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An Estimation Method for Ununiformity Degree of Fluid Flow Distribution in Fuel Cell Stacks

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Abstract: There is a non-uniform fluid flow distribution in fuel cell stacks. To gain an insight into this distribution an analytical model has been developed. In the model, the stack is viewed as a network of hydraulic resistances. The resistances both in manifold channels and flow field plate channels are included. The flow ununiformity degree (the flow difference between the average and the least flux of cell) can be solved from the model. The influence of the cross-sectional area of manifold channels and the number of cells on the flow distribution has been analyzed. In this manner, a very useful tool for fuel cell stacks design is obtained.

Key words: Fuel cell stacks, fluid distribution, fluid manifold

INTRODUCTION

Fuel cells are electrochemical devices that directly convert the chemical energy of a fuel into electricity. The high efficiency of fuel cells and the prospects of generating electricity without pollution have made them a serious candidate of power. Depending on the load, one single fuel cell produces 0.5~1 V. To yield a sufficiently high voltage, the cells are stacked. Stacking of the cells imposes some difficult technological problems. One of them is how well the reactant and refrigerant fluids distribute along a stack of fuel cells. A non-uniform distribution will cause differences in the performance of each cell, which is allowed within certain limits[1]. Important parameters in such a prediction are the actual fluid flows and the resistance that the fluid meets when flowing through the system.

The physical and chemical phenomena related to a fuel cell stack load following performance are considered by lots of researchers[2-6]. Fujimura *et al.*[2] provided a three-dimensional, steady-state temperature distribution across a stack without reforming, which takes into account of the heat transfer in the stacking direction and between the end plates and the surroundings. Shinoki *et al.*[3] also provided a steady-state model for a stack with internal reforming. Achenbach[4,5] provided the current and temperature distributions in an SOFC stack under both steady-state and transient conditions. In addition, there are simplified one-dimensional or lumped stack models for integration into the system models. However, some of the models are still based on the assumption of a uniform gas distribution along the stack direction and some of them are too complicated which are more suitable for process simulations or design verifications. Boersma *et al.*[6] provided a analytical model for stacks, but only U type of manifold has been considered and the resistances influenced by the consumption of fluid are not studied.

The model presented in this study provides an analytical approximation and is applicable for fuel cell stacks. The manifold of Z and U type Fig. 1 has been considered. The analytical approach has the disadvantage that simplifications need to be made to get a result. The advantage I s that under the simplifications, the result is verifiable, as opposed to a numerical analysis. Furthermore, the analytical approach clearly identifies the importance of each parameter in relation to the fluid distribution.

Model: The fluid flow distribution in a fuel cell stack is determined by the hydraulic resistances in the flow field plate and manifold channels. The resistances are: the resistance caused by the flow through the manifold and cell channels, which can be regard as pipes; the resistance caused by the turnings of the flow in the cell channels; the resistance that results from the pressure drop caused by splitting and combing of the flow in the manifold channels.

The main assumptions are:

- The resistance in the channels can inferred from laminar flow formulae[7]
- The consumption rate of the fluid due to the reaction is assumed to be positive correlation to the flow flux.

Flow results from a pressure drop. For a simple one-dimensional duct flow, the relation between flow and along the pipe pressure drop can be given by Department Hydraulic Engeering[8].

