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Multi-Scale Analyses of Damage Evolution in Plain Woven Composites

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Abstract: The multi-scale method is adopted in damage analysis of plain woven composites. The microscopic Repeated Unit Cell (RUC) model for yarn was studied firstly with appropriate failure criteria, thus elastic and strength properties were obtained. Then these results were applied to failure research in the mesoscopic-repeated unit cell model for woven composite. The predicted results by the present method are compared with the experimental data and good correlation is observed.

Key words: Plain woven composite, multi-scale analysis, damage evolution, finite element method

INTRODUCTION

Woven fabric is constituted with two vertical yarns and has advantages of better stability, impact resistance and low fabrication cost as compared with unidirectional fiber reinforced composites. Therefore, woven composites have been widely used as primary structural constituents in aerospace, automobile, marine and defense industries.

For effective use and design of woven composites, one should understand their mechanical behavior clearly. The complex architecture of the woven composites, however, makes the analysis of their mechanical behavior very difficult. In past decades, tremendous amount of works dedicated to predicting the elastic properties of the materials with the repeated unit cell approach. Ishikawa[1] and Chou (1982) (Naik and Shembekar, 1992) developed one-dimensional and two-dimensional models and obtained the elastic properties based on the classical lamination theory. Recently, with rapidly growing computational capability, three-dimensional models have been used to predict the overall properties (Tan *et al*., 1998; Barbero *et al*., 2006). Further more, textile composites have a feature of an inherent structural hierarchy in various length scales, i.e., a) the fiber diameter s cale; b) the yarn-diameter scale; c) the meso-repeated unit cell scale; and d) the macro-structural component scale. Swan and Kim (2002) (Wang *et al*., 2005) studied the elastic behavior of woven composite step by step based on the multi-scale analysis.

The works mentioned above were all contributed to the elastic constants. As is well known, it is very important to make clear of strength properties of woven composites. For damage analyses of composites (Zako *et al*., 2003; Zeng *et al*., 2004), there are two major issues, failure criterion and post-failure stiffness reduction. Among these analyses, generally speaking, yarns are considered as transversely isotropic material and the Maximum stress criterion (Zako *et al*., 2003; Tabeie and Ivanov, 2004), Hoffman criterion, Tsai-Hill criterion and Tsai-Wu tensor criterion (Bahei *et al*., 2004; Zeng *et al*., 2004) can be directly used. As to the isotopic matrix material, the Maximum principal stress criterion (Tabeie and Ivarov, 2004) or von Mises criterion (Bahei *et al*., 2004; Zeng *et al*., 2004) are adopted for predicting its failure. There are commonly two methods for post-failure stiffness reduction, the in directions and non-directions. The non-directions stiffness reduction assumes that the post-damage stiffness matrix is reduced to a near-zero value in all directions. This approach is often applied with the Hoffman criterion, Tsai-Hill criterion, Tsai-Wu tensor criterion and von Mises criterion which could not specified the failure direction. The in directions stiffness reduction only reduces the stiffness in the material's failure orientation and applied associated with the failure criterion which can specify the material's failure direction, such as the Maximum stress criterion. In the previous works done on woven composites, parts of them reduced the post-failure stiffness of yarns in directions (Zako *et al*., 2003; Tabeie and Ivarov, 2004), but none of them considered the stiffness reduction of matrix in this way.

There is still an obstacle on failure analysis of the woven composites that is the elastic and strength properties of the impregnated yarns. A lot of theoretical

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equations were built to predict the elastic constants of the unidirectional composites in the past decades such as the mixture rules and the Halpin-Tsai equations. But they all make use of the modification parameters which is determined by certain experiment to obtain the correct value. The modification parameters depend on geometrical and constitution which strictly limit the application of these equations. As to the strength properties, there is seldom equations can predict it correctly.

In this study, the multi-scale method was extended to the failure analyses of woven composites. Based on the properties of two basic constituents of the composite, fiber and matrix, the yarn properties, elastic constants and strengths, are first obtained through FE analysis on the micro-RUC model with certain failure criterion. This method can provide exact results without the costly experiments and can be used to any material and any fiber volume fraction. Next, the tensile properties of the composite are obtained through analysis of a meso-RUC model. The nonlinear response of the yarns is considered in this analysis. The stiffness reduction of matrix is considered in particular orientations in the micro- and meso-scale RUCs. The element disappear technique which is commonly used in previous researches is abandoned. The predicted results by the current method are compared with the experimental data of the plain woven composites and they are in good agreement.

