

Vibration Analysis of Rotating Machine by High Resolution Methods

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Abstract: In this study, a high resolution methods to detection, analysis and diagnosis of mechanical faults in rotating machinery is studied. For certain types of faults, for example, raceway faults in rolling element bearings, an increase in mass unbalance. The bearing defects induce vibration, this vibration can be characterized by his frequency and his amplitude.

Key words: Beaing fault diagnosis, detection, estimation, spectral analysis, high resolution, rotating machinery

INTRODUCTION

Rotating machines are used widely in industrial manufacturing plants. The industry's heavy reliance on these machines in critical applications makes catastrophic. Therefore, a significant amount of research effort is focused on the preventive maintenance of motors. Motor Vibration Signature Analysis (MVSA) provides an important way to asses the health of a machine. The present work deals with analysis and diagnosis of mechanical faults in rotating machines based on vibration analysis. The characteristic vibration frequencies due to bearing defects can be calculated from the rotor speed and bearing geometry. The typical rolling element bearing geometry is displayed in Fig. 1. the characteristic vibration frequencies, f_v can be calculated using (1)-(4)^[1].

The outer race defect frequency, f_{OD} . the ball passing frequency on the outer race, is given by:

$$f_{OD} = \frac{n}{2} f_m \left(1 - \frac{BD}{PD} \cos \phi \right) \quad (1)$$

Where ϕ is the contact angle, PD is the pitche diameter, BD is the ball diameter, n is the number of balls f_m and is the rotational speed.

The inner race defect frequency, f_D . the ball passing frequency on the inner race, is given by

$$f_{ID} = \frac{n}{2} f_m \left(1 + \frac{BD}{PD} \cos \phi \right) \quad (2)$$

The ball defect frequency, f_{BD} , the ball spin frequency, is given by

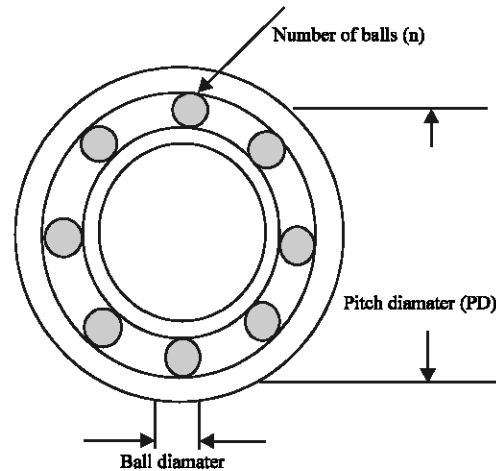


Fig. 1: Rolling element bearing geometry

$$f_{BD} = \frac{PD}{2BD} f_m \left(1 - \left(\frac{BD}{PD} \right)^2 \cos^2 \phi \right) \quad (3)$$

the train defect frequency, f_{TD} , caused by an irregularity in the train, is given by

$$f_{TD} = \frac{1}{2} f_m \left(1 - \frac{BD}{PD} \cos \phi \right) \quad (4)$$

In traditional MVSA. The Fourier transform is utilized to determine the vibration spectrum. Then, the bearing defect induced frequencies are identified and compared with initial measurement to detect any deterioration in

bearing health. The short coming of this study is that the Fourier analysis is limited to stationary signals and the vibration bearing is nonstationary by nature.

In this study, the high resolution methods based on the decomposition of the motor vibration on the q superposed sinusoids is proposed as an alternative approche^[2].

DESCRIPTION OF THE EXPERIMENT

Prony's method: To consider the frequencies characteristics of the anomalies affecting the rock, we choose a method of high resolution, because for example the acoustics characteristics of a pore or a slit can present very closed frequencies^[3,4].

Consideration of the prony method for real valued data is made first. The basic philosophy for the prony approach in this section.