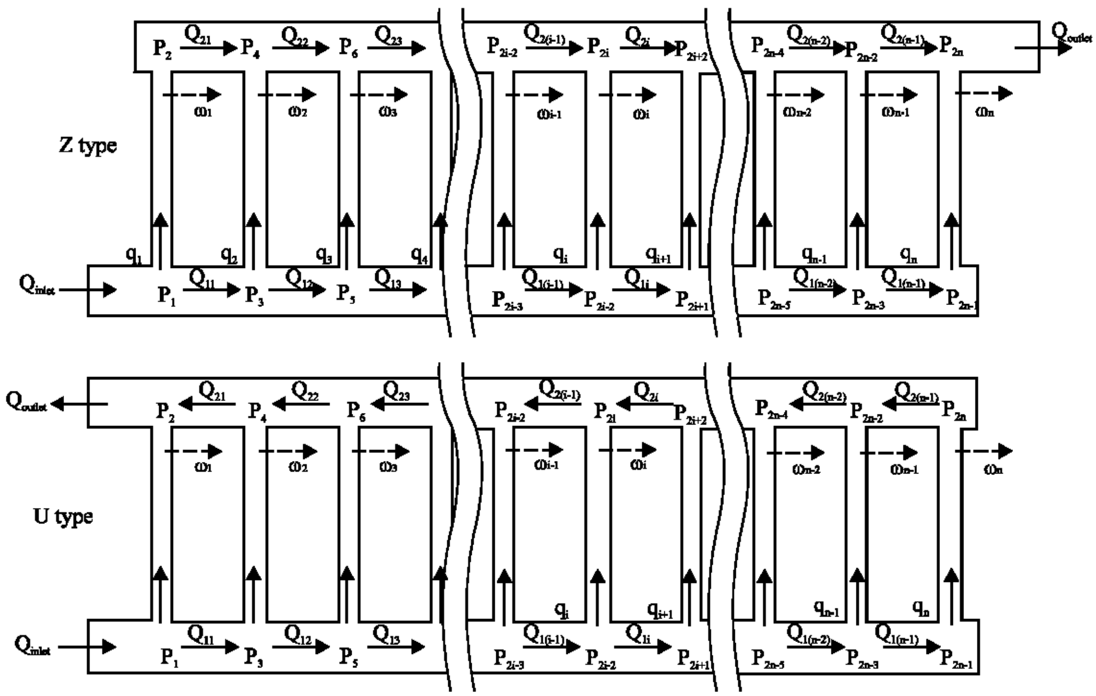

Fig. 1: Flow through a stack. Q is the flow flux of manifold, q is the flow flux of per cell and ω is the fluid assumption of one cell

$$\Delta p_{pl} = K\frac{1}{2D_{hc}}\rho v^2 \quad (1)$$

where Δp_{pt} is the along the pipe pressure drop, K the hydraulic resistance coefficient, ρ the density, v the flow velocity averaged over the cross-sectional area, l the flow length and D_{hc} is the hydraulic diameter of the channel.

The relation between flow and local pressure drop can be given by the local resistance coefficient method[8].

$$\Delta p_u = \frac{\xi}{2}\rho v^2 \quad (2)$$

where ξ is the local resistance coefficient.

The relation between flow and local pressure drop can also be given by the equivalent length method[8].

$$\Delta p_u = K\frac{l_e}{2D_{hc}}\rho v^2 \quad (3)$$

where l_e is the equivalent length.

Along the pipe pressure drops of the cell and manifold channels are computed by Eq. 1. The local resistances o f cell channels are computed by Eq. 3 and the local resistances of manifold channels are computed by Eq. 2.

The resistance in the cell channels: The flow in these channels is assumed to be laminar. This is valid for most cases and can be checked by calculating the actual Reynolds number for a specific case. For laminar flow, *K* is inversely proportional to v, that is $K = \frac{2f\mu}{\rho D_{hc}^2 v}$. Therefore, the pressure drop in Eq. 1 and 3 is linear with v and hence they can be shown that

$$\Delta p_{pl} = K' l_c q \quad (4)$$

$$\Delta p_u = K' l_e q \quad (5)$$

with

$$K' = \frac{f\mu}{A_c D_{hc}^2} \quad (6)$$

where μ is the fluid viscosity, k the number of channels in parallel that can be allocated to the manifold channel, A_c the cross-sectional area of one channel, l_c the channel length and q the flow that passes the cell area considered (which is equal to kvA_c); f is a friction factor that depends on the channel geometry. For a channel with a square cross section f = 28.4[9]. The mass balance equation of the lamina Δx at x of the channel can be given as

$$0 = q|_{x} - q|_{x+\Delta x} - rA_c\Delta x \quad (7)$$

where rA_c is the consumption rate of the fluid flow through a unit of length. Assume that r is linear with the flux, that is

$$r = w\frac{q}{A_c} \quad (8)$$

where w is consumption factor. Thus, substituting Eq. 8 to 7 results in

$$\frac{dq}{dx} = -wq \quad (9)$$

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$$x = 0,\ q = q_0 \quad (10)$$