Mutli-scale RUC: As explained in the section of introduction, two types of the RUC models developed.

Micro-RUC model of yarns: Yarns are considered as unidirectional fiber reinforced composite which is idealized as periodic array of fibers in the matrix with fiber volume fraction equal to the packing density. A great deal of mechanical models, such as rectangular cross section model, column model and hexagonal model, has been studied. Among them, the hexagonal model is widely quoted for modeling transversely isotropic material. Since the periodical boundary conditions cannot be used easily in the hexagonal model, a rectangular micro-RUC model containing the hexagonal model is adopted herein as shown in Fig. 1. Thus, the periodical boundary conditions can be easily applied to finite element analysis.

Meso-RUC of plain woven composite: The plain-woven composites, containing a single warp and weft layer, possess a periodic microstructure only in the plane of the woven layer. However, structural components that utilize plain-woven composites are fabricated as laminates from several plies as illustrated in (Fig. 2). The meso-geometry has a configuration with periodical in three orthogonal directions in this study. The rectangle area shown in (Fig. 2) is selected to be the meso-RUC by neglecting the effect of surface cells.

Fig. 1: Fiber distribution pattern: (a) fiber distribution; (b) micro-mechanical model of yarns

Fig. 2: Plane woven laminate structure

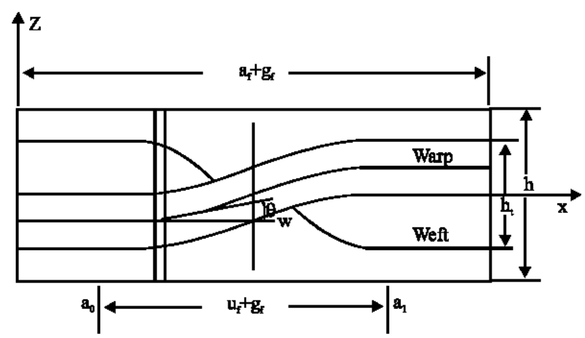

Fig. 3: The center path of warp yarn

The microstructure of the cell can be determined by textile parameters: a_w, the width of warp yarns; a_f, the width of weft yarns; g_w, the gap between two warp yarns; g_f, the gap between two weft yarns; u_w, the curve distances of warp yarns; u_f, the curve distances of weft yarns; h, the thickness of one lay; and h_t, the thickness of warp yarn and weft yarn in one layer. The center path of warp yarn shown in (Fig. 3) can be written as:

$$z = \begin{cases} -h_t/4 & (0 \le x \le a_0) \\ \sin((x - \frac{a_f + g_f}{2})\frac{\pi}{u_f + g_f})\frac{h_t}{4} & (a_0 \le x \le a_1) \\ h_t/4 & (a_1 \le x \le a_f + g_f) \end{cases} \quad (1)$$

Failure criteria and stiffness reduction: Two three-dimensional heterogeneous FE models at different scales, consist of fiber, matrix and fiber boundless, were employed. If the stress level satisfies the failure criterion of the material, the matrix, fiber, or tow would crack. Final failure corresponds to the rupture of fiber in the micro-RUC and to the break of the tow along the longitudinal direction in the meso-RUC. The criteria and stiffness reduction approach used for the three kinds of materials are summarized below.

Failure criteria and stiffness degradation of matrix: The matrix was treaded macroscopically an isotropic body in the micro-RUC and the meso-RUC. The Maximum principal stress theory is used for the damage prediction, which can be expressed as follows:

$$\sigma_I \ge X_m^t \quad (2a)$$

$$\sigma_{III} \le X_m^c \quad (2b)$$

$$\tau_{max} \ge S_m \quad (2c)$$

where X_m^t, X_m^c and S_m are the tensile, compressive and shear strengths of the material, respectively. σ_I and σ_{III} are the first principal stress and the third principal stress and τ_{max} is the maximum shear stress which can be obtained form:

