

## Modal Analysis of a Cavity by T.L.M. and Study of T.E. Modes

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**Abstract:** The objective of the work presented in this paper consists of evaluating, by a numerical method, the distribution of electromagnetic energy in the centre of a cavity. More precisely it is the observation of the electromagnetic field around a device under test. The numerical technique which we opted is Transmission Line Matrix (TLM) which is based on time domain. The possible targeted applications are, on one side, the analysis of a measurement techniques e.g. Mode Stirrer Reverberating Chamber (MSRC) and on the other side the analysis of electromagnetic environment of a system.

**Key words:** T.L.M., T.E. modes, M.S.R.C., numerical technique

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### INTRODUCTION

Transmission Line Modelling (TLM) or Transmission Line Matrix (TLM) Modelling is a general numerical simulation technique suitable for solving field problems. Its main application has been in electromagnetics, but it has also been applied to thermal or diffusion problems as well as acoustics.

The TLM method belongs to the general class of differential time-domain numerical modelling methods. The principles of the TLM time domain method have been introduced by Johns and Beurle<sup>[1]</sup>. Voltages and currents in this network are equivalent to electric and magnetic fields in electromagnetic systems. The main advantage of this method is the simplicity of formulation and programming for a large range of applications. The basic approach of the TLM method is to obtain a discrete model which is then solved exactly by numerical means; approximations are only introduced at the discretisation stage. This is to be contrasted with the traditional approach in which an idealized continuous model is first obtained and then this model is solved approximately.

In this study, the distribution of electromagnetic energy in the centre of a cavity is studied. More precisely it is the observation of the electromagnetic field around a device under test to be placed afterwards. The T.E. modes determined by TLM are then compared with the analytical results for a cavity. At the end we study the deformation of the distribution of modes by the change in the geometry. Mode Stirrer Reverberating Chamber (MSRC) and on the other side the analysis of electromagnetic environment of a system are the possible applications.

### THE TWO DIMENSIONAL TLM METHOD

Propagation and scattering of pulses in a network consists of interconnected ideal transmission lines. In two dimensional scheme, at each time step, every node receives incident voltage pulses and sends scattered pulses, as shown in Fig. 1.

The scattered pulses at time (t) become incident pulses on adjacent nodes at (t + Δt). The scattering matrix is computed from transmission lines theory<sup>[2]</sup>. For electromagnetic systems, the discrete model is formed by conceptually filling space with a network of transmission-lines in such a way that the voltage and current give information on the electric and magnetic fields. The point at which the transmission-lines intersect is referred to as a node and the most commonly used node for 3-dimensional work is the symmetrical condensed node.

At each time step, voltage pulses are incident upon the node from each of the transmission-lines. These pulses are then scattered to produce a new set of pulses which become incident on adjacent nodes at the next time step. The relationship between the incident pulses and the scattered pulses is determined by the scattering matrix, which is set to be consistent with Maxwell's equations. Additional elements, such as transmission-line stubs, can be added to the node so that different material properties can be represented<sup>[3, 4]</sup>. In order to appreciate the importance of dispersion, the process in Fig. 1 shows the response of TLM network to a single impulse which contains all frequencies. Thus harmonic solutions of a problem can be obtained by Fourier transform. Accurate solutions will be obtained only at frequencies for which the dispersion effect can be neglected.

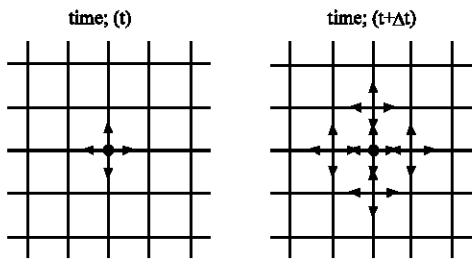


Fig. 1: Propagation of impulses in a two dimensional network<sup>[2]</sup>

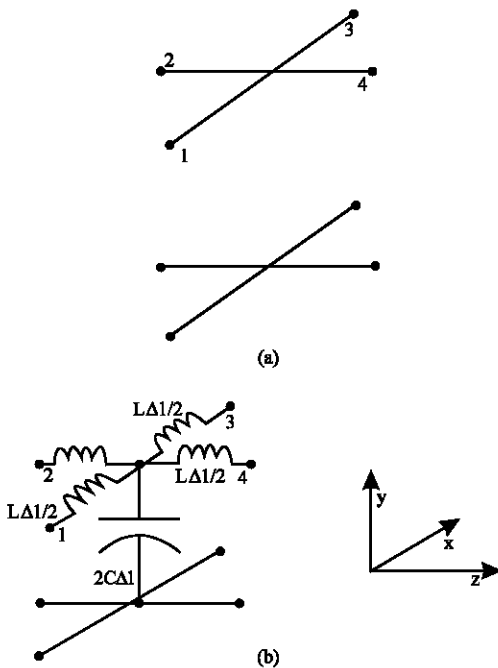


Fig. 2: (a) Shunt Node and (b) equivalent lumped element model in a 2-dimensional TLM network

