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Analysis of Different Solutions to Multivariable Constrained Predictive Control: Application to A Distillation Process

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Abstract: Generalised predictive control algorithms are powerful control design methods widely applied to industrial processes. This approach is applied for control of an industrial distillation column. However, there is no easy way to solve the problem of constraints. This study presents an application of different methods of optimisation to constrained generalised predictive control, in order to achieve optimal performance. We show how these optimisations methods can be applied to solve the constrained generalised predictive control efficiently and with less computation burden than that of other methods.

Key words: Conjugate gradient, constrained GPC, optimisation, adaptive control, algorithm

INTRODUCTION

Generalised Predictive Control (GPC) algorithms which are first proposed under various names by several groups of workers Cutler and Ramaker[1], Yedstie[2], Clarke *et al.*[3], de Keyzer and Van Cauwenberghe[4] constitute a class of powerful control algorithms that have been widely applied to industriel processes. The formulations of constrained optimization problem have been proposed by Chang and Seborg[5], Campo and Morari[6], Dave *et al.*[7], Ricker[8] and Kuznetsnov and Clark[9]. Shell Oil has used the direct approach, QDMC[10], for solving the constrained optimization problem involved in the control of petroleum processes. In recent year the predictive control has become a very important area of research because through it's good performance and robustness provided that the tuning parameters have been properly selected. Controllers based on this method are capable to control difficult processes, such as processes with long time delay, non-minimum phase and unstable processes. The controllers based on the prediction theory also exhibit remarkable robustness with respect to model mismatch[3,11]. But there is still a problem to assure a reasonable control action in the case of large non-linearity with changing parameters and technological restrictions.

In this study, we present an application of different solutions to the constrained model based predictive generalized control based on a multidimensional version of this method. Using constraints in order to tune specific controllers has already been considered in several works. Kuznetsov and Clarke[9] have used constrained GPC for improving controlled plants. Ishikawa *et al.*[12] considered a practical method of removing ill-conditioning in industrial constrained predictive control; the resulting controller can suppress the excessive input movements and improve the control performance. Abou-Jeyab *et al.*[13] also used constrained multivarible predictive control to eliminate the large cycling in the product composition in a distillation column. This list shows that CGPC is very important in the control arena.

The study is organised as follows. In the first section, we review one classical formulation of GPC. It should be pointed out, however, that similar ideas can also be applied to variants of this GPC formulation, as they have been proposed in other studies. In the second section, we describe the principle of constrained predictive as applied to control system, while the third section describes the application of the different methods of optimisation algorithm to handle the constraints in the GPC.

REVIEW OF GPC

Generalised Predictive Control(GPC): The GPC method was proposed by Clark *et al.*[3] and has been implemented together with other prediction control strategies in many industriel applications. In classical representation one supposes that a model of the linear (or linearised plant) is given in the following autoregressive moving average form with exogenous inputs:

$$A(z^{-1})\Delta(z^{-1})y(t) = B(z^{-1})\Delta(z^{-1})u(t-1)+C(z^{-1})e(t) \quad (1)$$

Where: u(t), y(t) and e(t) are the input signal, output signal and disturbance process, respectively, at time t $A(z^{-1})$, $B(z^{-1})$, $C(z^{-1})$ are polynomials in the unit delay

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operator z^{-1} with A and C monic. The role of the Δ operator ($\Delta = 1-z^{-1}$) is to ensure integral action of the controller in order to cancel the effect of step varying output disturbances. The GPC algorithm consists of applying a control sequence to minimise the following multistage cost function defined as follows:

$$J(u,t) = \epsilon\{ \sum_{j=N1}^{Ny} [y(t+j)-r(t+j)]^2+\lambda \sum_{j=N1}^{Nu} [\Delta u(t+j-1)]^2\} \quad (2)$$

Subject to $\Delta u(t+j+N_u) = 0$, $j = 0,1\ldots$. In this expression, r(t+j) describes the future reference trajectory, N_y is called the prediction horizon, $N_u \le N_y$ is the control horizon, λ is a parameter which wheights the relative importance of control effort with respect to output error and ε denotes the mathematical expectation in a stochastic framework.

