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Impact of Back Order Discounts on Length of Protection Interval Inventory Model

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Abstract: In the study, stochastic period inventory model analyzed with the back order ratio is fully depends on the length of protection interval, which is expressed as the sum of review period and lead-time. Also, back order discount and protection interval considered as decision variables, which control to wider applications for a periodic inventory model. Two parameters perfect information and partial information about the protection interval demand distributions have been discussed. We first assume that the protection interval demand follows a normal distribution and relax this assumption by considering the first and second moments of the probability distribution of protection interval demand are known.

Key words: Length of protection interval, back order ratio, periodic review, discounts on length, inventory model

INTRODUCTION

In the recent inventory control system literature deals the assumption that the demand during the stock out period in fully Back ordered or fully vanished out. The most of the inventory papers concentrated on the assumption that the back orders ratio is a prescribed constant, which leads to uncontrollable. In latest literature, some studies (Ben-Daya and Hariga, 1999; Chung *et al*., 1999; Ouyang and Wu, 1997; Ouyang *et al*., 1996) have been analyzed the continuous review model with the partial back order rate as a fixed constant.

In this study, we analyze the assumption that the back order ratio is dependent on the length of protection interval. In many practical situations where shortages are allowed, we can often observe that some customers may refuse the back order case.

When a shortage occurs, many factors may affect customer's willingness of accepting backorders. It is obvious that for well -formed products or fashionable goods such as certain brand gun shoes, hi-fi-equipment and clothes, customers may prefer to wait, in order to satisfy their remands. Besides the products themselves, there is a potential factor that may motivate the customers desire for backorders. The factor is some extent by offering a price discount on the stock out item by negotiation to secure more backorders it may take the customer more willing to wait for the desired items. In other words, the higher the price discounts from the supplier, the higher the advantage of the customers. And hence, a large no. of back order ratio may result. This phenomenon revolts that, as unsatisfied demands occur during the stock out period, how to find an optimal back order ratio by controlling a price discount from a supplier to minimize the relevant inventory total cost is a decision - making problem worth discussing.

In a recent study, Pan and Hsiao (2001) studied the continuous review inventory model with backorder discounts. The applications of periodic review inventory model can often be found in managing inventory cases such as smaller retail stores, drugs stores and grocery stores by Taylor (1996).

The applications of the periodic inventory models can often be found in managing inventory cases such as smaller retail stores, drugstores and grocery stares. For this, in contrast to the continuous review inventory model, we seek to investigate a periodic review model with back order discounts to accommodate more practical feature of the real production/ inventory systems. Consequently here we consider an option in which while a shortage occurs, a price discount can always be offered on the stock out them in order to secure more back orders for the periodic review inventory models. The main object of this study is to testing the effects of controllable back order discount on the periodic review inventory model (ie) the study proposes a general model which allows review period and backorder rate as decision variables to fit a more realistic inventory situation. We commence the inventory cycle with the protection interval demand that follows a normal distribution and finally we exclude this assumption by considering that the first and second moments of prob. distribution of the protection interval demand to be known and finite in the inventory situation then solve this problem by using the minimax distribution free approach.

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ASSUMPTIONS

The following are the assumptions, which are applicable for this proposed model.

- Back order ratio, which depends on the length of protection interval N.
- M = The target level can be explained as the sum of expected demand during the protection interval and safety stock sst.
  sst = $p \times \sigma \sqrt{t_1 + L}$, where p is the safety factor and satisfies the relation, P (M > N) – r = 0.
- The inventory level is reviewed every t_1 units of time.
- During the stockout time, the backorder ratio β_N is two variable functions of t_1 and L and is in proportion to the price discount offered by the supplier per unit π_1.

(i.e.) $\beta_N = \beta_1(t_1+L)\,\pi_1/\pi_2$ where $\beta_1 = [0, 1]$, $\pi_1 = [0, \pi_2]$.