$$\tau_{max} = \frac{1}{2}(\sigma_I - \sigma_{III}) \quad (3)$$

If the σ_1 exceeds the tensile strength of matrix material, a tensile crack would present in matrix. Except for the normal and shear stresses, all remaining stress components can be transferred across the crack. As is shown in Fig. 4a, σ_{11}, σ_{12} and σ_{13}, are degraded to near zero, subscript 1 denotes the Cartesian axis perpendicular to the plane of the crack while 2 and 3 are in the crack plane. The stiffness degradation of the matrix in the crack coordinate system is

$$[C]^{cr} = E_m \begin{bmatrix} d_m Z_1 & d_m Z_2 & d_m Z_2 & 0 & 0 & 0 \\ d_m Z_2 & Z_1 & Z_2 & 0 & 0 & 0 \\ d_m Z_2 & Z_2 & Z_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_m Z_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_m Z_3 \end{bmatrix} \quad (4)$$

If the maximum shear stress is great than shear strength of the matrix, the failure model is shown in (Fig. 4b) and only σ_{22} can be transferred. Here, axis 1, 2, 3 is the principal stress direction and the stiffness degradation of the matrix can be expressed as:

$$[C]^{cr} = E_m \begin{bmatrix} d_m Z_1 & d_m Z_2 & d_m Z_2 & 0 & 0 & 0 \\ d_m Z_2 & Z_1 & d_m Z_2 & 0 & 0 & 0 \\ d_m Z_2 & d_m Z_2 & d_m Z_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_m Z_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_m Z_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_m Z_3 \end{bmatrix} \quad (5)$$

In Eq. 4 and 5,

$$Z_1 = \frac{1 - \upsilon_m}{(1 + \upsilon_m)(1 - 2\upsilon_m)}, Z_2 = \frac{\upsilon_m}{(1 + \upsilon_m)(1 - 2\upsilon_m)},$$

$$Z_3 = \frac{1}{2(1 + \upsilon_m)}$$

and d_m is a small number representing the loss of the stiffness in the particular directions, taken to be 0.01 for the damaged state during numerical calculations.

The crack orientation in a RUC may vary at different locations. It is convenient to have the degraded stiffness matrix transferred to the global coordinate system with the transformation matrix, [T], between the local and global coordinate system (Warg *et al*., 2005). The post-damage stiffness matrix in the global coordinate system is

$$[C'] = [T]^T [C]^{cr} [T] \quad (6)$$

If Eq. 2b is satisfied, then the element presents the compressive failure. The non-directions stiffness reduction approach will be used. The stiffness is reduced to a near-zero value irrespective of the failure direction.

Failure criteria and stiffness degradation of fiber: There are many kinds of fibers that present different characteristic. For example, glass fiber and boron fiber are

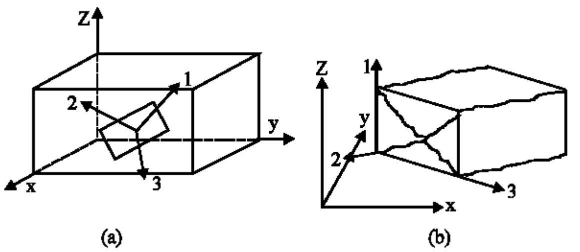

Fig. 4: Crack coordinate system: (a) tensile crack; (b) shear crack

isotropic materials, but carbon fiber and Kevlar fiber are transversal isotropic bodies. In this study, the Maximum principal stress criterion as expressed in Eq. (2) is applied to predict the failure of isotropic fibers. As to the transversal isotropic fibers, the Maximum stress criterion is adopted, which can be written as follows:

$$\sigma_i \geq X_{if}^t \quad (7a)$$

$$\sigma_i \leq X_{if}^c \quad (7b)$$

$$|\tau_{ij}| \geq S_{ijf} \quad (7c)$$

where X_{if}^t, X_{if}^c and τ_{ijf} are the tensile, compressive and shear strengths related to the three major material orientations.

In the micro-RUC of yarns, if one of Eq. (2) or Eq. 8 were satisfied, the fiber element would lose its load bearing capability completely. Therefore, the non-direction stiffness reduction is suitable for this condition.

