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Two-Scale Simulation for the Tensile Behavior of Unidirectional Hybrid Composites

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Abstract: In this study, a two-scale 3D model for the tensile behavior of unidirectional hybrid composites is proposed. In the fiber scale, the tows are defined as fiber/matrix system. In the tow scale, hybrid composites are regarded as combination of two (or more) types of tows. The strength estimation of fibers is statistically described by the two-parameter Weibull distribution. In both scales, analyses are based on shear slag model and taking the fiber/resin interfacial toughness into account. The proposed model is then used in simulation the tensile behavior of hybrid unidirectional composite bars reinforced by fibers of carbon and glass. Numerical simulations agree well with experiment results. Relations among the carbon fiber volume fraction and the stiffness, strength and tensile toughness are obtained.

Key words: Unidirectional hybrid composites, shear-lag model, Monte-Carlo method, tensile behavior, interfacial toughness

INTRODUCTION

Hybrid fiber reinforced composites contain two or more kinds of fibers with various stiffness, toughness and strength. A properly designed volume rations for each fiber and rational fiber/matrix interface strength can integrate advantages of each fiber in hybrid composites. Understanding the relations between the mechanical properties of constituents and those of hybrid fiber composites is necessary for practical applications or for developing new hybrid composite materials. To obtain these relations through experiment results a high cost in both time and materials. Therefore, an analytical model or numerical simulation is required.

As is known, the final failure of a hybrid composite usually undergoes complex micro damage processes. Under tensile loading, the fiber tow with low tensile toughness breaks earlier than those with higher toughness. The load carried by the broken tows transfers to the unbroken tows via shear. Thus, the composite can still carry load until all tows fails. Even in mono-type fiber tow, strengths of fibers may vary. The load transferred from broken fiber to unbroken ones through shear is similar to the case between tows of hybrid composite. The local stress concentration and released energy introduced by fiber breakage and fiber/matrix toughness all affect the micro damage. Shear-lag model (Okabe *et al*., 2001) is a candidate to simulate the failure processes for both tows and hybrid composites. It gives a good description of experimentaly observed fracture behavior of unidirectional composites (Ochiai *et al*., 2001). The ordinary shear-lag method has a limitation, that only the fibers carry the applied stress and matrix acts only as a stress-transfer-medium. So this method only applied for low-yield-stress matrix composites. Ochiai *et al.* (1997, 1999) extended this method as to describe the situation in which both fibers and matrix carry the applied stress and act as stress transfer media.

Since the strengths of individual components and their interfaces are stochastic, therefore, stochastic failure models have been extensively used in experimental (Hamada *et al*., 1997) and analytical (Roson, 1964; Zweben, 1977; Zweben and Rosen, 1970) analysis of failure process and the ultimate strength for hybrid fiber reinforced composites. Monte-Carlo simulation method is a good tool to predicting the tensile properties of unidirectional fiber reinforcing composites. Two and three-dimensional Monte-Carlo simulation techniques coupled with the classical or modified shear-lag models have been widely used for predicting the tensile properties of unidirectional fiber reinforcing composites (Oh, 1979).

In present study, a two-step three-dimensional modified shear-lag model coupled with Monte-Carlo simulation method is proposed. In the fiber scale, referred as scale one, tow is regarded as two-phase material of

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fiber and matrix. While in the tow scale, denoted as scale two, a hybrid composite is combination of two or more types of tows. In both scales, three-dimensional shear-lag model is adopted to simulate the tensile behavior of tow and hybrid composites, respectively. Monte-Carlo method with a two-parameter Weibull distribution is used to describe statistically the strengths of fibers and the two parameters are determined by experimental data. The strength of tows used in tow-scale model is calculated by the fiber-scale model. Interface toughness is taken into account in both models. The two-scale model is then applied to investigate the tensile behavior of a hybrid unidirectional composites bars. Numerical simulations are compared with experiment results to verify the proposed model. Relations are obtained between the carbon fiber volume fraction and the stiffness, strength and tensile toughness.