MODEL DESCRIPTION

By our assumption the protection interval demand X has a p.d.f $f_x(x)$ with finite mean $D(t_1+L)$ and S.D $\sqrt{t_1 + L}$. and the target level $M=D(t_1+L)+p\sigma \sqrt{t_1 + L}$.

As in montgomery for the periodic case, it can be found that the expected holding cost per year is,

$h[M\text{-}DL\text{-}Dt_1/2 + \{1\text{-}\beta(t_1+L)\}\ E(N\text{-}M)]$

and the expected stockout cost per year is,

$$\frac{1}{t_1}\left[\pi_1\beta(t_1 + L) + \pi_2\left(1 - \beta(t_1 + L)\right)\right]E(N - M)$$

Where E (N – M) is the expected demand short at the end of the cycle.

Since the total expected annual cost is the sum of the ordering cost + holding cost and stock-out cost, we have to minimize the expect annual cost (i.e.,)

$$EC(t_1, \beta) = B/t_1 + h[M\text{-}DL\text{-}Dt_1/2 + \{1\text{-}\beta(t_1+L)\}\ E(N\text{-}M)] +$$

$$\frac{1}{t_1}\left[\pi_1\beta(t_1 + L) + \pi_2\left(1 - \beta(t_1 + L)\right)\right]E(N - M) \qquad (1)$$

Now using the value of $\beta = \beta_N\ \pi_1/\pi_2$ in the above, we get,

$$EC(t_1, \beta) = \frac{B}{t_1} + h[M\text{-}DL\text{-}\frac{Dt_1}{2} + \{1\text{-}\ \frac{\beta_N\pi_1}{\pi_2}(t_1+L)\}\ E(N\text{-}M)] +$$

$$\frac{1}{t_1}\left[\pi_1\frac{\beta_N\pi_1}{\pi_2}(t_1 + L) + \pi_2\left(1 - \frac{\beta_N\pi_1}{\pi_2}(t_1 + L)\right)\right]E(N - M) \qquad (2)$$

$$= \frac{B}{t_1} + h[M\text{-}DL\text{-}\frac{Dt_1}{2}] + [h\{1\text{-}\ \frac{\beta_N\pi_1}{\pi_2}(t_1+L)\}$$

$$+ \frac{1}{t_1}\pi_2\left(1 - \frac{\beta_N\pi_1}{\pi_2}(t_1 + L)\right)\Bigg]E(N - M)$$

The model proceed further by the assumption that the protection interval N follows a normal distribution with $d_1\ (t_1 + L)$ and S.D. $\sigma\sqrt{t_1 + L}$.

Since $N = d_1(t_1 + L) + p\sigma \sqrt{t_1 + L}$, the expected shortage quantity E(N–M) at the end of the cycle it can be expressed as,

$$E(N\text{-}M) = \int_M^\infty (x - M) = fN(x)\,dx = \sigma\sqrt{t_1 + L}\,\alpha(p)$$

Where, $\alpha(p) = \delta(p) - p[1\text{-}\gamma(p)]$, $\delta(p)$ and $\gamma(p)$ denotes the p.d.f and distribution function of a standard normal, respectively.

Substituting the values of E(N-M) in (2), we get the revised total annual cost and it becomes,

$$EC(t_1, \pi_1) = \frac{B}{t_1} + h[\frac{Dt_1}{2} + p\sigma\sqrt{t_1 + L} +]$$

$$[h\{1 - \frac{\beta_N\pi_1}{\pi_2}(t_1 + L)\} + \frac{1}{t_1}$$

$$\left(1 - \frac{\beta_N\pi_1^{\,2}}{\pi_2}(t_1 + L) + \pi_2 - \beta_N\pi_1\right)\Bigg]\sigma\sqrt{t_1 + L}\ \alpha(p)$$

For obtaining the optimal values of t_1 and π_1 we will find the minimization of $EC(t_1, \pi_1)$.