Failure criteria and stiffness degradation of yarn: For yarns of transversal isotropic material, failure and stiffness reduction are orientation dependent. For example, the transverse failure can occur without breaking the longitudinal fibers and do not completely lose the stiffness of the material. Therefore, the Maximum stress criterion is used to predict the failure of yarn, as expressed in Eq. 8. The stiffness reduction scheme for post-failure yarns in (Zhang *et al.*, 2005) is adopted and the stiffness reduction factors are list in (Table 1). The failure in the axial direction of the yarn leads to the ultimate failure of the composite material.

To model the degradation of the yarn material, the stiffness matrix is written as follows (Tabeie and Ivarov, 2004):

$$[C_y] = [S_y]^{-1} = \begin{bmatrix} \frac{1}{E_1} & -\sqrt{\frac{\upsilon_{12}}{E_1}\frac{\upsilon_{21}}{d_2E_2}} & -\sqrt{\frac{\upsilon_{12}}{E_1}\frac{\upsilon_{21}}{d_3E_2}} & 0 & 0 & 0 \\ -\sqrt{\frac{\upsilon_{12}}{E_1}\frac{\upsilon_{21}}{d_2E_2}} & \frac{1}{d_2E_2} & -\frac{\upsilon_{23}}{\sqrt{d_2E_2d_3E_3}} & 0 & 0 & 0 \\ -\sqrt{\frac{\upsilon_{12}}{E_1}\frac{\upsilon_{21}}{d_3E_2}} & -\frac{\upsilon_{23}}{\sqrt{d_2E_2d_3E_3}} & \frac{1}{d_3E_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{d_4G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{d_5G_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{d_6G_{12}} \end{bmatrix}^{-1} \quad (8)$$

Table 1: Failure criteria and degradation scheme for yarn material

	Reduction coefficients				
Failure mode	d_2	d_3	d_4	d_5	d_6
Longitudinal tension	Ultimate failure				
Longitudinal compression	Ultimate failure				
Transverse tension, 2-direction	0.01	1.00	0.20	1.00	0.20
Transverse compression, 2-direction	0.01	1.00	0.20	1.00	0.20
Transverse tension, 3-direction	1.00	0.01	0.20	1.00	0.20
Transverse compression, 3-direction	1.00	0.01	0.20	1.00	0.20
Longitudinal shear, 12-plane	0.01	1.00	0.01	1.00	1.00
Transverse shear, 23-plane	0.01	0.01	0.01	0.01	0.01
Longitudinal shear, 31-plane	1.00	0.01	1.00	1.00	0.01

where d_i, I = 2, 3, …,6, are the reduction factors for yarn material, initially all of them equal unity.

The degraded stiffness matrix of yarn material is defined in the material principal coordinate system. It should be transferred to the global coordinate system with the transformation matrix, [T].

RESRLTS AND DISCUSSION

Predicted material properties for yarn: The literature (Warg *et al*., 2005) studied the elastic properties of yarn with same micro-RUC model based on periodical boundary conditions. The predicted results kiss well with data computed by equations with a suitable parameter determined by experiment. Thus the correctness of the model and the method built to predict the constants of yarns are verified. As to strength properties, Zhang *et al*., 2005) predicted the strength of E-glass/epoxy unidirectional laminate under off-axis loading by the smeared crack method and the computed results are in good agreement with the test data performed on a similar composite system. But it was not considered all the strengths of the unidirectional composite integrated.

The impregnated yarn constituted of E-glass and epoxy with the fiber volume fraction of 70% (Tabeie and Ivarov, 2005) has been studied and the material properties are list in (Table 2). Finite element model of micro-RUC for yarn is shown in (Fig. 5) with a total number of 16762 elements and 8612 nodes. Eight-node brick elements and six-node wedge elements are adopted. The mesh in the opposite boundary surfaces of the micro-RUC is exactly the same for applying the periodic boundary conditions. The dimension of the RUC is 5×346.4×200 and only one layer of element is required in the thickness direction (x-direction, Fig. 5).

There are six strengths of the unidirectional composite identified as: longitudinal tension strength, longitudinal compression strength, transverse tension strength, transverse compression strength, longitudinal shear strength and transverse shear strength. For each case, the appropriate periodic boundary condition is applied.

The described micro-mechanical material model of woven composite materials is programmed as a UMAT in ABAQUS commercial finite element code. The boundary conditions are applied using the equations. Under incrementally applied loadings, stress analyses were first carried out and used to identify the damage location by appropriate failure criterion. Once damage occurs, post-failure stiffness reduction is then applied for further stress analyses. The analyses keep going on until the final failure of the entire micro-RUC.