The GPC strategy may appear at first sight as an open loop control policy, since Nu future control increments are computed explicitly through the minimisation of Eq. 2, However at time t one solves this optimisation problem with criterion J (u, t) for control strategy $\{\Delta u(t+j)$ $j = 0,\ldots N_u-1$ $\}$but one applies only the first element, $u(t) = u(t-1)+\Delta u(t)$ and the control optimization process is carried over again at time t+1 with the criterion j(u, t+1). This is called a receding horizon control strategy.

CONSTRAINED GPC

In constrained control, a set of inequalities may be added to the control objective to limit the variations of certain variables to a given range.

$$var_{min} \le var(t+j) \le var_{max}, j = N_1,\ldots,N_J \quad (3)$$

where var(t+j) is a variable under restriction.

The main objectives of constrained predictive control are set point tracking and prevention/reduction of constrained transgressions.

The constraints can be equalities and/or inequalities:

- Equality Constraints are usually end-point constraints and are generally used to ensure stability in certain stabilizing strategies such as Clark and Scattolini[14] or Kouvaritakis *et al.*[15].
- Inequality constraints, which are usually interval constraints used to represent input/output limitations and performance specification. The introduction of inequality constraints raises important difficulties, such as how to calculate the constrained optimal control, since there is no more analytical solution to the problem of the cost function minimization subject to these inequalities. There are several algorithms solving constrained problems exactly and within a finite number of steps. They can be divided into those based on the computation of Lagrange multipliers[16,17] and those reducing a QP problem to non-negative least squares[18]. A review of the different approaches to tackle constrained GPC is given[9].

DIFFERENT SOLUTIONS PROPOSED FOR MULTIVARIABLE CASES

Constrainedgeneralised predictive control using Dichotomy method.[16]: Using the dichotomy method in constrained predictive control, first we must find the feasible region. Only those constraints that limit the feasible region of the space need to be taken into account. The method used for constraints reduction is given in[19].

Proposed Algorithm:

- **Step1:** Compute diophantine, G, fco.
- **Step 2:** Determine the feasible region for ΔU, the min and max limits of Δu, $\Delta u=1\ldots N_u$.
- **Step 3:** Take the median of each interval and get 2x N_u intervals, $[x_{1min}x_{1int}]$, $[x_{1int}x_{1max}]$, $[x_{2min}x_{2int}]$,$[x_{2min}x_{2int}]$ …
- **Step 4:** Compute the averages of each interval obtained.
- **Step 5:** Permute the averages obtained and get a matrice of dimension: ($2^{\text{number of variables}}$) x nvar the number of permutations is equal to the number of transfer functions i.e,. $2^{\text{number of variables}}$.
- **Step 6:** At each line of the matrix, compute the cost function (2): We obtain a vector that contains the values of J, for ΔU_i i=1...2^{NU}
- **Step 7:** At time t, take Δu that gives the better minimun. See in to interval ΔU belongs.
- **Step 8:** Go to $Step_3$ with Δu_i. The number of loops depends on the precision we want and the value of x_{min}, x_{max}.
- **Step 9:** Take $\Delta U = [\Delta u(t)_i \Delta u(t+1)_{i\ldots} \Delta u(t+N_U-1)]$ and compute the control input signal $u(t)=u(t-1)+\Delta u(t)$ to feed the system.

The Zoutendjik method: We proposed an iterative approach based on the method of Zoutendjik including the following points[9]:

- For every sampling period, the starting point is chosen so that all required constraints become saturated.
- We use a search direction ensuring a good

orientation of the search vector (d) toward the minimum of the objective function f (x).

- The step of displacement is calculated so that it leads every time to the decrease of the objective function.

