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Practical Statistical Methods for Predicting Life and Reliability of Fine Pitch Gear Pairs

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Abstract: Practical experience with the modern gear analysis methods (e.g., ISO /6336, BS 436, DIN/3990) suggests that they may be too pessimistic for fine-pitch gears in which the effects of gear inaccuracies are considerable. These inaccuracies may contribute significantly to the variability's and uncertainties of the applied and permissible stresses. In this research all the individual manufacturing errors are treated as random variables. The applied torque and other factors which in practice may also be uncertain are taken here as constants. Two probabilistic methods are developed in order to establish a statistical analysis for the applied and permissible stresses used in the ISO/6336 procedure. These give a more realistic estimate of the actual stresses and strengths for given (or assumed) random distributions errors of the individual manufacturing errors. The two statistical methods developed in this work have shown substantial agreement in predicting the Life and the reliability of fine pitch gear pairs.

Key words: Gear, fatigue, stress, uncertainties, random, statistics, life, reliability

INTRODUCTION

To be considered as satisfactory, gear drives must operate as predicted at the design stage. Poor gear performance generally results in gear failures which may be originally due to:

- Pitting fatigue caused by contact stresses exceeding the strength of the material.
- Tooth breakage caused by bending stresses exceeding the bending fatigue strength of the material.
- Other surface failures associated with poor lubrication conditions such as scuffing, scoring and wear.

The methods currently used in practice for predicting the stresses present during the operation of gears are usually those given by one of the common gear rating standards such as ISO/6336 (1996) , BS 436 (1986) , DIN/ 3990 (1986) and AGMA 2101 (1995). All these above standards adopt a fundamentally similar approach and so ISO/6336 will be taken as an example.

Stresses in real gears: The Hertzian and root bending stresses applied to a real gear can be written as follows:

$$\sigma_H = \underbrace{[Z_E]}_{\text{Material factor}}.\underbrace{[F_t.K_A]}_{\text{Applied load factor}}.\underbrace{\left[\sqrt{\frac{1}{d_1.b}}\right]}_{\text{Nominal gear size factor}}.\underbrace{\left[(Z_H.Z_\varepsilon.Z_\beta).\sqrt{\frac{u+1}{u}}\right]}_{\text{Pitting grometry factors}}.\underbrace{\sqrt{[K_V.K_{H\beta}.K_{H\alpha}]}}_{\text{Gear accuracy factors}} \quad (1)$$

$$\sigma_F = \underbrace{[F_t.K_A]}_{\text{Applied load factor}}.\underbrace{\left[\frac{1}{b.m_n}\right]}_{\text{Nominal gear size factor}}.\underbrace{[Y_F.Y_S.Y_\beta]}_{\text{Bending geometry factors}}.\underbrace{[K_V.K_{F\beta}.K_{F\alpha}]}_{\text{Gear accuracy factors}} \quad (2)$$

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It is difficult to identify accurately all factors that contribute to increase the uncertainty and the variability in the applied contact and bending stresses. However providing the applied load is known and constant (which is the case in this research) and since the uncertainties of the geometric factors (with the exception of (Y_S)) are neglected (Pennell, 1987), the variability and uncertainty in σ_H and σ_F can then only arisedue to the gear accuracy factors (K_V, $K_{H\beta}$, $K_{H\alpha}$, $K_{F\beta}$ and $K_{F\beta}$). In the standards (ISO/6336, 1996; BS 437, 1986; DIN 3990, 1986; AGMA 2101, 1995) mentioned above, the fundamental accuracy of each gear is defined by the accuracy of profile, lead and pitch of the teeth relative to some defined axis and the mounting accuracy of the gear relative to its (working) axis of rotation. Error tolerances are then given by the accuracy grades defined in the gear accuracy standards (ISO 1328, 1995; BS 436, 1972; DIN 3962, 1978). However, the combined lead error of the teeth is usually the most critical accuracy factor in determining the load capacity of gear pair especially for fine pitch gears. Since, for similar gears of the same material (i.e., the same stress level), the ratio of tooth misalignment to the elastic deflection ($f_{\beta y}/\delta_e$ used for $K_{H\beta}$ calculation) (ISO 6336, 1996), is higher for fine pitch gears. This makes their face load distribution inherently worst and consequently explains why careful analysis of manufacturing errors is particularly necessary for fine pitch gearing.

In general, geometric errors those affect gear rating resolve themselves into three types; errors affecting the dynamic factor, those affecting face load distribution factors and those affecting load sharing between adjacent tooth pairs. In addition to these geometric errors, variations in material quality (composition, cleanliness hardness, etc.) and in surface finish also affect the fatigue strength and hence the gear rating.

We are thus concerned with the general problem of determining the statistics of functions (σ_H, σ_F) of n random variables (K_V, $K_{H\beta}$, $K_{H\alpha}$, $K_{F\beta}$ and $K_{F\beta}$), whose statistics can be calculated in their turns. This problem has been solved both analytically and numerically as set out below.