Fig. 5: Finite element model of yarns

Figure 6 shows the stress-strain curves obtained using the finite element simulation with the post failure response. It is seen that, before reaching the failure stress, all curves are elastic, whereas upon exceeding the failure stress, the curves of longitudinal compression, transverse tension and transverse compression drop dramatically. But, the stress-strain curve of longitudinal tensile has a transition when the strain reaches 0.032 because of the failure of the matrix and shows the nonlinear response, to be taken into accounted in the analyses of the meso-RUC model. The curves of longitudinal shear and transverse shear undulate obviously after the initial damage occurs. But the maximum stresses are slight higher than the stress where the initial damage presented. Therefore, the strengths of longitudinal shear and transverse shear are taken to be the stresses when initial damage occurred. The elastic and strength properties of the yarns are listed in (Table 3) except the longitudinal module. The elastic property of longitudinal can be expressed as:

$$E_1 = \begin{cases} 52.155\text{GPa} & (\varepsilon_1 \le 0.032) \\ 51.136\text{GPa} & (\varepsilon_1 > 0.032) \end{cases} \quad (9)$$

The relationship between stress and strain of longitudinal tension determined by Eq. 9 is also shown in (Fig. 6a). It can be seen that the simulation curve fit well with the FE result and that the simple form can be easily implemented in the meso-scale analysis of plain-woven model.

Table 2: Mechanical properties of fiber and matrix (15)

Mechanical properties	Young's modulus (GPa)	Shear modulus (GPa)	Poisson's ratio	Tensile strength (MPa)	Compressive strength (MPa)	Shear strength (MPa)
E-glass	73	30	0.22	2760	2208	-
Epoxy resin	3.5	1.30	0.35	112	241	89.6

Table 3: Mechanical properties of yarns

Mechanical properties	E_2 (GPa)	G_{12} (GPa)	G_{23}(GPa)	ν_{12}	X_t(Mpa)	X_c (MPa)	Y_t (MPa)	Y_c (MPa)	S_1 (MPa)	S_t(Mpa)
	16.792	6.170	5.782	0.245	1935.1	1431.1	109.15	171.73	69.10	62.78

Fig. 6: The stress-strain curves of yarns: (a) longitudinal tension and compression; (b) transverse tension and compression; (c) longitudinal and transverse shear

Predicted material properties for plain woven composite: The four-node linear tetragonal elements were used to model the yarn and matrix for all cases and the number of total nodes and elements of model were 30000 and 280000, respectively (shown in Fig. 7). The volume fraction of fibers in the composite material is 47.19%. Since the yarns fiber volume fraction is 70%, the volume fraction of the impregnated yarn material in the meso-RUC model is 67.41%. Material properties of tows vary along the orientation of the path curve; a user subroutine UEXTERNALDB is developed to capture the orientations change tow elements. The periodical boundary conditions are again adopted in this analysis.

Figure 8 shows the computed result for deformation and distribution of normal stress in loading direction before the initial damage occurs. It is observed that the warp yarns are straightened due to the tensile loading and the weft yarns are waved more deeply. Figure 9 shows the damage development and the state of the inner fiber bundles. The initial damage is the transverse cracks along the edge of weft yarns and appears when the loading strain reaches 0.7% and progresses from the edge to the center gradually along weft yarns with the increase of the tensile strain. With further gradual increases of loading, these cracks extend to the matrix regions and matrix cracks spread along the transverse direction. When the strain reaches about 1.8%, fiber damages appear thus the composite is broken.

To verify the predicted results, test samples are fabricated. The fabric is constituted of 24Tex E-glass thread with density of 20 ends per centimeters. The area density of the fabric is 100 g m^{-2}. The matrix is epoxy resin. There are total 34 layers to build the 3 mm thickness laminate. The fiber volume fraction of the samples is 46.7%. The tensile experiments were performed on the WDW-E2000 testing machine. Predicted and experimental stress-strain curves are shown in (Fig. 10). Good correlation is observed between the predicted and experimental data. The possible reasons to cause the slight difference are perhaps due to neglecting the distinguishing between the surface layer and interior layers and the layers in the specimen is not strictly periodic in the thickness direction.

