Journal of Engineering and Applied Sciences 2 (9): 1383-1386, 2007
ISSN: 1816-949X

Study of the Optical Bistability in a Laser Containing a Saturable Absorber

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Abstract: In this study, we develop a simple mathematical model to describe the action of a saturable absorber in a Laser cavity (LSA). We will study theoretically the effect of optical bistability in a laser saturable with homogeneous broadening for a Fabry-Perot resonator. On the basis of the system of Rate Equation Approach (REA), we returned this system without dimension then analytically solved this type of equations for the stationary case. We have numerically solved this type of equation for stationary case. We elaborated a program in Matlab allowing to determine the effect of the optical bistability, to draw the curves representing the density of the photons in the pumping function of the active medium and to analyze the linear stability of the obtained solutions.

Key words: Optical Bistability (OB), Laser containing a Saturable Absorber (LSA), homogeneous broadening, fabry-perot cavity

INTRODUCTION

The optical bistability has known a great development these last years. The majority of the optical flip-flops developed until now utilize two states of transmission which result in levels "high" and" low" of the energy transmitted by the device Bistable systems are possible when setups such as the Fabry Perot etalon or ring cavity are used in combination with a non-linear medium (Hong and Ha, 1991; Hong and Bao, 1999; Bao *et al*., 1999a, b). Two types of non-linear optical elements can be used:

Absorptive non-linear elements: The absorption coefficient, a, is a function of the optical intensity. In Absorptive Element, a saturable absorber has an absorption coefficient which is a non-linear function of density of photons. The cavity is setup for resonance: At small intensities, the absorption due to the element is high and the output is low and as the intensity is increase beyond intensity of threshold, the absorption rapidly decreases and the output goes high.

Dispersive elements: The refractive index, n, is a function of the optical intensity.

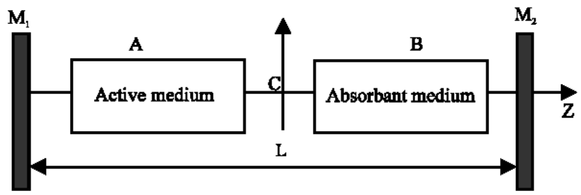

Fig. 1: Device of a laser containing a saturable absorber. L is the cavity length; Z is the direction of propagation. M1 and M2 are plane mirrors

From the solution of these Rate Equations Approach (REA) of a LSA with Fabry-Pérot by considering a homogeneous broadening for the stationary case, we could show the existence of the effect of phenomenon of the optical bistability while varying the different physical parameters. We finished our study by the analysis of the stability of the stationary solutions.

The device of a laser containing a saturable absorber is shown by Fig. 1.

SYSTEM OF RATE EQUATIONS APPROACH (REA)

The system of a laser with saturable absorber where broadening is homogeneous consists of the following

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three equations equations: One equation giving the density of photons n_j in the cavity resonator, Two further rate equations determine the variation of the difference of population of the active medium N_a and that of the absorbing medium N_b. According to Hong and Ha (1991), Hong and Bao (1999), Baoand *et al.* (1999a), Bao *et al.* (1999b) and Djabi (1993) the system of equations is written as follows:

$$\left.\begin{aligned}
&\frac{dn_j}{dt} = -x_j n_j + \frac{B}{L} g\left(n_j + 1\right) \\
&\left[\int_{z_2-\frac{1}{2}}^{z_1+\frac{1}{2}} N_a \sin^2\left(\frac{\pi}{L}qz\right)dz - \int_{z_2-\frac{1}{2}}^{z_1+\frac{1}{2}} N_b \sin^2\left\{\left(\frac{\pi}{L}qz\right)dz\right.\right] \\
&\frac{dN_a}{dt} = \frac{R_a}{L} - N_a\left(Bgn_j + \gamma_a\right) \\
&\frac{dN_b}{dt} = \frac{R_b}{L} - N_b\left(Bgn_j + \gamma_b\right)
\end{aligned}\right\} \quad (1)$$

Where,

x_j : Represent the coefficients of the losses of the resonator for the mode j.

B : Is the coefficient of Einstein

L, q : Are the cavity length and is the mode index, respectively

γ_a, γ_b : Are the damping coefficients of the active medium and absorber respectively

We assume $\gamma_b = \xi\gamma_a = \xi\gamma$ with ξ a saturation coefficient: $(0 \le \xi \le 1)$.

Absorption and emission line shapes are supposed top have a Lorentzian shape g with

$$g = \frac{\Gamma^2}{\Gamma^2 + \Delta_j^2}$$

Where, Γ is the homogeneous width of line and $\Delta_j = |\omega_j - \omega_0|$ the detuning from the central frequency ω_0.