TWO-SCALE 3D MODIFIED SHEAR LAG MODEL

In general, a unidirectional hybrid composite is composed by two kinds of fibers with different volume fractions for each fiber. A tow composed of matrix and fiber is a two-phase material. Since the structure for hybrid composites and tow is similar and differs only in dimensions, therefore, they are all regarded as two-phase materials in the analysis.

A 3D model for a two-phase unidirectional material is schematically shown in Fig. 1. For convenience, a rectangular cross section is adopted. Each bar with a square cross section and its axis along the fiber direction contains only a single-phase material. For example, each white bar and shaded bar represents materials with different phase. Then, all bars are uniformly discretized by many elements along its axial direction (fiber direction). Thus, the two-phase unidirectional material is modeled byrectangular hexahedron, shown in Fig. 1. Various volume fractions of each phase material and the hybrid manner can be conveniently modeled by the ratio and distribution of the two colored bars with rectangular cross sections. When the two-phase material is under uni-axial loads, the stress acting on an element based on the shear-lag theory is shown in Fig. 2. Thus, equilibrium equation for each element can be obtained namely,

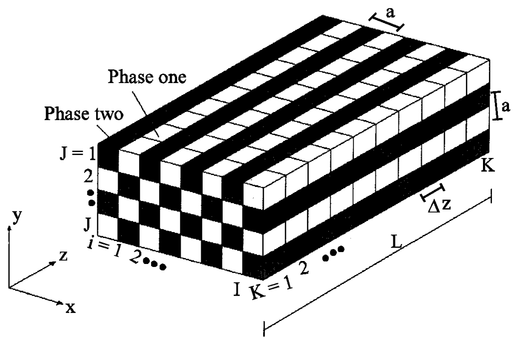

Fig.1: A 3D two-phase model for a unidirectional material

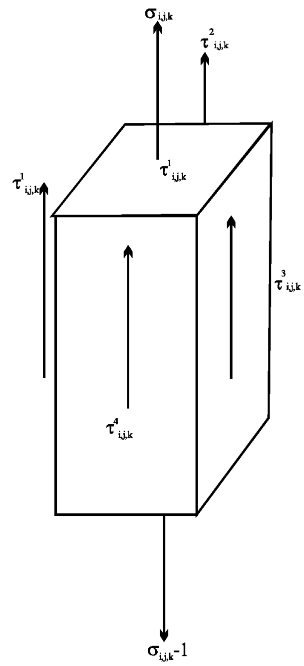

Fig. 2: Tensile and shear stresses on element surfaces

$$EA\frac{d^2u}{dz^2} + a\sum_{l=1}^{4}\tau^l = 0 \quad (1)$$

Where E, A and α denote the axial elastic modulus, cross-sectional area and the side length of the across section, respectively; u s the displacement in the z-direction; τ^l is the shear stress on the interface determined by the displacement difference of two adjacent elements the superscript l represents the 4 side surface and takes the value of 1 to 4 accordingly. The shear stress τ between two adjacent surfaces can be written as

$$\tau = \frac{\bar{G}}{a}(u_1 - u_2) \quad (2)$$

Where

$$\bar{G} = \frac{1}{l_1 + \eta}\left(\frac{1}{G_1} + \eta\frac{1}{G_2}\right) \quad (3)$$

$$\eta = \frac{1}{2}\left(1 + \frac{G_2}{G_1}\right) \quad (4)$$

while andrepresent the center displacement of the axis at two adjacent elements; $G_1$and G_2 are the shear moduli of these two elements.

Equation 1 is to be solved by the finite difference method. For interior elements, one has

$$\frac{d^2u_{i,j,k}}{dz^2} = \frac{u_{i,j,k-1} - 2u_{i,j,k} + u_{i,j,k+1}}{(\Delta z)^2} \quad (5)$$

$$\begin{aligned} \tau^1_{i,j,k} &= \bar{G}(u_{i,j,k} - u_{i-1,j,k}) \\ \tau^2_{i,j,k} &= \bar{G}(u_{i,j,k} - u_{i+1,j,k}) \\ \tau^3_{i,j,k} &= \bar{G}(u_{i,j,k} - u_{i,j-1,k}) \\ \tau^4_{i,j,k} &= \bar{G}(u_{i,j,k} - u_{i,j+1,k}) \end{aligned} \quad (6)$$

For surface elements, shear stress acts only on three faces. For corner elements, only two surfaces have shear stress.