The process being analyzed is assumed to consist of M sinusoidal components:

$$\begin{aligned}
 x(n\Delta T) &= \sum_{i=1}^M A_i \cos(2\pi f_i \Delta T + \phi_i) \\
 &= \sum_{i=1}^M \left[\frac{A_i}{2} \exp(j2\pi f_i \Delta T + j\phi_i) + \frac{A_i}{2} \exp(j2\pi f_i + j\phi_i) \right] \\
 &= \sum_{i=1}^M [C_i \Phi_i^n + C_i^* \Phi_i^{*n}] \tag{5}
 \end{aligned}$$

where

$$C_i = \frac{A_i}{2} \exp(j\phi_i), \Phi_i = \exp(j2\pi f_i \Delta T)$$

And * denotes complex conjugate. The equation (5) may be define by this expression:

$$\prod_{i=1}^M (\psi - \Phi_i)(\psi - \Phi_i^*) = \sum_{i=1}^{2M} a_i \psi^{2m-1} = 0 \tag{6}$$

with a₀=1 where the {Φ_i} are the roots of unity.

If we consider that the sum of the m sinusoidal can be modeled with error ε_i so ε_i = ∑_{k=0}^{2M} a_kx^{2m+1-k}

for

$$I = 1, 2, \dots, N-2M \tag{7}$$

The roots of (5) also satisfy the polynomial equation:

$$\sum_k^{2M} a_k \psi^k = 0 \quad a_{2M} = 1 \tag{8}$$

we can extract α_j = α_{2M-j} with α₀ = α_{2M} and the number of coefficients to be determined is reduced by one half.

$$\epsilon_i = \sum_{i=0}^{2M} (a_k X_{2m+1-k} + X_{2M-k+i}) \tag{9}$$

The coefficients α₁, α_M are determined in least square fashion by minimizing the total square error

$$E = \sum_{i=0}^{N-2M} \epsilon_i^2 \tag{10}$$

which yields the normal equation:

$$\sum_{j=0}^M [\sum_{i=0}^{2M} (X_{2m-k} + X_{k+i})(X_{2M-j+i} + X_{j+i})] = 0 \tag{11}$$

for k = 1,, M from the estimated {α_i} values, the {φ_i} are determined using equation (6). This gives the frequencies estimates. To obtain the {C_i} a second set off normal equation is solved.

$$\sum_{j=0}^M \left[C_i \sum_{j=0}^N \phi_k^j \phi_i^j + C_i^* \sum_{j=0}^N \phi_k^{*j} \phi_i^{*j} \right] = \sum_{j=0}^N 2 \left(\text{real} \phi_i^j \right) x_i \tag{12}$$

for k = 0, 1,, M. the {C_i} provide both amplitude A (or power) and phase {φ_i}.

PISARENKO METHOD^[5]

Given the discrete-time noisy sinusoidal measurement:

$$x(n) = s(n) + b(n), n = 1, 2, \dots, N-1 \tag{13}$$

where

$$\begin{aligned}
 s(n) &= -\sum_{m=1}^{2N} a_m s(n-m) \\
 &\text{and} \tag{14}
 \end{aligned}$$

$$x(n) = -\sum_{m=1}^{2N} a_m s(n-m) + b(n)$$

We substituted s(n-m) by, x(n-m) - b(n-m) it gives x(n) - b(n) =

$$-\left(\sum_{m=1}^{2N} a_m x(n-m) - \sum_{m=1}^{2N} a_m b(n-m)\right) \quad (15)$$

$$\sum_{m=0}^{2N} a_m x(n-m) = \sum_{m=0}^{2N} a_m b(n-m) \quad (16)$$

this equation (16) can be written into matrix form as

$$\begin{aligned} X^T(n)A &= B^T(n)A \\ \text{with} & \\ X^T(n) &= [x(n), x(n-1), \dots, x(n-2N)]^T \\ B^T(n) &= [b(n), b(n-1), \dots, b(n-2N)]^T \end{aligned} \quad (17)$$

and

$$A = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ \vdots \\ a_{2N} \end{bmatrix}$$

We multiply the equation (17) with X (n) it gives

$$[X(n)X(n)^T]A = [X(n)B(n)^T]A \quad (18)$$

and we take the mean of (18) we obtain

$$\begin{aligned} E[X(n)X(n)^T]A &= E[X(n)B(n)^T]A \\ E[X(n)X(n)^T]A &= E[(S(n) + B(n))B(n)^T]A \end{aligned} \quad (19)$$