Are the pumping rates of the active and the absorbing medium respectively,

The term

$$\left[\int_{z-\frac{1}{2}}^{z+\frac{1}{2}} N_a \sin^2\left(\frac{\pi}{L}qz\right)dz - \int_{z-\frac{1}{2}}^{z+\frac{1}{2}} N_b \sin^2\left(\frac{\pi}{L}qz\right)dz\right]$$

describes the effect of interference produced in the Fabry-Pérot resonator. It can take various values which depend on the position of the two nonlinear media in the optical resonator. In general, if $l >> \lambda$, (where λ is the wavelength of the laser modes) the populations N_a and N_b are independent of position z inside the cavity and consequently this term can be written as:

$$(N_a - N_b)\left[\int_{z-\frac{1}{2}}^{z+\frac{1}{2}} \sin^2\left(\frac{\pi}{L}qz\right)dz\right] = \left(N_a - N_b\right)\frac{A_0}{2}$$

For the case where the nonlinear media are placed in the middle of the cavity, A_0 takes the value

$$\left[1 - \frac{L}{\pi q} \sin\frac{\pi q l}{L}\right]$$

STUDY OF THE OPTICAL BISTABILITY IN A LSA

Study of the effect of the optical bistability: To study the optical bistability, us let us consider a stationary situation to this end while posing:

$$\frac{dn_j}{dt} = \frac{dN_a}{dt} = \frac{dN_b}{dt} = 0$$

To simplify and facilitate calculations, we write the system of Eq. 1 in a form which uses only adimensionnées sizes, namely (Tarassov, 1985):

- Time standardized compared to the rate of relieving: $t' = t\gamma$.
- Density of photons standardized compared to the density of photons to saturation:
  $$Q_j = \frac{n_j}{n_{jsat}} \quad \text{with} \quad n_{jsat} = \frac{1}{\frac{B}{\gamma}g}$$
- Coefficient of the losses standardized at the rate of relieving:
  $$\xi_0 = \frac{x_j}{\gamma}$$
- Densities of inversion of population of the active medium and absorbent standardized compared to the density of inversion of population of system to the threshold:
  $$n_a = \frac{N_a}{(N_a - N_b)_{seuil}} \text{ and } n_b = \frac{N_b}{(N_a - N_b)_{seuil}}$$
  $$\text{with } \left(N_a - N_b\right)_{seuil} = \frac{x_j}{\frac{A_0}{2L}Bg}$$

Pumping of the active medium and absorbent standardized compared to the pumping of the threshold:

$$r_a = \frac{R_a}{R_{seuil}} \text{ and } r_b = \frac{R_b}{R_{seuil}} \quad \text{with } R_{seuil} = L\gamma\left(N_a - N_b\right)_{seuil}$$

By making this change, after simple transformations, the system of equations becomes:

$$\left.\begin{aligned} \frac{dQ_j}{dt'} &= \left[-Q_j + \left(Q_j + \frac{1}{n_{jsat}}\right)(n_a - n_b)\right] \\ \frac{dn_a}{dt'} &= r_a - n_a\ (Q_j + 1) \\ \frac{dn_b}{dt'} &\ \xi r_b - n_b\ (Q_j + \xi) \end{aligned}\right\} \qquad (2)$$

The equation becomes, after simplification, a cubic equation in following form:

$$A_4 Q_j^3 + A_3 Q_j^2 + A_2 Q_j + A_1 = 0 \qquad (3)$$

Where,

$$A_4 = 1,\ A_3\ (1+\xi) - (r_a - \xi r_b).$$

$$A_2 = \xi - \xi(r_a - r_b) - \frac{1}{n_{jsat}}(r_a - \xi r_b)$$

And

$$A_1 = \frac{-1}{n_{jsat}}\xi(r_a - r_b)$$

The Eq. 3 is a cubic equation of which it last term tends towards zero. However ($A_1 << 1$).

$$A_1 = \frac{-1}{n_{jsat}}\xi(r_a - r_b) \text{ with } \frac{1}{n_{jsat}} \approx 10^{-12}$$

We can apply a variationnel method. The Eq. 3 can be put in the following form:

$$\left(Q_j + \frac{A_1}{A_2}\right)(Q_j^2 + A_3 Q_j + A_2) = 0 \qquad (4)$$

In short, to have at least two positive solutions i.e. the effect of the optical bistability, it is necessary that the pumping of the active medium (r_a) checks at the same time the following conditions:

$$\begin{cases} r_a < 1 + r_b = r_{a2} \\ r_a > \dfrac{-(2\xi - 2\xi r_b - 2) + \sqrt{\delta}}{2} = r_{a1} \\ r_a < \dfrac{-(2\xi - 2\xi r_b - 2) - \sqrt{\delta}}{2} = r_{a3} \\ r_a > 1 + \xi(1 + r_b) = r_{a4} \\ r_a > r_b = r_{a5} \end{cases} \qquad (5)$$

To determine the interval of the optical bistability, it is necessary to compare the values of pumping of the active medium (r_a) between them.

One finds the interval following: $I_{BO} = [r_{a1}, r_{b2}]$ what implies that $r_{a1} < r_a < r_{a2}$.

