used in demography in Bangladesh. Moreover, model is mainly two types: Stochastic and deterministic. A deterministic model has been tried to build up between them.

Therefore, the fundamental objectives of this study are to establish a mathematical model to population growth rate and increase food rate of Malthus's Theory of population, that is, his assumptions and to apply CVPP to the model to test the validity of the model.

MATERIALS AND METHODS

Using the scattered plot of rate of increase of population growth by rate of increase of food (Fig. 1), it is observed that rate of increase of population growth is positive exponentially distributed with respect to rate of increase food. Therefore, in this case, a positive exponential model is considered and the structure of the model is[2]

$$y = e^{(a+bx)} + u$$

where, x represent rate of increase of population growth; y represent rate of increase of food; a, b are parameters and u is the disturbance term of the model.

The model has been estimated using the software STATISTICA.

Checking model validation: To check how much the model is stable, the Cross Validity Prediction Power (CVPT), ρ^2_{cv} is applied here. The formula is given below:

$$\rho^2_{cv} = 1 - \frac{(n-1)(n-2)(n+1)}{n(n-k-1)(n-k-2)}(1-R^2)$$

where, n is the number of cases, k is the number of predictors in the model and the cross validated R is the correlation between observed and predicted values of the dependent variable. The shrinkage of the model is the positive value of the difference of and R^2. Moreover, the stability of R^2 of the model is equal to (1- shrinkage)[3].

F-test: The F-test is applied to the model in this study to verify the measure of the overall significance of the model as well as the significance of R^2. The formula for F-test is as follows:

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

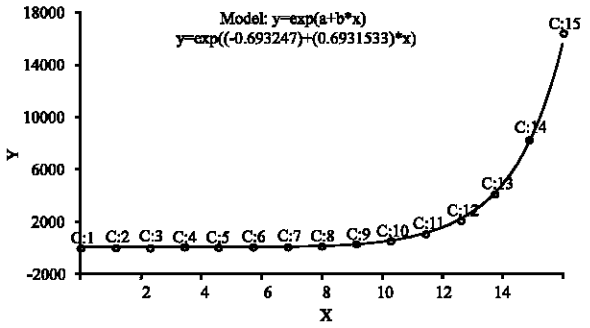

Fig. 1: Observed and fitted Malthus's assumption's of rate of increase of population growth and rate of increase of food. X: Rate of increase of food and Y: Rate of increase of population growth

with (k-1, n-k) degrees of freedom (d.f.);
where k is the number of parameters is to be estimated, n is the number of classes and R^2 =the coefficient of determination in the model[4].

RESULTS AND DISCUSSION

The fitted mathematical model between rate of population growth and rate of increase of food of Malthus's Theorem is:

y=exp(-0.693247+0.6931533x)
t-stats (-8648.54) (136978.3)
p-value (0.00) (0.0)

with coefficient of determination (R^2) is 100% and ρ^2_{cv} is also 100%. In this model shrinkage is zero (0).

From this statistics we see that the fitted model is highly cross-validated and its shrinkage is 0. These imply that the fitted model is 100% stable. Moreover, all the parameters of the fitted model are also highly statistically significant with 100% of variance explained. Moreover, the stability of R^2 of the model is equal to 100%. In this study the calculated value of F-test is infinite, that is, large quantity which means that the fitted model is overall highly significant.

It is to be mentioned here that usual models, i.e. Gompertz model, Makeham model, logistic model, negative exponential model, log-linear model and semi-log linear model were also applied but those are worst fitted with respect to their coefficient of determination and shrinkages. Therefore, the results of those models were not presented here.

CONCLUSIONS

Malthus's assumptions of rate of increase of population growth and rate of increase of food follows positive exponential model. It is also in fact a relation between arithmetic ratio and geometric ratio of positive integers.

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