By solving Eq. 9 the fluid flow distribution along the channels can be given as

$$q = q_0 e^{-w\cdot x} \quad (11)$$

from Eq. 4 we get

$$-dp_{p1} = K'qdx \quad (12)$$

Using Eq. 5, 11 and 12, the total pressure drop can be given as

$$\Delta p = \Delta p_{p1} + \sum_{i=1}^{m}\Delta p_u(i) = -\int_0^{l_c} K'qdx - \sum_{i=1}^{m} K'l_e q(x_i) \quad (13)$$

Substituting Eq. 11 to 13 results in

$$\Delta p = K'q_0\left(\frac{1-e^{-wl_c}}{w} + \sum_{i=1}^{m} e^{-\frac{iwl_c l_e}{m+1}}\right) \quad (14)$$

The consumption factor of fluid can be given by Faraday's law

$$IS = n'F\frac{q_0(1-e^{wl_c})}{22.4\times10^{-3}} \quad (15)$$

where I is average current density, S the cell area and F is the Faraday constant.
Eqation 14 will then become

$$\Delta p = K_c q_0 \quad (16)$$

with

$$K_c = \left(\frac{1-e^{-wl_c}}{w} + \sum_{i=1}^{m} e^{-\frac{iwl_c l_e}{m+1}}\right)\frac{f\mu}{A_c D_{hc}^2} \quad (17)$$

The resistance in the manifold channels: The pressure drop between two adjacent cells in the inlet and outlet manifold channels Δp_{mi} and Δp_{mo} respectively, can be written as[9]

$$\Delta p_{mi} = K_1 Q_1^2 \quad (18)$$

$$\Delta p_{mo} = K_2 Q_2^2 \quad (19)$$

where Q_1 and Q_2 are the flows in the manifold channels of the inlet and outlet, respectively.

K_1 represents the sum of the pipe flow and the resistance due to splitting, while K_2 represents the sum of the resistance of the pipe flow and the resistance due to combining and are described by Eq. 20 and 21

$$K_1 = \left(K_{mi} + K_p\frac{l_m}{D_{hm}}\right)\frac{1}{2}\frac{\rho}{A_m^2} \quad (20)$$

$$K_2 = \left(K_{mo} + K_p\frac{l_m}{D_{hm}}\right)\frac{1}{2}\frac{\rho}{A_m^2} \quad (21)$$

where K_{mi} and K_{mo} are the resistance coefficients for splitting and combination of the flow, respectively and K_p is the resistance coefficient for the pipe flow of the manifold channel. These coefficients are dimensionless and are tabulated in handbooks[7] or have to be inferred from measurements. D_{hm} is the hydraulic diameter of the manifold channel, l_m is the distance between two adjacent cells, measured between the centers of the plates and A_m is the area of a manifold channel. It is assumed that the inlet and out let diameters are equal, so we can assume that $K_1 = K_2$.

Estimation for fluid distribution: The stack can be represented as a network of hydraulic resistances as depicted in Fig. 1.

- Using Eq. 16, 18 and 19, for Z type of manifold channels, the pressure drop across the cell, with index i, can be written as

$$\Delta p_i = K_c q_1 + K_2(Q_{21}^2 + Q_{22}^2 + \ldots + Q_{2(i-1)}^2) - K_1(Q_{11}^2 + Q_{12}^2 + \ldots + Q_{1(i-1)}^2) \quad (22)$$

and

$$\Delta p_i = K_c q_i \qquad i = 2,3,4 \tag{23}$$

Thus, combining Eq. 22 and 23 gives

$$q_i = q_1 + \frac{K_1 + K_2}{2K_c}[(Q_{21}^2 - Q_{11}^2) + (Q_{22}^2 - Q_{12}^2) + \ldots + (Q_{2(i-1)}^2 - Q_{1(i-1)}^2)] \tag{24}$$