Differentiate (3) partially w.r.t 't_1' and π_1 we get

$$\frac{\partial EC(t_1,\pi_1)}{\partial t_1} = -\frac{B}{t_1^{\,2}} + h\left(\frac{d_1}{2} + \frac{p\sigma}{2\sqrt{t_1 + L}}\right) - \frac{C(\pi_1)\sigma\sqrt{t_1 + L}\alpha(p)}{t_1^{\,2}} + \frac{\left[h(1 - \frac{\beta_N\pi_1}{\pi_2}) + \frac{C(\pi_1)}{t_1}\right]\sigma\alpha(p)}{2\sqrt{t_1 + L}} \qquad (4)$$

$$\frac{\partial}{\partial t_1}EC(t_1,\pi_1) = \left(\frac{2\frac{\beta_N\pi_1}{\pi_2} - \beta_N}{t_1} - \frac{h\beta_N}{\pi_2}\right)\sigma\sqrt{t_1 + L}\alpha(p) \qquad (5)$$

Where $C(\pi_1) = \pi_2 - \beta N\pi_1 + \frac{\beta_N \pi_1^2}{\pi_2} > 0$ since $\frac{\pi_1}{\pi_2} > \beta N(1-\frac{\pi_1}{\pi_2})$.

Next we will prove that $EC(t_1, \pi_1)$ is convex function in (t_1, π_1) which satisfies

$$\frac{\partial EC(t_1,\pi_1)}{\partial t_1} = 0 = \frac{\partial EC(t_1,\pi_1)}{\partial \pi_1}$$

Proof: First we obtain the Hessian matrix

$$H = \begin{bmatrix} \frac{\partial^2 EC(t_1,\pi_1)}{\partial t_1^2} & \frac{\partial^2 EC(t_1,\pi_1)}{\partial t_1 \partial \pi_1} \\ \frac{\partial^2 EC(t_1,\pi_1)}{\partial \pi_1 \partial t_1} & \frac{\partial^2 EC(t_1,\pi_1)}{\partial \pi_1^2} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{11} \\ a_{22} & a_{22} \end{bmatrix}$$

From the above matrix H, finding the principal Minor of H, then we continue our procedure for convexity of EC (t_1, π_1).

$$\text{Since } \frac{\partial EC(t_1,\pi_1)}{\partial t_1} = -\frac{B}{t_1^2} + h\left(\frac{d_1}{2} + \frac{p\sigma}{2\sqrt{t_1+L}}\right) - \frac{C(\pi_1)\sigma\sqrt{t_1+L}\alpha(p)}{t_1^2} + \frac{\left[h(1-\frac{\beta_N\pi_1}{\pi_2}) + \frac{C(\pi_1)}{t_1}\right]\sigma\alpha(p)}{2\sqrt{t_1+L}} \quad (4.1)$$

$$\text{Put } \frac{\partial EC(t_1,\pi_1)}{\partial t_1} = 0 \Rightarrow$$

$$\frac{B}{t_1^2} = \frac{hd_1}{2} + \frac{h\sigma}{2\sqrt{t_1+L}}\left[p + \left(1 - \frac{\beta_N\pi_1}{\pi_2}\right)\alpha(p)\right] - \frac{C(\pi_1)\sigma\sqrt{t_1+L}\alpha(p)}{t_1^2} \quad (4.2)$$

Where $C(\pi_1) = \pi_2 - \beta_N\pi_1 + \frac{\beta_N\pi_1^2}{\pi_2} > 0$.

$$\text{(i.e.)} \frac{B}{2t_1^2(t_1+L)} = \frac{hd_1}{4(t_1+L)} + \frac{h\sigma}{4(t_1+L)^{3/2}}\left[p + (1-\frac{\beta_N\pi_1^2}{\pi_2})\,\alpha(p)\right] + \frac{C(\pi_1)\,\sigma\,\alpha(p)}{4t_1(t_1+L)^{3/2}} - \frac{C(\pi_1)\,\sigma\sqrt{t_1+L}\,\alpha(p)}{2t_1^2(t_1+L)} \quad (4.3)$$