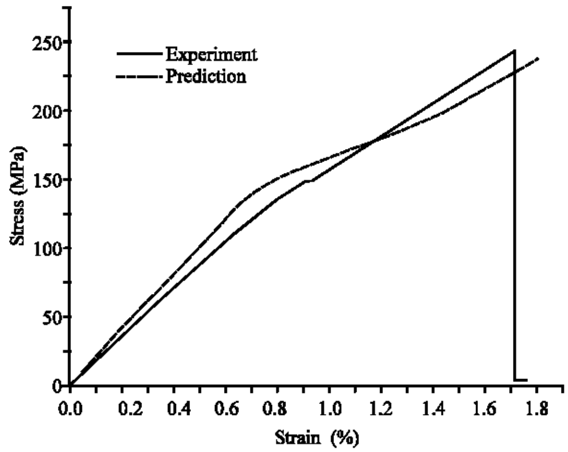

Fig. 10: The normal stress-strain curves of experiment and prediction

CONCLUSION

The multi-scale technique is introduced in the failure analysis of plain-woven composites. The micro-RUC of yarns was researched with three-dimensional finite element method. The yarns properties, elastic constants and strengths, were obtained and then used in the analysis of meso-RUC model of plain-woven composites. The stiffness reduction of both matrix and yarns are considered in particular orientations. The tensile experiments were performed and the predicted stress-strain failure history agrees well with experimental results.

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REFERENCES

Blackketter, D.M., D.E. Walrath and A.C. Hansen, 1993. Modeling damage in a plain weave fabric-reinforced composite material. J. Compos. Tech. Res, 15: 136-142.

Bahei-El-Din, Y.A., A.M. Rajendran and M.A. Zikry, 2004. A micromechanical model for damage progression in woven composite systems. Intl. J. Solids and Struct., 41: 2307-2330.

Barbero, E.J., J. Trovillion, J.A. Mayugo and K.K. Sikkil, 2006. Finite element modeling of plain weave fabrics from photomicrograph measurements. Composite Structures, 73: 41-52.

Chen, S., 1990. Mechanical of composite materials, The Aeronautical Industry Publishing, Beijng, pp: 50-62.

Ishikawa, T. and T.W. Chou, 1982. Elastic behavior of woven hybrid composites. J. Composite Materials, 16: 2-19.

Ivanov, I. and A. Tabiei, 2001. Three-dimensional computational micro-mechanical model for woven fabric composites. Composite Structures, 54: 489-496.

Karkkainen, R.L. and B.V. Sankar, 2006. A direct micromechanics method for analysis of failure initiation of plain weave textile composites. Composites Sci. Tech., 66: 137-150.

Naik, N.K. and P.S. Shembekar, 1992. Elastic behavior of woven fabric composites: I-lamina analysis. J. Composite Material, 26: 2196-2225.

Swan, C.C. and H. Kim, 2002. Multi-Scale Unit Cell Analyses of Textile Composites. 15th ASCE Engineering Mechanics Conference, New York, pp: 1-8.

Tan, P., L. Tong and G.P. Steven, 1998. A flexible 3D FEA modeling approach for predicting the mechanical properties of plain weave unit cell. Proc. ICCM, 11: 67-76.

Tabeie, A. and I. Ivanov, 2004. Materially and geometrically non-linear woven composite micro-mechanical model with failure for finite element simulations. Intl. J. Non-linear Mechan., 39: 175-188.

Wang, X.F., G.M. Zhou, C.W. Zhou and X.W. Wang, 2005. Multi scale analyses of woven composite based on periodical boundary conditions. J. Nanjing University of Aeronautics and Astronautics, (In Chinese), 37: 370-375.

Zako, M., Y. Uetsuji and T. Kurashiki, 2003. Finite element anayysis of damaged woven fabric composite materials. Composites Sci. Tech., 63: 507-516.

Zeng, T., D.N. Fang, M. Li and L.C. Guo, 2004. Predicting the nonlinear response and failure of 3D braided composites. Materials Lett., 58: 3237-3241.

Zhang, Y., Z. Xia and F. Ellyin, 2005. Nonlinear viscoelastic micromechanical analysis of fiber-reinforced polymer laminates with damage evolution. Intl. J. Solids Struct., 42: 591-604.