To approximate the sum of the terms in Eq. 24, it is assumed that the distribution is rather good. Eq. 24 will then become

$$q_i \approx q_1 + \frac{K_1 + K_2}{2K_c}\bar{q}^2 \{(1-\frac{\omega}{\bar{q}})^2[1+2^2+3^2+\cdots+(i-1)^2] - [(n-1)^2+(n-2)^2+\cdots+(n-i+1)^2]\} \qquad i = 2,3,4 \tag{25}$$

where $\bar{q}$ is the average inlet flow in cells i to n. Because the utilization of fuel may be 70~80%, we get

$$1 - \frac{\omega}{\bar{q}} = 0.09\text{~}0.04$$

From Eq. 25, we knows that cell n gets the least flow flux. Thus, the flow ununiformity degree (the flow difference between the average and the least flux of cell) can be given as

$$\gamma = \frac{|q_n - \bar{q}|}{\bar{q}} \tag{26}$$

Thus, combining Eq. 25 and 26 gives

$$\gamma \approx \frac{1}{4}\frac{K_1 + K_2}{K_c}\bar{q}[1 - (1-\frac{\omega}{\bar{q}})^2]\sum_{i=1}^{n-1} i^2 \tag{27}$$

- For the U type manifold channels, the flow flux of cell i can be given as

$$q_i = q_n + \frac{K_1 + K_2}{2K_c}[(Q_{1i}^2 + Q_{1(i+1)}^2 + \ldots + Q_{1(n-1)}^2) + (Q_{2i}^2 + Q_{2(i+1)}^2 + \ldots + Q_{2(n-1)}^2)] \tag{28}$$

To approximate the sum of the terms in Eq. 28, it is assumed that the distribution is rather good. Eq. 28 will then become

$$q_i \approx q_n + \frac{1}{6}\frac{K_1 + K_2}{K_c}\bar{q}^2\left[\left(1 - \frac{\omega}{\bar{q}}\right)^2 + 1\right](n-i+1)(n-i+2)(2n-2i+3) \tag{29}$$

Assume that the average flow for the entire stack is found in cell J and then it can be written as

$$q_J = \bar{q} = \frac{\sum_{i=1}^{n} q_i}{n} \tag{30}$$

Substituting Eq. 29 results in

$$q_J = q_n + \frac{1}{12}\frac{K_1 + K_2}{K_c}\bar{q}^2\left[\left(1 - \frac{\omega}{\bar{q}}\right)^2 + 1\right]\left[n\left(n^2 - 1\right)\right] \tag{31}$$

By comparing Eq. 29, for J=i and Eq. 31, the cell that gets the average flow is found. For stack height of interest (n>100) this is the study at approximately 38% of the stack height. The flow ununiformity degree can be given as

$$\gamma \approx \frac{1}{12}\frac{K_1 + K_2}{K_c}\bar{q}^2\left[\left(1 - \frac{\omega}{\bar{q}}\right)^2 + 1\right](0.62n+1)(0.62n+2)(1.24n+3) \tag{32}$$

Case study: To show the relevance of Eq. 27 and 32 a case study has been undertaken. Table 1 shows the data used for the calculations.

Table 2 shows the results calculated by Eq. 27 and 32 and the results calculated by numerical methods. It is found that the error of estimation method increases as the number of cells increases, but the estimation results are safer.

The influence of the cross-sectional area of the manifold channel A_m on the ununiformity degree of fluid distribution is shown in Fig. 2. The stack which comprises manifold channels of larger cross-sectional area has better fluid distribution. Because the larger cross-sectional area produces smaller resistance, the flow resistances of stacks are mainly caused by fluid flow through cells, the hydrogen then can distribute better. In addition, fluid distribution is better in U type of manifold channels than Z type of manifold channels.