- **Stage 1:** Initialization:
- **Stage 2:** The direction of search d_k is obtained from: for j = 1,2,...,n: (d_k):Min $d^T \nabla^2 f(x_k)\, d + \nabla^T f(x_k)\, d + f(x_k)$
- **Stage 3:** The step of displacement is obtained from: (α_k): Min f $(x_k+\alpha d_k)$ Subject to: $0 \le \alpha \le 1/2$. And the new solution is obtained by: $x_{k+1} = x_k + \alpha_k d_k$
- **Stage 4:** if $|f(x_{k+1})| < |f(x_k)|$, put k=k+1 and to go to the stage 01 Otherwise end. (x_k in this case is the solution of the problem proposed).

Economical methods: Using economical method we can give also a solution to the constrained predictive control for multidimensional systems [16]. This method is used for obtaining an approximate optimum with respect to α of the function $g(\alpha) = f(x_k+\alpha d_k)$ arising in the gradient minimisation of f(x).

GRADIANT CONJUGATE METHOD

Conjugate Direction Methods: The general principle, we are dealing with are iterative methods which, when applied to a quadratic function of n variables, lead to the optimum in a most n stages.

Application to the predictive control: Using the conjugate gradient method in constrained predictive control, first we must find the feasible region[16].

At each sampling period, we compute the control increment using the conjugate gradient method, if the solution satisfies the constraints it is implemented, otherwise it is replaced by the appropriate limits obtained from the constrained reduction method.

The proposed Algorithm can be described as follows:

- **Step 1:** Initialization of y and u, we give the values of N_1, N_2, N_U, λ
- **Step 2:** Compute diophantine, G, fco.
- **Step 3:** Determine the feasible region for ΔU Compute Δu by gradient conjugate method.
- **Step 4:** At each line of the matrix, compute the cost function (2):
- **Step 5:** At time t, take Δu that gives the better minimum.
- **Step 6:** Go to $Step_3$ with Δu_i. The number of loops depends on the precision we want and the value of x_{min}, x_{max}.
- **Step 7:** take $\Delta U=[\Delta u(t)_i\ \Delta u(t+1)_{i...}\Delta u(t+N_U-1)]$ and compute the control input signal $u(t) = u(t-1)+\Delta u(t)$ to feed to the system

The Fibonacci method: The solution based on this method is an approach without calculation of the derivative, based on an extension the method of Fibonnaci for the convex functions to several variables. This approach treats the constraints in an explicit manner while considering them directly in the space of search of the increment of orders[20].

RESULTS AND DISCUSSION

We propose to test the feasibility of the proposed methods on a typical example of multi variable system with constraints. The system is a distillation column with two inputs and two outputs, proposed as a benchmark problem in[21]. The two inputs are the reflux and vapor boilup rate and the outputs are the distillate and the bottom product. The aim of the tests will then be to check whether the all proposed methods are able to solve the problems of constraints and lead GPC to the optimal performance and to compare between all the methods. The model of the system

$$G_{11}(s) = \frac{0.878K_1e^{-\theta_1}}{75s+1}; \qquad G_{12}(s) = \frac{-0.864K_2e^{-\theta_2}}{75s+1}$$

$$G_{21}(s) = \frac{1.082K_1e^{-\theta_1}}{75s+1}; \qquad G_{22}(s) = \frac{-1.096K_2e^{-\theta_2}}{75s+1}$$

$K_i \in [0.8\ 1.2]$; et $\theta_i \in [0.0\ 1.0]$

Under matrix form:

$$G(s) = \frac{1}{75s+1}\begin{bmatrix} 0.878 & -0.864 \\ 1.082 & -1.096 \end{bmatrix}\begin{bmatrix} K_1e^{-\theta_1} & 0 \\ 0 & K_2e^{-\theta_2} \end{bmatrix}$$

The given specifications are:

- For the unit step input on the first channel of the column at t = 0, the output y_1 (t) and the output y_2 (t) must satisfy:
  - $t \ge 30$ mn $y_1(t) \ge 0.9$
  - $\forall$ t $y_1(t) \le 1.1$: maximum overshoot 10%
  - $t \to \infty$: $0.99 \le y_1(t) \le 1.1$
  - $\forall$ t: y_2 (t) 0.5
  - $t \to \infty$: $-0.01 \le y_2(t) \le 0.01$
- Closed loop stability
- The control signal is limited to 200.