The noise is white, Gaussian and independent to the measurement. The equation (19) can be rewritten as

$$\begin{aligned} E[X(n)X(n)^T]A &= E[(S(n) + B(n))B(n)^T]A \\ &= (E[S(n)B(n)^T] + E[B(n)B(n)^T])A \end{aligned} \quad (20)$$

$$R_{2N+1}A = E[S(n)]E[B(n)^T] + \sigma^2 IA$$

$$R_{2N+1}A = \sigma^2 A$$

Where A is the minimum eigenvector corresponding to the smallest eigenvalue of the matrix $R=[x(n)x(n)^T]$. the Z transformation of equation (14) is given by

$$S(Z) \left(1 - \sum_{m=1}^{2N} a_m Z^{-m} \right) = 0 \quad (21)$$

In order to obtain the N frequencies of the measurement, we must solve the equation (21). First we have two components of sinusoid:

$$\begin{aligned} s(n) &= A_1 \cos w_1 + A_2 \cos w_2 \\ r(n) &= E[s(n)s(n+\tau)] \\ r(n) &= E \begin{bmatrix} (A_1 \cos w_1 + A_2 \cos w_2)x \\ (A_1 \cos w_1(t+\tau) + A_2 \cos w_2(t+\tau)) \end{bmatrix} \\ r(n) &= A_1^2 E[\cos w_1 \cos w_1(t+\tau)] \\ &+ A_1 A_2 E[\cos w_1 \cos w_2(t+\tau)] + \\ &A_2 A_1 E[\cos w_2 \cos w_1(t+\tau)] \\ &+ A_2^2 E[\cos w_2 \cos w_2(t+\tau)] \end{aligned} \quad (22) \\ r(n) &= \frac{A_1^2}{2} \cos w_1 \tau + \frac{A_2^2}{2} \cos w_2 \tau \end{aligned}$$

Then for N components of sinusoid we will have^[6].

$$\begin{aligned} \tau = 1, r(1) &= P_1 \cos w_1 + P_2 \cos w_2 + \dots + P_n \cos w_n \\ \tau = 2, r(2) &= P_1 \cos 2w_1 + P_2 \cos 2w_2 + \dots + P_n \cos 2w_n \end{aligned} \quad (23)$$

where is $P_n = \frac{A_n^2}{2}$ the power of the nth amplitude of sinusoid component.

This equation (23) can be written into matrix form as

$$\begin{bmatrix} \cos w_1 \dots \cos w_2 \dots \dots \dots \cos w_n \\ \cos 2w_1 \dots \cos 2w_2 \dots \dots \dots \cos 2w_n \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \cos nw_1 \dots \dots \dots \cos nw_n \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ \vdots \\ P_n \end{bmatrix} = \begin{bmatrix} r(1) \\ r(2) \\ \vdots \\ \vdots \\ r(n) \end{bmatrix} \quad (24)$$

If we have the W_i pulsations and $r(i)$ autocorrelation coefficients, we can solve these equations to obtain the P_i power corresponding to f_i identified.

RESULTS AND DISCUSSION

The tested observances used are the signals of bearing healthy, a bearing defect with different levels, vibration signal without clumsy and vibration with clumsy. These signals are used to realize a class of references observances. The simulation begins with rotating machine in the normal operating mode and defective mode (Fig. 3 and Fig. 8). Then, we compute the DSP from a normal and defective operating mode.

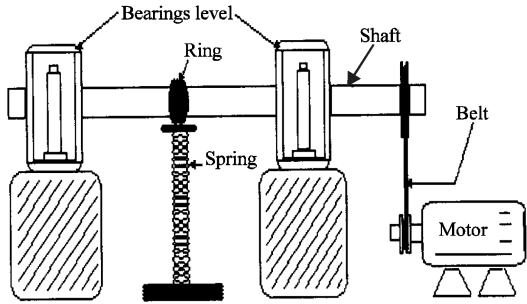


Fig. 2: Experiment set-up

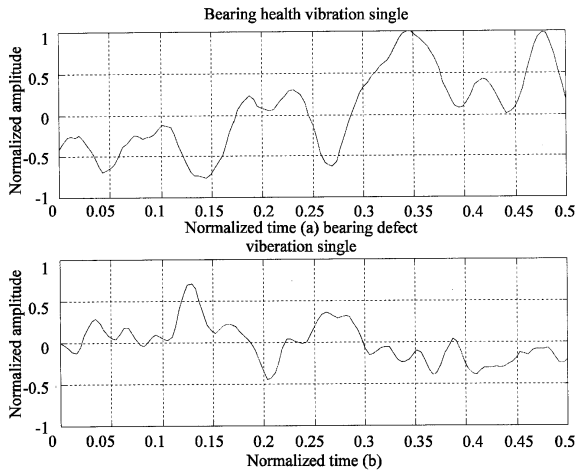


Fig. 3: Temporel representation of vibration with (a): bearing healthy (b): bearing defect