Differentiating (4.1) partially w.r.t 't_1'

$$\frac{\partial^2 EC(t_1,\pi_1)}{\partial t_1^2} = -\frac{2B}{t_1^3} + \left(-\frac{C(\pi_1)}{t_1^2\sqrt{t_1+L}} + \frac{2C(\pi_1)\sqrt{t_1+L}}{t_1^3} - \frac{C(\pi_1)}{4t_1(t_1+L)^{3/2}}\right) \alpha(p)\frac{h\sigma}{4(t_1+L)^{3/2}}\left[p + (1-\frac{\beta_N\pi_1}{\pi_2})\alpha(p)\right] \quad (4.4)$$

$$\text{Let } f(t_1) = -\frac{2B}{t_1^3} + \left(-\frac{C(\pi_1)}{t_1^2\sqrt{t_1+L}} + \frac{2C(\pi_1)\sqrt{t_1+L}}{t_1^3} - \frac{C(\pi_1)}{4t_1(t_1+L)^{3/2}}\right)\alpha(p) \quad (4.5)$$

$$\text{and } g(t_1) = \frac{B}{2t_1^2(t_1+L)} - \frac{C(\pi_1)\sigma\alpha(p)}{4t_1(t_1+L)^{3/2}} + \frac{C(\pi_1)\sigma\sqrt{t_1+L}\alpha(p)}{2t_1^2(t_1+L)} \quad (4.6)$$

From (4.3), which shows that,

$$g(t_1) > \frac{h\sigma}{4(t_1+L)^{3/2}}\left[p + (1-\frac{\beta_N\pi_1}{\pi_2})\,\alpha(p)\right] \quad (4.6)$$

By using (4.5) and (4.6) we get,

$$\frac{\partial^2 EC(t_1,\pi_1)}{\partial t_1^2} > f(t_1) - g(t_1)$$

$$> \frac{B(3t_1+4L)}{2t_1^3(t_1+L)} + \frac{(t_1+4L)C(\pi_1)\sigma\alpha(p)}{2t_1^3\sqrt{t_1+L}} > \frac{B(3t_1+4L)(t_1+4L)C(\pi_1)\sigma\alpha(p)}{2t_1^3(t_1+L)} \quad (4.7)$$

Hence, $|a_{11}|$ of $H > 0$.
Next, we will prove that $|a_{22}| > 0$.

$$\text{Now} \frac{\partial EC(t_1,\pi_1)}{\partial t_1} = \left[\frac{2\frac{\beta_N\pi_1}{\pi_2} - \beta_N}{t_1} - \frac{h\beta_N}{\pi_2}\right]\sigma\sqrt{t_1+L}\ \sigma(p) \quad (4.8)$$

$$\text{and} \frac{\partial^2 EC(t_1,\pi_1)}{\partial \pi_1^2} = \frac{2\sigma\beta_N\sigma(p)\sqrt{t_1+L}}{\pi_2 t_1} \quad (4.9)$$

$$\frac{\partial^2 EC(t_1,\pi_1)}{\partial_1 t_1 \partial\pi_1} = -\frac{\left(2\frac{\beta_N\pi_1}{\pi_2}-\beta_N\right)\sigma\alpha(p)\sqrt{t_1+L}}{t_1^2} + \frac{\left(\frac{2\frac{\beta_N\pi_1}{\pi_2}-\beta_N}{t_1} - \frac{h\beta_N}{\pi_2}\right)\alpha(p)}{2\sqrt{t_1+L}} = \frac{-\beta_0(2\pi_1-\pi_2)\sigma\alpha(p)\sqrt{t_1+L}}{\pi_2 t_1^2}$$