Table 1: Gives a summary of the results where the unsatisfactory results are in bold

		Tracking			Interaction	
Inputs	The Methods	.t = 30	Max	.t = 100	Max	.t = 100
	Zoutendijk	1.13	1.10	1.00	0.41	-0.002
	Economical	0.91	**1.21**	1.00	**0.58**	-7.4_x10^{-4}
Set point $y_1 = 1$	Qp	**0.57**	**1.48**	**1.35**	**0.54**	**-0.33**
	Gradient	0.91	**1.21**	1.00	0.86	-7.5_x10^{-4}
Set point $y_2 = 0$	Dichotomie	**0.88**	**1.28**		0.99	0.24-0.001
	Fibonacci	**0.87**	**1.28**	1.00	0.24	0.005
	Zoutendijk	1.10	1.10	1.00	0.50	-0.01
	Economical	0.93	**1.17**	1.00	0.44	-6_x10^{-4}
Set point y_1=0	Qp	1.10	**1.54**	**1.12**	0.42	-0.19
	Gradient	0.93	**1.17**	1.00	0.43	-6_x10^{-4}
Set point y_2 =1	Dichotomie	0.95	1.10	0.99	0.38	-0.002
	Fibonacci	0.96	**1.11**	0.99	0.38	-0.011

Table 2: gives a summary of the results where the unsatisfactory results are in bold

		Tracking			Interaction	
Inputs	The methods	.t = 30	max	.t = 100	Max	.t = 100
	Zoutendijk	1.06	1.10	1.02	0.42	-0.01
	Economical	0.96	**1.21**	0.99	**0.57**	2.4_x10^{-4}
Set point y_1=1	Qp	**0.68**	**1.47**	**0.86**	**0.52**	**0.38**
	Gradient	0.95	1.20	0.99	**1.14**	2.5_x10^{-4}
Set point y_2 =0	Dichotomie	**0.67**	1.05	1.04	0.34	**-0.02**
	Fibonacci	**0.89**	1.15	1.03	0.33	**-0.02**
	Zoutendijk	1.09	1.10	1.01	0.48	-0.02
	Economical	0.97	**1.17**	0.99	0.44	1.9_x10^{-4}
Set point y_1=0	Qp	**0.71**	**1.48**	**0.57**	0.46	**0.167**
	Gradient	0.97	**1.17**	0.99	0.42	1.9_x10^{-4}
Set point y_2 =1	Dichotomie	**0.85**	1.02	1.01	0.57	**-0.03**
	Fibonacci	0.94	1.09	1.09	0.53	**-0.17**

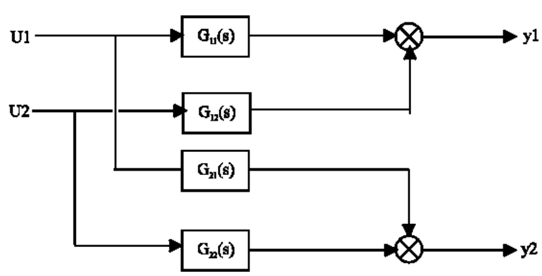

Fig. 1: Bloc diagram of the system

- The same specifications are given for the input of the second channel.

The sampling period is T_e = 0.4 min.

The constraints acting on the control increments are: $-12 \le \Delta u_i \le 12$; $|u_i| \le 200$ with i = 1, 2.