And the roots of the Eq. 4 giving the densities of photons are:

$$Q_0 = -\frac{A_1}{A_2},\ Q_1 = \frac{-A_3 - \sqrt{\Delta}}{2} \text{ and } Q_2\ \frac{-A_3 - \sqrt{\Delta}}{2} \qquad (6)$$

With:

$$\Delta = A_3^2 - 4A_2$$

Curves representing the optical bistability according to pumping of active medium: To plot the curves which represent the densities of photons Q_0, Q_1 and Q_2 according to pumping of the active medium (r_a) in the interval of the optical bistability obtained and more particularly to examine the influence of the various parameters of the L.S.A the coefficient of saturation ξ and the pumping of the absorbing medium r_b, we drew up a program in MATLAB in order to determine the intervals of the optical bistability and the values of (Q_0, Q_1, Q_2) then, we plotted the curves of hysteresis representing the densities of photons according to the pumping of the active medium (r_a) for the parameters ξ and r_b (Fig. 2).

Linearization of the nonlinear system of equations in the vicinity of the stationary solution: The linear analysis of stability consists in determining the evolution of small variations of balances. If the balance is stable, their variations diminish in time. On the other hand, they diverge in the case of an unstable balance. One linearizes the system of differential equations in the vicinity of the stationary solutions and considers for this purpose (Francon, 1986; Tarassov, 1985; Tachikawa and Josa, 1987; Dangoisse *et al*., 1987):

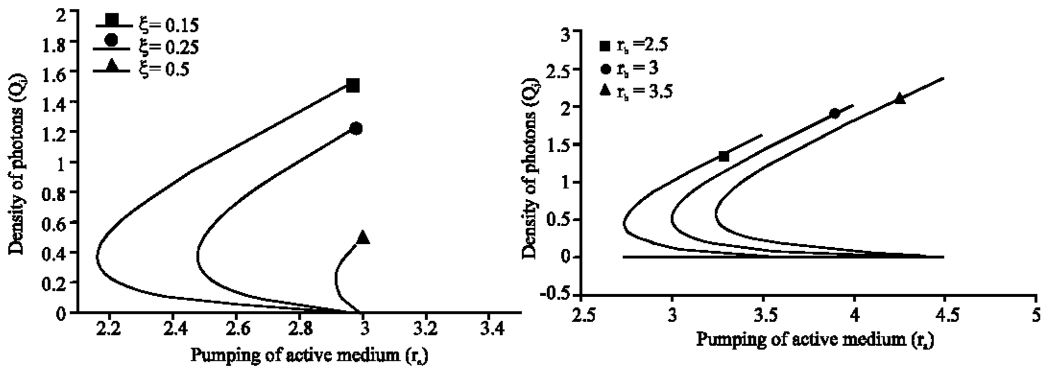

Fig. 2: Density of photons Qj according to the pumping of the active medium (r_a)

$Q_j = Q_{js} + \Delta Q_j(t')$, $n_a = n_{as} + \Delta n_a(t')$ and $n_b = n_{bs} + \Delta_{nb}(t')$

Q_{js}, n_{as}, n_{bs} : Are stationary values.
ΔQ_j, Δn_a, Δn_b : Are small corresponding variations of quantity with respect to their stationary values.

The system becomes:

$$\begin{bmatrix} \frac{d\Delta Q_j}{dt'} \\ \frac{d\Delta n_a}{dt'} \\ \frac{d\Delta n_b}{dt'} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \Delta Q_j \\ \Delta n_a \\ \Delta n_b \end{bmatrix} \tag{7}$$

Where,

$$a_{11} = \xi_0 (n_{as} - n_{bs} - 1),\ a_{12} = \xi_0 \left(Q_{js} + \frac{1}{n_{jsat}} \right),$$

$$a_{13} = \xi_0 \left(Q_{js} + \frac{1}{n_{jsat}} \right),\ a_{21} = -n_{as},\ a_{22} = (Q_{js} + 1),$$

$$a_{23} = 0,\ a_{31} = -n_{bs},\ a_{32} = 0 \text{ and } a_{33} = -(Q_{sj} + \xi)$$

To obtain the values of λ, we calculate the determinant of $(A + \lambda I) = 0$. After calculation, one obtains a cubic equation in following form:

$$\lambda^3 + C_1 \lambda^2 + C_2 \lambda + C_3 = 0 \tag{8}$$

With

$C1 = a_{11} + a_{22} + a_{33}$, $C_2 = a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} - a_{12}a_{21} - a_{13}a_{31}$ and $C_3 = a_{11}a_{22}a_{33} - a_{12}a_{21}a_{33} - a_{13}a_{31}a_{22}$

It is noticed that Eq. 8 is an algebraic equation of the third degree. To know if the state is stable or unstable, its roots should be determined.

CONCLUSION

According to the obtained results in the analysis of the optical bistability, we note that the branches representing Q_{j1}, Q_{j0} and Q_{j2} are unstable because they have two positive roots and a negative root of the eigenvalues some are the parameters of the L.S.A (the coefficient of saturation ξ and the pumping of the absorbing medium r_b taken in the interval of the optical bistability.

In conclusion, the release of a laser per insertion of a saturable absorber in the cavity has many advantages:

- Possibility of obtaining short impulses.
- Release of the laser without contribution of external energy.

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