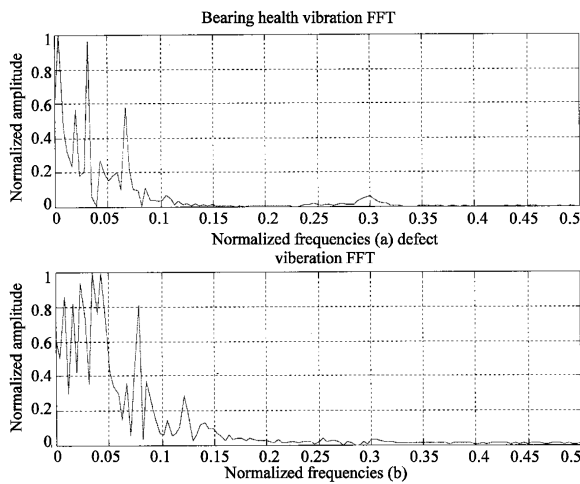


Fig. 4: FFT of vibration with (a): bearing healthy (b): bearing defect

The estimate DSPs are shown on the same Fig. (Fig. 4, Fig. 9).

The PSD also is calculated starting from the coefficients of the AR filter (Fig. 5, Fig. 10). We can

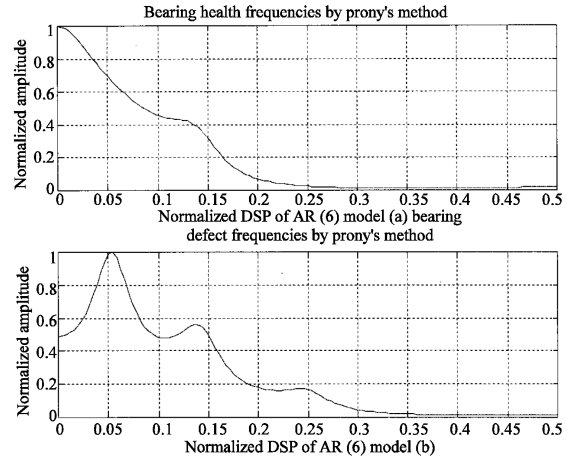


Fig. 5: DSP of AR(6) model vibration with (a): bearing healthy (b): bearing defect

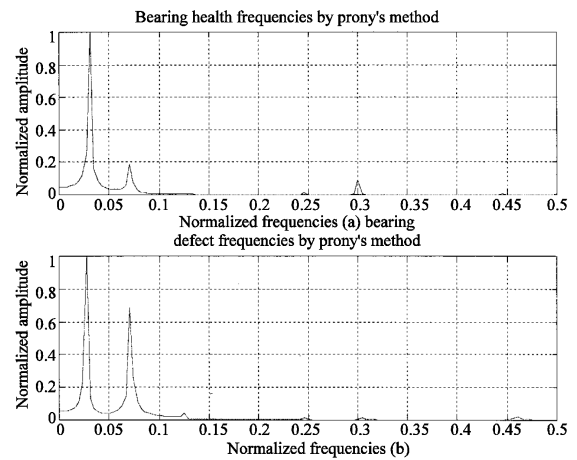


Fig. 6: Frequencies characteristics of vibration with (a): bearing healthy (b): bearing defect