The design parameters were chosen as: $(N_2^1, N_u^1, N_2^2, N_u^2, \lambda^1, \lambda^2) = (8, 2, 10, 2, 10^{-3}, 10^{-4})$

For nominal case: $K_1 = K_2 = 1$: The references are $[y_{c1}\ y_{c2}] = [1\ 0]$ then $[y_{c1}\ y_{c2}] = [0\ 1]$. Figure 1 and 2 show the results of the simulation in the nominal case for each of the method studied. The output is y1 and y2.

As it can be seen on the figures above, the tracking of set point by the output using Fibonacci, Gradient, Dichotomie, Economical and Zoutendijk method is the same, we can see that this result is not obtained by GPC method using QP. The constraints are satisfied by all methods, the Zoutendijk method gives the best result with low increment and control action.

The comparison of the methods used is given in Table 1, in bold are the non satisfactory results.

For the case K_1 = 1.2 K_2 = 0.8: References are $[y_{c1}\ y_{c2}] = [1\ 0]$] then $[y_{c1}\ y_{c2}] = [0\ 1]$, Fig. 3 show the simulation results.

The same remarks as in the nominal case hold here. The Fibonacci method and the gradient method give a better static error (Table 2).

For $K_1 = K_2 = 1.2$: The two input references $[y_{c1}\ y_{c2}] = [1\ 0]$ and] then $[y_{c1}\ y_{c2}] = [0\ 1]$, Fig. 4 and 5 show the simulation results (Table 3).

The results for this case show that the stability and robustness is ensured for all optimisation methods.