Since when $\frac{\partial EC(t_1,\pi_1)}{\partial\pi_1}=0, \Rightarrow \frac{2\frac{\beta_N\pi_1}{\pi_2}-\beta_N}{t_1} - \frac{h\beta_N}{\pi_2} = 0$

$$= -\frac{h\beta_N\sigma(p)\sqrt{t_1+L}}{\pi_2 t_1} \qquad \text{[From (7)]}$$

$$\text{Now, } |a_{22}| = \begin{vmatrix} \frac{\partial EC(t_1,\pi_1)}{\partial t_1^2} & \frac{\partial^2 EC(t_1,\pi_1)}{\partial t\partial\pi_1} \\ \frac{\partial^2 EC(t_1,\pi_1)}{\partial\pi_1\partial t_1} & \frac{\partial^2 EC(t_1,\pi_1)}{\partial\pi_1^2} \end{vmatrix}$$

$$> \frac{B(3t_1+4L)+(t_1+L_1L)C(\pi_1)\sigma\alpha(p)}{2t_1^3(t_1+L)} \left[\frac{2\sigma\beta_N\alpha(p)\sqrt{t_1+L}}{\pi_2 t_1}\right] - \left[\frac{-h\beta_N\sigma\alpha(p)\sqrt{t_1+L}}{\pi_2 t_1}\right]^2$$

Using (4.7), (4.8) and (4.9)

$$> \left[\frac{(t_1+L)C(\pi_1)\sigma\alpha(p)}{2t_1^3\sqrt{t_1+L}}\right]\left[\frac{2\sigma\beta_N\alpha(p)\sqrt{t_1+L}}{\pi_2 t_1}\right] - \left[\frac{h\beta_N\sigma\alpha(p)\sqrt{t_1+L}}{\pi_2 t_1}\right]^2 = \left[\frac{\beta_N\alpha(p)\sqrt{t_1+L}}{\pi_2 t_1}\right] - \left[\frac{(t_1+L)C(\pi_1)\sigma\alpha(p)}{t_1^3\sqrt{t_1+L}} - \frac{h^2\beta_N\sigma\alpha(p)\sqrt{t_1+L}}{\pi_2 t_1}\right]$$

$$\frac{\beta_N\sigma^2\alpha^2(p)\sqrt{t_1+L}}{\pi_2^2 t_1^4}\left[\pi_2^2(1-\beta_N)+3\beta_N\pi_1(\pi_2-\pi_1)\right] > 0$$

$$\Rightarrow |a_{22}| > 0$$

Hence, it is clearly $EC(t_1, \pi_1)$ is a convex function in (t_1, π_1). By examining the 2nd order sufficient conditions, we have

$$\frac{\partial EC(t_1,\pi_1)}{\partial t_1} = 0 = \frac{\partial EC(t_1,\pi_1)}{\partial\pi_1}$$

$$\Rightarrow \frac{B}{t_1^2} = \frac{hd_1}{2} + \frac{h\sigma}{2\sqrt{t_1+L}}\left[p+\left(1-\frac{\beta_N\pi_1}{\pi_2}\right)\alpha(p)\right] - \frac{C(\pi_1)\sigma\sqrt{t_1+L}\alpha(p)}{t_1^2} + \frac{C(\pi_1)\sigma\alpha(p)}{2t_1\sqrt{t_1+L}} \tag{6}$$

$$\text{and } \pi_1 = \frac{t_1 h + \pi_2}{2} \tag{7}$$

Substituting (7) in (6), we get,

$$\Rightarrow \frac{B}{t_1^2} = \frac{hd_1}{2} + \frac{h\sigma}{2\sqrt{t_1+L}}\left[p+\left(1-\frac{\beta_N}{\pi_2}\left(\frac{t_1 h+\pi_2}{2}\right)\right)\alpha(p)\right] - \frac{H(t_1)\sigma\sqrt{t_1+L}\alpha(p)}{t_1^2} + \frac{H(\pi_1)\sigma\alpha(p)}{2t_1\sqrt{t_1+L}} \tag{8}$$

Where,

$$H(t_1) = C(\frac{t_1 h+\pi_2}{2}) = \pi_2 - \beta_N(\frac{t_1 h+\pi_2}{2}) + \frac{\beta_N}{\pi_2}\left(\frac{t_1 h+\pi_2}{2}\right)^2$$

Now we will discuss, about obtaining the optimal values of t_1 and π_1. Based on the following algorithm we can find the optimal values.