The shunt node and equivalent lumped element model of two dimensional TLM network is shown in Fig. 2.<sup>[4]</sup>

L and C are the inductances and capacitances per unit length for the transmission lines. The distance between any two adjacent nodes is constant and is equal to  $\Delta l$ .

TLM mesh can be extended to three dimensions giving complex network containing series and shunt nodes. Each of the six field components is simulated by a voltage or a current in that mesh.

### WAVE PROPAGATION PROPERTIES

In a free space medium  $\epsilon = \epsilon_0$ ,  $\mu = \mu_0$  and the wave propagates with the velocity of light,  $c$  :

$$c = \frac{1}{\sqrt{\mu\epsilon}} \tag{1}$$

For the elementary transmission line and for  $\epsilon_r = \mu_r = 1$ , the inductance and capacitance per unit length are related by:

$$c = \frac{1}{\sqrt{LC}} \tag{2}$$

However as,  $\mu = L$  and  $\epsilon = 2C[1]$ , we can say that as if voltage and current waves, on each transmission line component, propagate at the speed of light  $c$ , the complete network of intersecting transmission lines represents a medium of relative permittivity twice that of free space. It shows that on the network of the transmission lines, the speed of propagation is slowed down, slow wave propagation. This is a fundamental property of TLM method in two dimensional case.

### RESULTS AND DISCUSSION

**Principle of calculation of modes:** To validate the calculation by the TLM method, we compare the given distribution with the analytical calculation. The walls are modelled as perfect conductor. The network of TLM is excited (for TE modes) locally, at a point arbitrarily selected, with  $E_z=0$ ,  $H_x=H_y=0$  where  $H_x$  and  $H_y$  are the components of magnetic field and  $E_z$  is the component of electric field. The system is excited with a Dirac unit pulse at time  $t = 0$ . At any point in the cavity, the electromagnetic wave can be obtained under the form of succession of pulses with the following equation:

$$Y(t) = \sum_{n=1}^N y(n) \cdot \delta(t - n\Delta t) \tag{3}$$

where,  $\Delta t$  is the incremental time step.

Application of Fourier transform on equation (3) will give the spectral response. However, it is evident that the observation of signal will be effected over a finite time period. On the other side, to distinguish very close modes, this observation window should be sufficient. We used the Hanning window to limit the phenomena of Gibbs. We know that the dimensions of discretisation should be smaller than the wavelength of operation. Successive measurements show that  $\lambda_{min}/10$  is sufficient to get a very good precision for the resonant frequencies<sup>[1]</sup>.

**Distribution of Transverse Electric modes:** Figure 3 shows the geometrical structure for the purpose of the study of the influence of the geometrical modification. Fig. 4 shows some of the resulted TE modes obtained from the numerical calculations performed using the TLM method. As can be seen from the figure,

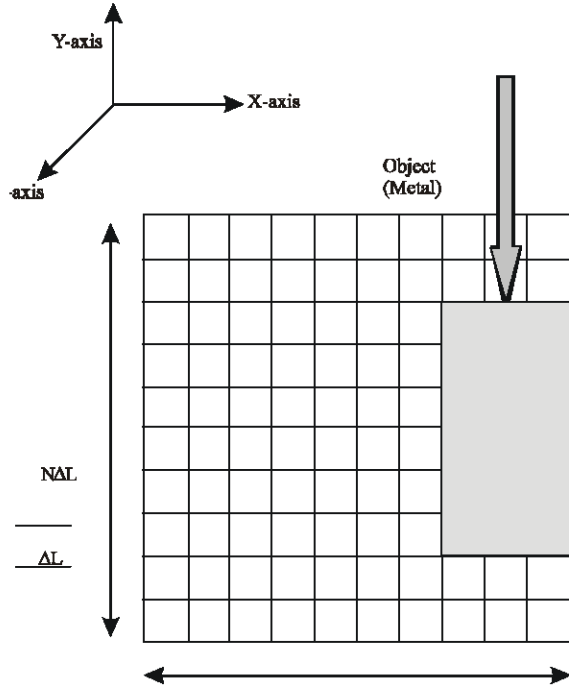


Fig. 3: Geometrical structure for studying the influence due to the object

the modes are easily extracted using the Fourier transform. For verification purposes, the numerical results are compared with analytical ones calculated and summarized in Table 1.