If the element normal stress along the axial direction, $\sigma_{i,j,k}$, exceeds its tensile strength $X_{i,j,k}$, namely,

$$\sigma_{i,j,k} \geq X_{i,j,k} \quad (7)$$

The element failures and is assumed to break at its center cross section. Therefore, if element (I, j, k-1) failures, $u_{i,j,k} = u_{i,j,k-1}$. Eq. 5 may be re-written as

$$\frac{d^2u_{i,j,k}}{dz^2} = \frac{u_{i,j,k+1} - u_{i,j,k}}{(\Delta z)^2} \quad (8)$$

If the shear stress between two adjacent elements' faces exceeds its shear strength, i.e.,

$$\tau^1_{i,j,k} \geq \tau_0 \quad (9)$$

These two faces will de-bond and the friction force between the de-bonded faces will not be considered in this study. In other words, the shear stress is zero once de-bonding occurs. The strain energy will be released once fiber breaks. The released strain energy may cause further damages. It is assumed that the broken fiber surfaces separate apart to absorb the strain energy. Thus,

$$\frac{A_0 \Delta u}{2}(\bar{\sigma} - \bar{\sigma}_d) = 4aL_dG_{IIC} \quad (10)$$

Where $\bar{\sigma}$ and $\bar{\sigma}_d$ are the average stress in the cross sections before and after the occurrence of the damage;A_0 and are the cross-sectional area and the displacement increments between two ends of the model, whereis the cross sectional area of tows in the analysis of fiber scale and the cross sectional area of the test specimen of hybrid composites in the analysis of tow scale: L_d is the length of de-bonding face; G_{IIc} is the II-type fracture toughness of the interface. Although there is an I-type fracture component near the broken face, but its effect is neglected in the analysis since it is far more less than the II-type fracture component. The quantity on the left hand side of Eq. 10 is the leased strain energy due to the broken of the fiber or the fiber tow, the quantity on the right hand side of Eq. 10 is the energy absorbed by separation of the interfaces.

For convenience, $\xi = \sqrt{(EAd/Gh)}$ introduce as the characteristic length. Thus, the displacement for element (i,j,k) becomes

$$U_{i,j,k} = \frac{1}{C_1 + C_2C_3\Delta Z^2}\Big[H_{i,j,k}U_{i,j,k-1} + + H_{i,j,k+1}U_{i,j,k+1} + C_2\Delta Z^2\left(P^1_{i,j,k}U_{i+1,j,k} + P^2_{i,j,k}U_{i-1,j,k} + P^3_{i,j,k}U_{i,j-1,k} + P^4_{i,j,k}U_{i,j+1,k}\right)\Big] \quad (11)$$

In which

$$H_{i,j,k} = \begin{cases} 0 & \sigma_{i,j,k} > X_{i,j,k} \\ 1 & \sigma_{i,j,k} \leq X_{i,j,k} \end{cases} \quad (12)$$

$$P^1_{i,j,k} = \begin{cases} 0 & |\tau^1_{i,j,k}| > \tau_0 \\ 1 & |\tau^1_{i,j,k}| \leq \tau_0 \end{cases} \quad (13)$$

Where $U_{i,j,k} = (u_{i,j,k} / \xi)$ is the non-dimensional element displacements; $\Delta Z = (\Delta z/\xi)$ represents the non-dimensional element length along the longitudinal direction; $C_1$and C_2 are parameters describing element failures; $C_1 = 2$ and $C_2 = 1$ for undamaged element (i,j,k-1) $C_1 = 1$ and $C_2 = 0.75$ if the element failures; C_3 is the parameter describing the failure of interface between two adjacent elements, it takes 1 for perfect bonds; otherwise 0 for damaged bonds.