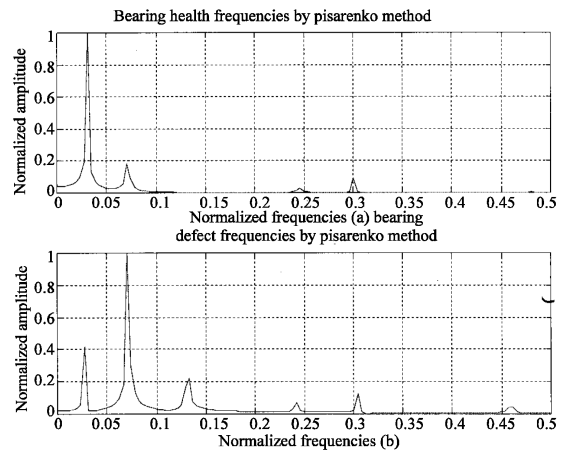


Fig. 7: Frequencies characteristics of vibration with (a): bearing healthy (b): bearing defect

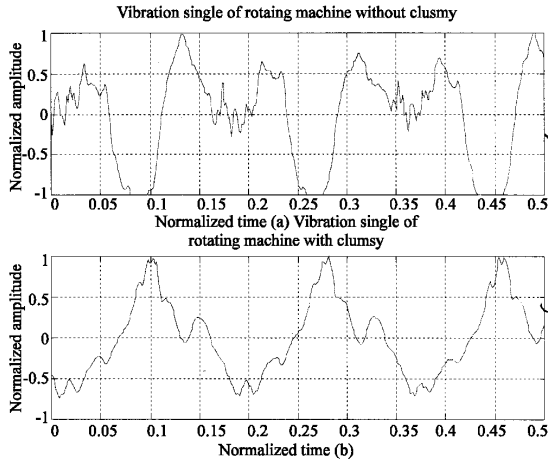


Fig. 8: Temporal representation of vibration (a): without clumsy (b): with clumsy

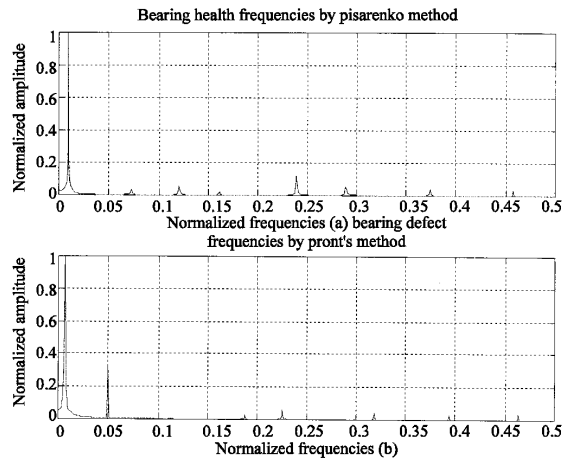


Fig. 11: Frequencies characteristics of vibration (a): without clumsy (b): with clumsy

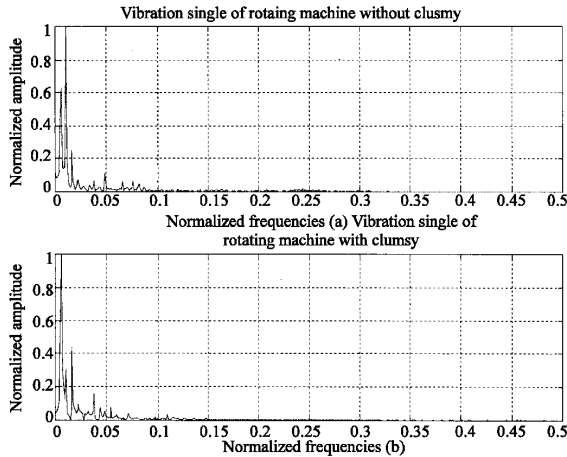


Fig. 9: FFT of vibration of rotating machine (a): without clumsy (b): with clumsy

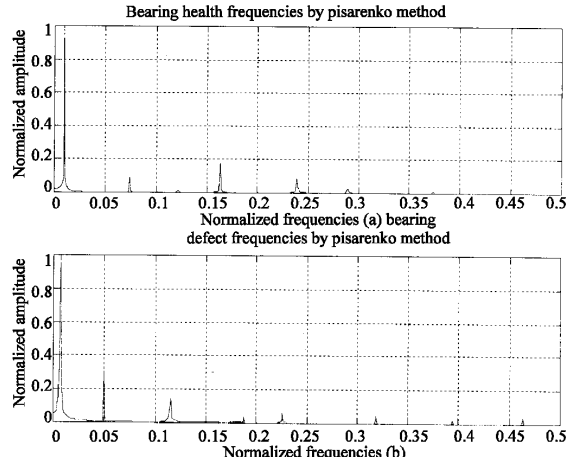


Fig. 12: Frequencies characteristics of vibration (a): without clumsy (b): with clumsy