The obtained modes well agree with the analytical calculations. Recalling few equations used for the calculation are:

$$[S] = \begin{bmatrix} +1/2 & -1/2 & -1/2 & -1/2 \\ -1/2 & +1/2 & -1/2 & -1/2 \\ -1/2 & -1/2 & +1/2 & -1/2 \\ -1/2 & -1/2 & -1/2 & +1/2 \end{bmatrix} \quad (4)$$

$$H_z = \frac{1}{Z_0} (-V_1^i + V_2^i + V_3^i + V_4^i) \quad (5)$$

$$E_x = -V_2^i + V_4^i \quad (6)$$

$$E_y = +V_1^i - V_3^i \quad (7)$$

and

$$\begin{aligned} V_1^i &= 1/2 [+E_x \cdot Z_0 + 1/2 * H_z] \\ V_2^i &= 1/2 [-E_y \cdot Z_0 + 1/2 * H_z] \\ V_3^i &= 1/2 [-E_x \cdot Z_0 + 1/2 * H_z] \\ V_4^i &= 1/2 [+E_y \cdot Z_0 + 1/2 * H_z] \end{aligned} \quad (8)$$

Table 1: Analytical and numerical results for different modes

S. No.	Type of TE <sub>mn</sub> modes	Analytical calculation (Ghz)	Numerical results TLM(GHz)
1	TE <sub>01</sub>	0.150	0.150
2	TE <sub>11</sub>	0.212	0.212
3	TE <sub>20</sub>	0.300	0.301
4	TE <sub>21/12</sub>	0.335	0.335
5	TE <sub>22</sub>	0.424	0.424
6	TE <sub>30</sub>	0.450	0.449
7	TE <sub>31</sub>	0.474	0.474
8	TE <sub>32</sub>	0.540	0.540
9	TE <sub>40</sub>	0.600	0.597
10	TE <sub>41</sub>	0.618	0.620
11	TE <sub>53</sub>	0.636	0.636
12	TE <sub>42</sub>	0.670	0.671
13	TE <sub>43</sub>	0.750	0.750

Other details:

$$NAL = 1m, \Delta L = 4cm$$

$$N\delta t = 180nS, N_x = N_y = 25$$

$$\text{Max frequency} = 1GHz, \text{Excitation Node} = 53(3*3)$$

$$\text{Output Node} = 523(32*21)$$

$$\text{Number of iterations} = 1909$$

$$E_y(\text{excit}) = 0, E_x(\text{excit}) = 0, H_z(\text{excit}) = 1$$

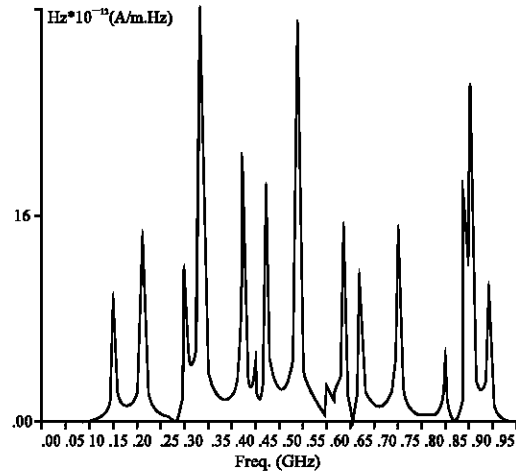


Fig 4: Response of the magnetic field component

## CONCLUSION

This paper represents the TLM method and its comparison for calculation of modes for a structure. Like other methods, this one can be used to take into account the lossy medium or non homogenised medium. We can also look into with variable discretisation of the mesh. The method is based on the symmetrical condensed nodes and permits to calculate the EM field in a cavity along with the presence of metallic obstacles. After the application of Fourier transform, the method can be used to evaluate the resonant frequencies of the structure.

In future, we can look into the obstacles with small dimensions and develop the code which can incorporate these issues of EMC. Another objective is the characterization of MSRC (Mode Stirrer Reverberating

Chamber) and study of an object inside MSRC and consequently the influence of this on the homogenisation of EM field.

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