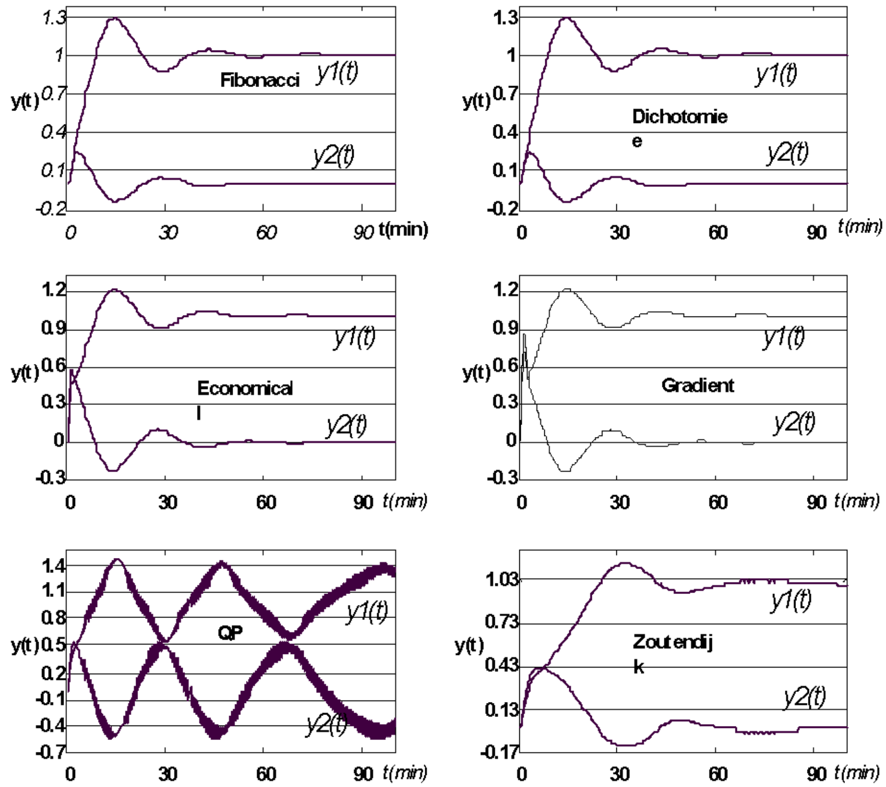

Fig. 2: Outputs y1 and y2 for the six methods with $K_1 = K_2 = 1$

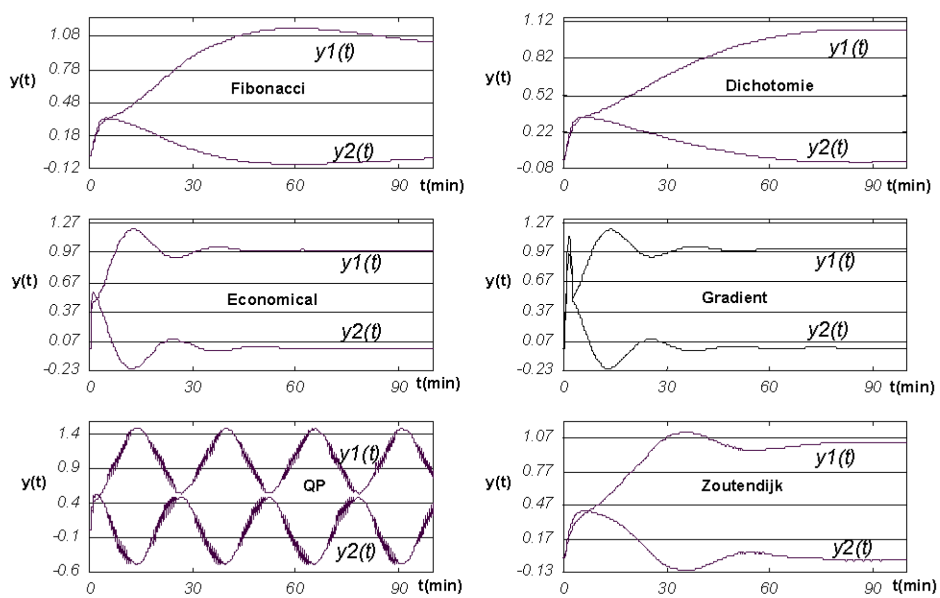

Fig. 3: Output y1 and y2 for the six methods $K_1 = 1.2$ $K_2 = 0.8$

However the performance robustness is satisfactory for the method of Zoutendijk.

For the case $K_1 = 0.8$-$K_2 = 1.2$: The two references $[y_{c1}\ y_{c2}] = [1\ 0]$ and $[y_{c1}\ y_{c2}] = [0\ 1]$ in the second case