ALGORITHM

Step 1: For a given q, determine t_1, from (8) and then calculate π_1 from Eq. (7) and compare π_1 and π_2.

- If $\pi_1 < \pi_2$, π_1 is feasible and the optimal solutions is $(t_1^*, \pi_1^*) = (t_1, \pi_1)$ then go to step 2.
- If $\pi_1 > \pi_2$, π_1 is not feasible. Let $\pi_1 = \pi_2$ and calculate the corresponding value of t_1 from Eq. 6 the optimal solution is $(t_1^*, \pi_1^*) = (t_1, \pi_1)$ then go to step 2.

Step 2: Compute the corresponding minimum expected total annual cost EC (t_1^*, π_1^*) and hence the optimal target level $M^* = d(t_1^* + L^*) + p\sigma\sqrt{t_1^* + L^*}$. And the optimal back order ratio is

$$\beta_N^* = \frac{\beta_N\pi_1}{\pi_2}(t_1^* + L^*).$$

In many practical situations, the distributional information of the protection interval demand is often quite limited. Hence, we exclude the assumption about the normal distribution of the protection interval demand and only assume that the protection interval demand X has given finite first 2 moments: (i.e.) the p.d.f $f_x \in \Omega$, of class of p.d.f's with finite mean $d(t_1+L)$ and standard deviation $\sigma\sqrt{t_1+L}$. Since the probability distribution of X is unknown, we cannot find the exact value of E(N-M). We propose to apply the minimax distribution free procedure for our problem. The minimax distribution free approach for this problem is to find the p.d.f. f_x in Ω for each (t_1, π_1) and then minimize the expected total annual cost over (t_1,π_1).

We need the following propositions to solve the problems of find optimal values of t_1 and π_1.

Proposition I:
For any $f_x \in \Omega$,

$$E(N-M) \le \frac{1}{2}\left[\sqrt{\begin{matrix}\sigma^2(t_1+L)+\{M-\\ D(t_1+L)^2-\{M-D(t_1+L)\}\end{matrix}}\right] \quad (10)$$

Proof: Since $M = d_1(t_1+L) + p\sigma\,\sigma\sqrt{t_1+L}$ for any probability distribution of the protection interval demand X the above inequality always hold. Then, using inequality (10), it suffixes to minimize.

$EC^*(t_1\text{-}\pi_1) =$

$$\Rightarrow \frac{B}{t_1}+h\left[\begin{matrix}\frac{dt_1}{2}+p\sigma\sqrt{t_1+L}+\frac{1}{2}\left(1-\frac{\beta_N\pi_1}{\pi_2}\right)\\ \sigma\sqrt{t_1+L}\,(\sqrt{1+p^2}-p\end{matrix}\right] + \frac{C(\pi_1)\sigma\sqrt{t_1+L}}{2t_1}\left(\sqrt{1+p^2}-p\right) \quad (11)$$