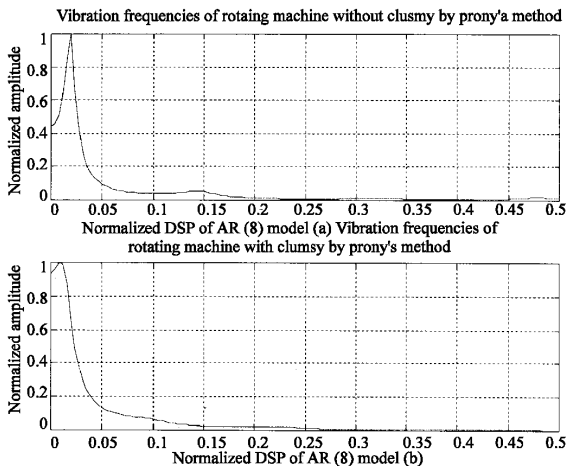


Fig. 10: DSP of AR(8) model vibration of Rotating machine (a): without clumsy (b): with clumsy

conclude from the results obtained that these methods are incapable to distinguish between two nearest frequencies. The coefficients, which the number indicates the order of the filter, are calculated by an adaptive algorithm.

The proposed methods (Prony and Pisarenko) are capable of resolving closely-spaced frequencies and providing satisfactory frequency estimates. The figures (Fig. 6, Fig. 7, Fig. 11, Fig. 12) provide the remedy for classic spectral analysis. We can conclude that the results obtained are satisfactory for the PSD estimation by the high resolution methods (Prony and Pisarenko).

In fact, the diagnosis can be done by the spectra (or PSD) when they present quick changes which indicate the existence of defect operating system. From this spectra, a localization of different faults can be detected.

The Table 1, 2, 3 and 4 give the characteristics frequencies calculated by the Prony and Pisarenko

Table 1: Frequencies estimated by Prony's method of the healthy bearing and defect

fi	Healthy bearing	Defect bearing
f0	0.0305	0.0267
f1	0.0715	0.0714
f2	0.2460	0.1263
f3	0.3006	0.2450
f4	0.4459	0.3029
f5	0.5000	0.4594

Table 2: Frequencies estimated by Pisarenko method of the healthy bearing and defect

fi	Healthy bearing	Defectbearing
f0	0.0306	0.0270
f1	0.0717	0.0712
f2	0.2450	0.1310
f3	0.3005	0.2412
f4	0.4809	0.3053
f5	0.5000	0.4591

Table 3: Frequencies estimated by Prony's method of the Rotating machine without and with clumsy

fi	Without clumsy	With clumsy
f0	0.0096	0.0066
f1	0.0728	0.0498
f2	0.1205	0.1147
f3	0.1607	0.1878
f4	0.2396	0.2253
f5	0.2895	0.3183
f6	0.3738	0.3935
f7	0.4570	0.4628

Table 4: Frequencies estimated by Pisarenko method of the Rotating machine without and with clumsy

fi	Without clumsy	With clumsy
f0	0.0097	0.0066
f1	0.0742	0.0498
f2	0.1207	0.1148
f3	0.1630	0.1878
f4	0.2395	0.2253
f5	0.2896	0.3183
f6	0.3737	0.3935
f7	0.4574	0.4628

algorithm. The normalized estimated frequencies obtained by different methods are shown on these Tables. From these frequencies, a considerable amount of information which can be used in identification of different faults existing in the rotating machine.

CONCLUSION

Motor Vibration Signature Analysis (MVSA) provides an important way to assess the health of a machine. The high resolution frequencies analysis by Prony's and Pisarenko method ensure a good analysis and diagnosis for the rotating machine. This approach can solve the problem of quantitative and classic analysis of these systems and guaranteed a preventive maintenance of motors. From these characteristics vibration, a localization of different faults and defects of the rotating machine can be detected.

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