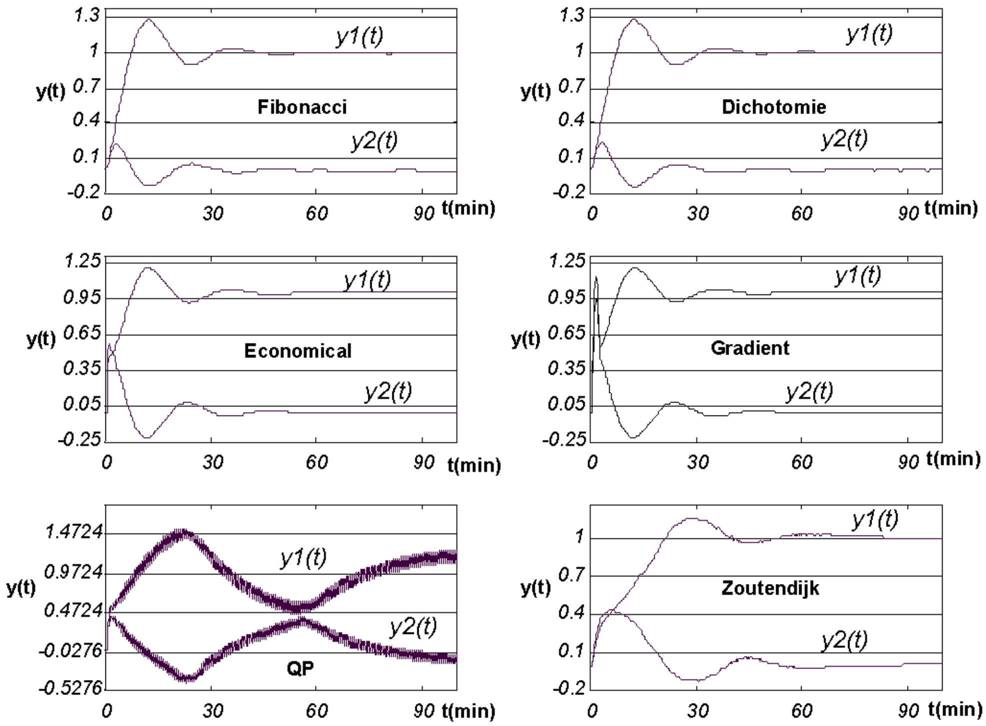

Fig. 4: Outputs y1 and y2 for the six methods $K_1 = K_2 = 1.2$

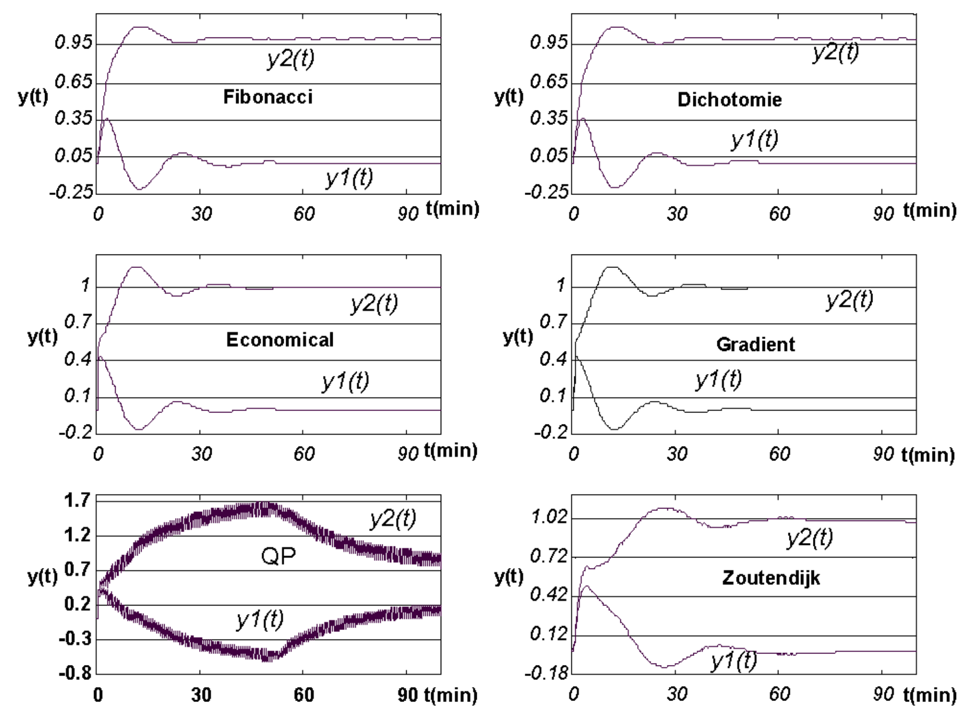

Fig. 5: Outputs y1 and y2 for the six methods $K_1 = 0.8$ $K_2 = 1.2$

Table 3: a summary of the results where the unsatisfactory results are in bold

		Tracking			Interaction	
Inputs	Methods	.t = 30	max	.t = 100	Max	.t = 100
	Zoutendijk	**1.15**	**1.15**	0.99	0.42	0.004
	Economical	0.98	**1.20**	0.99	0.57	2.2_x10^{-5}
Set point y_1=1	Qp	**-0.22**	**0.42**	**-0.19**	**1.54**	**1.12**
	Gradient	0.98	**1.20**	0.99	**1.13**	4.3_x10^{-5}
Set point y_2 =0	Dichotomie	0.96	**1.27**	1.00	0.22	0.001
	Fibonacci	0.96	**1.27**	0.98	0.22	-0.005
	Zoutendijk	1.08	1.10	0.99	0.49	0.01
	Economical	0.99	**1.17**	0.99	0.44	3.4_x10^{-5}
Set point y_1=0	Qp	1.34	**1.69**	**0.74**	0.44	**0.04**
	Gradient	0.99	**1.17**	0.99	0.43	4.2_x10^{-5}
Set point y_2 =1	Dichotomie	0.98	1.10	0.99	0.35	-0.001
	Fibonacci	0.98	1.10	1.00	0.35	-0.005