Introduce the boundary conditions into Eq. 11. The displacements can be obtained by solving Eq. 11 with over-relaxation iteration method. In the present study, the hybrid composite is under tensile load in the fiber direction, thus, the boundary conditions are

$$u_{i,j,1} = 0, \quad u_{i,j,k} = u_0 \quad (14)$$

Where u_0 is the free end displacement with the other end fixed. The iteration formulations for obtaining the displacements can be written as

$$U^n_{i,j,k} = \lambda U^n_{i,j,k} + (1-\lambda)U^{n-1}_{i,j,k} \quad (15)$$

in which λ is the relaxation factor for accelerating the convergence speed, which takes the value between 1 and 2 and is the number of iterations.

Once the displacement is obtained, the axial stress for each element can be computed by

$$\sigma_{i,j,k} = E\frac{(u_{i,j,k} - u_{i,j,k-1})}{\Delta z} \quad (16)$$

The average axial stress for the unidirectional hybrid composites is computed by

$$\sigma_{app} = \frac{1}{I \times J}\sum_{i=1}^{I}\sum_{j=1}^{J}\sigma_{i,j,k} \quad (17)$$

The strength of fiber tows are described by a two-parameter Weibull distribution function, namely,

$$X_{i,j,k} = \sigma_0\left[-\frac{l_0}{\Delta z}\ln(1-F)\right]^{1/\beta} \quad (18)$$

Where the distribution functionis F represents by random numbers in the range of 0-1; $\sigma_0$and β represent the scale parameter and shape parameter, l_0 is the gage length of the tensile specimen; Δz and is the element length, respectively.

The analysis procedures in two scales are as follows: Firstly, establish the 3D shear-lag model in scale 1 (fiber scale); simulate the damage process of fiber tow under tensile load by the above mentioned theory. The strength parameters of fibers $\sigma_0$and β can then be determined by comparing the simulated results with the measured tensile strength of mono fiber composites with the same material system. Secondary, establish the 3D shear-lag model for the two- or multiple-phase composites in scale 2; each element in the model corresponds to a section of fiber tow whose strength is determined by the model in scale 1. Suppose that there are N_{tr} elements in the model in scale 2 and N_{fr} elements in the model in scale 1, then it is required to produce a random series with length of $N_{tr}\times N_{fr}$. In computing the strength of the n^{th} element in the model in scale 2, pick up elements (n - 1)N_{fr} to nN_{fr} in the series to the model in scale 1 during computation. Since each time use model in scale 1 during computation, the random numbers used are different; therefore, the element strength in the model in scale 2 varies randomly.

RESULTS AND DISCUSSION

Experiments were performed on unidirectional carbon/glass hybrid composites with various fiber volume fractions. The material of carbon fiber is T700, glass fiber is E glass and the matrix is Ethylene resin. Their material properties and interface fracture toughness are listed in Table 1 and 2. The fiber volume fractions of specimens made of pure glass fibers and pure carbon fibers are 52 and 54%, respectively. In the hybrid composites, the volume fractions of glass fibers and carbon fibers are considered the same as that of specimens made of pure glass fibers and pure carbon fibers. Figure 3 shows a cross section of hybrid composite, the part in dark color is carbon fiber and the other is the glass fiber. The volume fraction of carbon fibers and glass fibers are 21.6 and 31.2%.

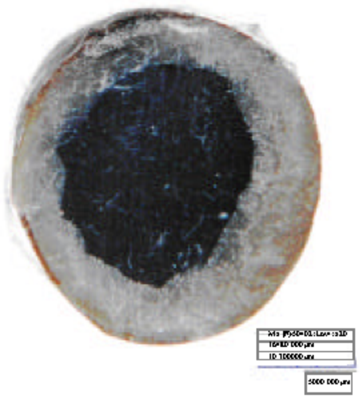

Fig. 3: A photo of the cross section of a hybrid composite bar

Table 1: Tensile moduli of components

	T700 carbon fiber	E glass fiber	Ethylene resin
Tensile modulus/GPa	245	72	3.6

Table 2: Model II interface toughness

	Glass fiber/resin	Carbon fiber/resin	Glass tow/carbon tow
G_{II}/KJ/m^2	59.42	36.13	42.03