Once again the approach employed is utilized to solve (11). It can be shown that EC (t_1,π_1) which satisfies,

$$\frac{\partial EC(t_1,\pi_1)}{\partial t_1}=0=\frac{\partial EC(t_1,\pi_1)}{\partial \pi_1}$$

$$\text{(i.e.)}\frac{B}{t_1^2}=\frac{hd_1}{2}+\frac{h\sigma}{2\sqrt{t_1+L}}\left[p+\frac{1}{2}\left(1-\frac{\beta_N\pi_1}{\pi_2}\right)\left(\sqrt{1+p^2}-p\right)\right]$$

$$-\frac{C(\pi_1)\sigma}{4t_1\sqrt{t_1+L}}\left(\sqrt{1+p^2}-p\right)+\frac{C(\pi_1)\sigma\sqrt{t_1+L}}{2t_1^2}\left(\sqrt{1+p^2}-p\right) \quad (12)$$

$$\text{and } \pi_1=\frac{t_1h+\pi_2}{2} \quad (13)$$

Using (13) in (12), we get,

$$\Rightarrow\frac{B}{t_1^2}=\frac{hd_1}{2}+\frac{h\sigma}{2\sqrt{t_1+L}}\left[\begin{matrix}p+\frac{1}{2}\left(1-\beta_N\frac{t_1h+\pi_2}{2\pi_2}\right)\\ \left(\sqrt{1+p^2}-p\right)\end{matrix}\right]$$

$$+\frac{H(t_1)\sigma}{4t_1\sqrt{t_1+L}}\left(\sqrt{1+p^2}-p\right)+\frac{H(t_1)\sigma\sqrt{t_1+L}}{2t_1^2}\left(\sqrt{1+p^2}-p\right) \quad (14)$$

From the Eq. 13 and 14 we can obtain the optimal (t_1, π_1) and the corresponding expected total annual cost $EC(t_1, \pi_1)$ can be obtained. However in practice, since p.d.f $f_x(x)$ is unknown even if the value of q is given we cannot get the exact value of p.

Therefore, in order to find the value of p, we need one more proposition.

Proposition 2: Let N represent the protective interval which has a p.d.f $f_x(x)$ with finite mean $d_1(t_1+L)$ and standard deviation $\sigma\sqrt{t_1+L}$ then for any real number $b > 0$.

$$P(N>b) \le \frac{\sigma^2(t_1+L)}{\sigma^2(t_1+L)+[b-d_1(t_1+L)]^2} \quad (15)$$

Proof: Since the target level $M = d_1(t_1+L) + p\ \sigma\sqrt{t_1+L}$ as mentioned earlier, if we take M instead of b in (15), we get,

$$P(N>M) \le \frac{1}{1+p^2} \quad (16)$$

Since it is assumed that the permissible stockout probability q during the protection is termed, is given, (i.e.) q = P(N>M), thus from (16). We can obtain

$$0 \le p \le \sqrt{\frac{1}{q}-1}$$

It is easy to verify that $EC^*(t_1, \pi_1)$ has a smooth curve for $p\in$

$$[0,\ \sqrt{\frac{1}{q}-1}]$$

CONCLUSION

In this study, we present a new stochastic periodic review inventory model involving controllable backorder discount. In the real market as unsatisfied demands occur, the higher the price discount from a supplier, the higher the advantage of the customers and hence, a large no. of backorders may result considering the reason, we assume that the backorder ratio is dependent on the amount of price discount from a supplier. In this study, we first assume that the protection interval demand follows a normal distribution and then exclude their assumption and apply the minimax distribution free procedure to solve the problem.

Notations:

B	:	Fixed ordering cost per order.
h	:	Inventory holding cost/unit/year.
M	:	Target level.
t_1	:	Length of a review period
L	:	Length of lead-time
N	:	The protection interval, which can be expressed as the sum of length of review period of length of lead-time length of lead-time = t_1 + L, demand which a probability density function (p.d.f) f_x with finite mean D(t_1+L) and S.D. $\sigma\sqrt{t_1 + L}$.
β_N	:	$\beta(t_1 + L)$ back order ratio which depends on the length of protection interval.
β_N^+	:	Upper bound of the back order ratio.
π_1	:	Back order price discount offered by the supplier per unit.
π_2	:	Marginal profit.
EXP(N)	:	Mathematical expectation.

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