Table 4: Gives a summary of the results where the unsatisfactory results are in bold

		Tracking			Interaction	
inputs	methods	.t = 30	max	.t = 100	Max	.t = 100
	Zoutendijk	1.07	**1.13**	1.02	0.40	-0.01
	Economical	0.94	**1.22**	0.99	**0.58**	0.002
Set point y_1=1	Qp	**1.11**	**1.32**	**0.93**	**0.61**	**-0.06**
	Gradient	0.94	**1.22**	0.99	**0.64**	0.002
Set point y_2 =0	Dichotomie	**0.63**	1.05	1.04	0.35	-0.02
	Fibonacci	**0.89**	**1.15**	1.01	0.35	-0.003
	Zoutendijk	1.08	1.09	1.01	0.49	-0.02
	Economical	0.94	**1.18**	0.99	0.44	0.001
Set point y_1=0	Qp	**1.26**	**1.41**	**1.14**	0.44	**-0.19**
	Gradient	0.94	**1.18**	0.99	0.43	0.001
Set point y_2 =1	Dichotomie	**0.86**	1.01	1.01	0.50	**-0.03**
	Fibonacci	0.90	1.05	1.05	0.50	**-0.11**

From the above results we can draw the following conclusions:

- For the proposed parameters (Nu, N2, λ), the GPC algorithm using the proposed methods gives more efficient results that QP.
- One can see that the gradient method is the fasted with satisfying convergence for most values of gain. The obtained output with this method have a very high overshoot, this degrades performance robustness, the stability robustness obtained with this method is better than the other methods of optimisation. The economical method gives the solution to this problem without taking into account the imposed constraints i.e the minimum of the objective function can be found out of the region of constraints, in this case the minimum of f(x) takes maximum or minimum of this region.
- For the economical method: the convergence of the method to the solution is ensured, but with an important computation time compared to the gradient method, the obtained point x_k is very sensitive and conditioned by the choice of the initial interval [α_{min}, α_{max}], a bad choice of this interval can lead to the divergence of the method.
- Fibonacci and Dichotomy are two methods not based on the calculus of the derivative of the objective function, the tracking problem is very good, but still fluctuates in an interval of time causing the instability of the system. This two methods do not converge to the minimum of f(x) if the feasibility region of solutions is not found by the method of Sugie. For the precedent system the Fibonacci method gives better results than Dichotomie method. For the performances and stability robustness, these two methods give better results than other the ones but with gains values K_1, K_2.
- For Zoutendjik method we obtained good results, the convergence is ensured, but with a longer time of computation compared to the other methods. The control increment and control signal are always found in the feasible region with very small values. For the performances and stability robustness, this method gives better results whatever the variations of K_1, K_2 in the both directions

CONCLUSION

In this study, different methods requiring modest computational ressources and easy to implement have been introduced and applied to solve multivariable constrained generalised predictive control problem.The constraints on the manipulated and other process variables need to be satisfied. For comparison the proposed methods have performed well on the binary distillation column. From the above analysis one can draw the following results: the proposed methods are reliable in the sense that the number of iterations is controllable, this is particularly important in real time applications. The manipulated variables do not violate the constraints. The results can suggest a systemic way of selecting a method. For the systems with parameters varying rapidly the gradient is preferable methods if the solution obtained lies in the constraint region. Likewise, Fibonacci and Dichotomie methods areacceptable if the region of feasibility exists. For the process with rapid dynamic, the Zoutendjik method is the most interesting.

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