As is know, the strength of fiber in the composite material, difficult to be measured directly, is lower than its original strength which can be directly measured. Thus an inverse method is adopted to obtain the strength of fibers in the composite material. Firstly, establish the 3D model in scale 1 (fiber scale) and simulate the tensile tests of pure glass fiber specimen and pure carbon fiber specimen to determine the strength parameters of the corresponding fiber tow based on the theory in this study. The mesh of cross section isand there are 20 equal sections along the axial direction. The cells representing the fibers are determined randomly. The total volume of cells representing the glass and carbon fibers are 52 and 54% of the volume of the model, respectively, thus, the fiber volume fraction is exactly the same as the actually measured one. Figure 4 shows the fiber distributions in models of glass fiber tow and carbon fiber tow. In the simulation, β is set to 60, the strength of glass fiber and

Table 3: Average tensile properties of carbon fiber tow and glass fiber tow

	Tensile modulus /Gpa		Tensile strength /Mpa		Critical tensile rate/%	
	Experiment	Calculation	Experiment	Calculation	Experiment	Calculation
Carbon tow	137.0	139.8	1838.3	1801.2	1.36	1.33
Glass tow	36.9	38.6	871.2	886.8	2.41	2.36

Table 4: Tensile behaviors of hybrid composites with various hybrid ratios

	T700 5.4%, Eg 46.8%		T700 10.8%, Eg 41.6%		T700 16.2%, Eg 36.4%		T700 21.6%, Eg 31.2%	
	Experiment	Calculation	Experiment	Calculation	Experiment	Calculation	Experiment	Calculation
Modulus/GPa	50.70	48.64	59.05	59.44	65.26	67.58	76.07	76.80
Strength/MPa	682.46	641.34	760.41	749.34	788.85	822.56	868.69	911.43
Critical tensile rate/%	1.52	1.54	1.48	1.47	1.23	1.22	1.18	1.24

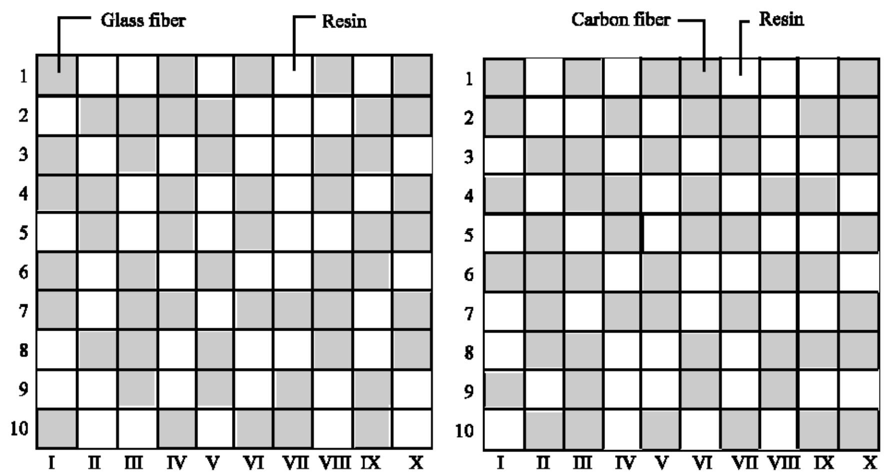

Fig. 4: Fiber distributions in models of glass fiber tow and carbon fiber tow (a) Cross section of glass fiber tow model (Glass fiber volume fraction is 52%) (b) Cross section of carbon fiber tow model (Carbon fiber volume fraction is 54%)

carbon fiber in the composite material are 980 MPA and 2050Mpa and the interface shear strength for glass fiber/matrix and carbon fiber/matrix is 530 MPa, respectively. The simulated tensile strength and maximum elongation coincide well with experimental results, as can be clearly seen from Table 3.