Figure 3 shows how the number of cells of a stack influent on the ununiformity degree. It can be seen that

Table 1: Data used in the model

Symbol	Name	Value	Unit	References
μ	viscosity	1.01×10^{-5}	Pa•s	[6]
f	friction factor	28.4	-	[9]
A_c	cross-sectional area of one gas channel	1×10^{-6}	m^2	[6]
D_{hc}	hydraulic diameter of one gas channel	1×10^{-3}	m	
l_c	length of gas channel	0.3	m	
l_e	equivalent length	0.03	m	[8]
	average flow flux of per cell	6×10^{-5}	$m^{3/s}$	
A_m	cross-sectional area of one manifold channel	3.5×10^{-4}	m^2	[6]
D_{hm}	hydraulic diameter of one manifold channel	0.02	m	
l_m	distance between two adjacent cells	0.005	m	
K_p	resistance coefficient for the pipe flow of the manifold channel	0.14	-	[9]
K_{mi}	resistance coefficients for splitting of the flow	0.16	-	[9]
K_{mo}	resistance coefficients for combination of the flow	0.16	-	[9]
ρ	density	0.069	kg m^{-3}	
n	number of cells	200	-	
S	cell area	0.04	m^2	

Table 2: The results calculated by Eq. 27 and 32 and the results calculated by numerical methods

	Z type of manifold channels		
n	|γ(estimation)	|γ(numerical)	Error
100	0.0015	0.0011	0.0004
200	0.0126	0.0074	0.0052
300	0.0420	0.0265	0.0155
U type of manifold channels			
100	0.0009	0.00084	0.00006
200	0.007	0.0056	0.0014
300	0.023	0.019	0.004

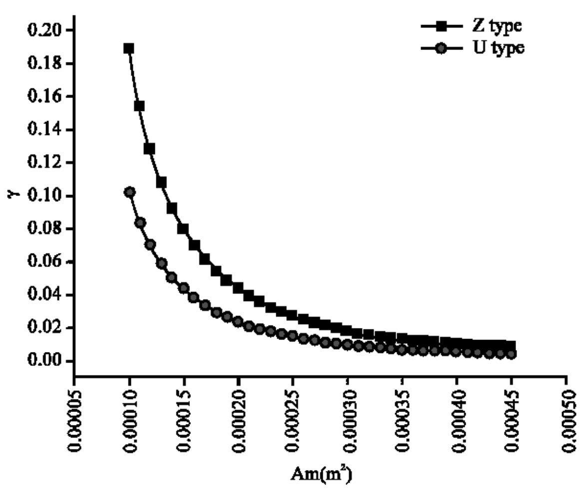

Fig. 2: The influence of the cross-sectional area of the manifold channel on the ununiformity degree of fluid distribution

for a smaller stack the fluid is evenly distributed. Therefore, cell channels of bigger resistances or manifold channels of smaller resistances should be used in the stack of more cells.

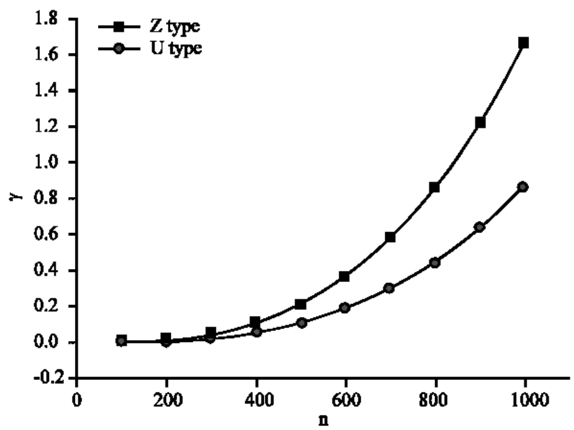

Fig. 3: The influence of the number of cells of a stack on the ununiformity degree of fluid distribution

CONCLUSION

- The flow distribution for fuel cell stacks can be given in analytical form, thereby a very useful tool is obtained for stack design.
- The influence of the cross-sectional area of manifold channels and the number of cells on the flow distribution has been analyzed.

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