Table 4 lists the simulated and experimentally determined mechanical behavior of hybrid composites with various hybrid ratios. The experimental data are the average ones. It can be seen again that the coincidence between the simulated results and tested data is good. Figure 5 shows the fiber distributions in the model of the hybrid composite with volume fraction of 31.2% glass fiber tow and of 21.6% carbon fiber tow. Figure 6 shows the simulated and tested tensile stress-strain curves for a hybrid composite bar with low volume fraction of carbon fibers. Similar tensile stress-strain curves for a hybrid composite bar with high volume fraction of carbon fibers are shown in Fig. 7. It is seen that from Fig. 6 that there are two obvious damage instants for hybrid composite bar with low volume fraction of carbon fibers. Once the first

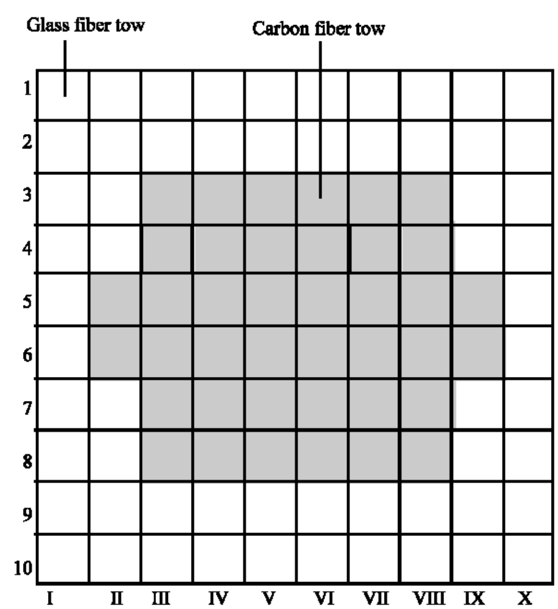

Fig. 5: Distributions of carbon fiber tows and glass fiber tows

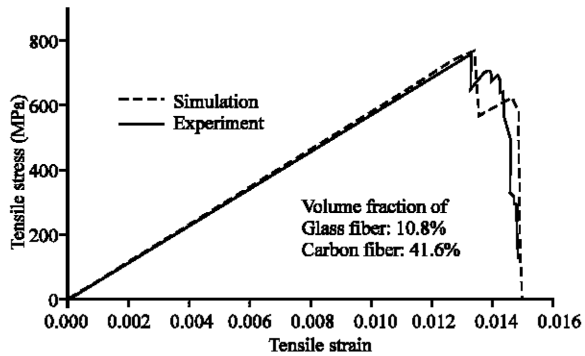

Fig. 6: Experimental stress-strain curve of hybrid composite bar with low volume fraction of carbon fibers

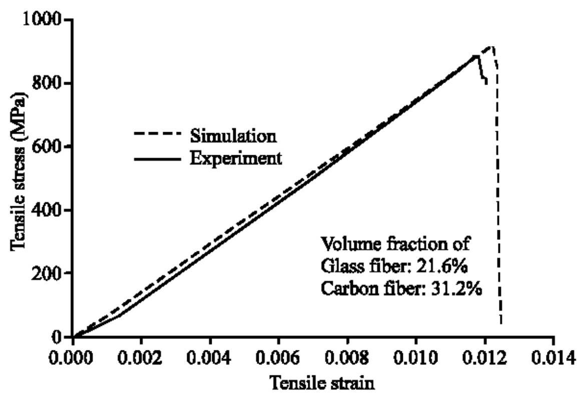

Fig. 7: Experimental stress-strain curve of hybrid composite bar with high volume fraction of carbon fibers

damage occurs, the specimen can undertake further load until the second damage occurs. This phenomenon does not exhibit for hybrid composite bar with high volume fraction of carbon fibers, as is shown in Fig. 7. In such cases, once the material damages, it failures completely. This may be an important finding to guide an proper design of hybrid composite bars in practice.

CONCLUSION

A two-scale 3D shear-lag model is proposed for predicting the tensile behavior of unidirectional hybrid composites. In the fiber scale, the tows are regarded as fiber/matrix material system. While in the tow scale, hybrid composites are regarded as the combination of two (or more) different types of tows. The fiber strength is simulated by the Monte-Carlo method. Tensile tests are also performed on the carbon/glass hybrid composites with various volume fraction ratios. It is found that the hybrid composites exhibit typical characteristics of failure twice when the volume fraction of carbon fibers is small (say 10% or less) for the material tested, otherwise, the materials failure only once. The simulations by the proposed model compares well with experiment data. Relations among the carbon fiber volume fraction and the stiffness, strength and maximum elongation are obtained. Thus the model can be used to predict the tensile mechanical behavior of hybrid composites.

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