Statistical analysis by using the analytical method: To devise a rigorous analytical procedure for calculating the mean and standard deviation of general functions of n random variables is practically not possible. It is, in fact, very difficult to find the probability density function, even for a general function of more than two random variables. However, for reliability problems (where the random variations of each variable can be assumed to be "small») the Taylor series approximation (Zhang, 2003; Michalec, 1966; Haugen, 1980) can be used as an adequate method for combining the random variables.

The general expression for a function of several independent variables $X_1, X_2, \ldots, X_n$ can be written as:

$$Y = (X_1, X_2, \ldots\ldots\ldots, X_n) \quad (3)$$

Expanding this by means of a Taylor series in the region "near" the mean values, of the n variables, we obtain:

$$Y = f(\bar{X}_1, \bar{X}_2, \ldots, \bar{X}_n) + \sum_{i=1}^{n} \frac{\partial Y}{\partial X}\Big|_{x=\bar{x}_i} .(X_i - \bar{X}_i) + \frac{1}{2!}\sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial Y}{\partial X_i . \partial X_j}\Big|_{x_i=\bar{x}_i} .(X_i - \bar{X}_i).(X_j - \bar{X}_j) + \ldots \quad (4)$$

In most engineering error problems, values of the higher order terms in the expansion are considered to be small compared with the mean values and first order terms, are therefore neglected. Using standard procedures, (Kapur and Lambertsonb, 1977). We then obtain the mean as:

$$\bar{Y} = f(\bar{X}_1, \bar{X}_2, \ldots\ldots \bar{X}_n) \quad (5)$$

while the (small) random variation of Y is given by:

$$\Delta Y = \sum_{i=1}^{n} \frac{\partial Y}{\partial X_i} . \Delta X_i \quad (6)$$

where $\Delta\bar{X}_i = (X_i - \bar{X}_i)$, is the variation of X_i from its mean. Thus ΔY is a linear function of the ΔX_i so, using standard theorems (DIN 3962M, 1978; Zhang, 2003; Michalec, 1966), the resultant standard deviation of Y is given by:

$$\sigma_Y = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial Y}{\partial X_i}\Big|_{x_i=\bar{x}_i} . \sigma_{X_i}\right)^2} \quad (7)$$

It would thus appear that all that is needed is to compute K_V, $K_{H\beta}$, $K_{H\alpha}$, etc., using the ISO calculation procedure (ISO 6336, 1996). These values could then be used, in turn, to compute (using Eq. 1,2) the statistics of σ_H, σ_{HP} etc.

A direct computation of $\sigma(\sigma_H)$, $\sigma(\sigma_{HP})$, $\sigma(\sigma_F)$, and $\sigma(\sigma_{HP})$ from the above equation is not, however, possible, since the factors K_V, $K_{H\beta}$ and $K_{H\alpha}$, ($K_{F\beta}$, $K_{F\alpha}$) are NOT independent variables (since $K_{H\alpha}$ depends on K_V and $K_{H\beta}$, in turn, depends on. K_V.

Nevertheless, since Sandler (1984) has shown that the variability of these factors is, in fact, small (Table 1), it seems worthwhile pursuing this approximate method.

Table 1: Estimated Uncertainties in ISO and AGMA Gear Rating Factors from (13)

±Δx	±1%		±2%	±5%	±7.5%		±10%	±15%		±20%		±40%
Method		ISO	AGMA	ISO	ISO	AGMA	ISO	AGMA	ISO	AGMA	BOX	BOX
X_i	Z_H	I	Z_N	$K_{H\alpha}$	C_m	$\delta_{H\lim}$	S_{ac}	K_A	C_{SF}	n	x	
	Z_E		Z_V	$K_{H\beta}$	C_L	K_V	C_V	S				
				Z_L			C_P					
				Z_R								
				Z_W								

One possibility is to ignore the dependence of the factors K_V, $K_{H\beta}$, $K_{h\alpha}$ 1996. This leads to a simple practical method for computing the mean and standard deviation of each factor affected by manufacturing tolerances and thus, the statistics of the resultant stresses and strengths in the calculation procedure (ISO/6336, 1996). This method is presented in the following section, in which K_V, $K_{H\beta}$ and $K_{H\alpha}$, are considered totally independent. On the other hand, an exact derivation of the standard deviations is also possible, although more tedious. The author had done this, where the interaction between K_V, $K_{H\beta}$, $K_{H\alpha}$ is fully taken into account.

Mean and standard deviation have σ_h, σ_f: Assuming that are K_V, $K_{h\beta}$ and $K_{h\alpha}$ independent, the mean and standard deviation of σ_H, σ_F can then be derived from Eq. 5 and 7 as:

$$\bar{\sigma}_H = Z_H.Z_E.Z_\varepsilon.Z_\beta.\sqrt{K_A.\bar{K}_V.\bar{K}_{H\alpha}.\bar{K}_{H\beta}.\frac{F_t}{b.d_1}.\frac{u+1}{u}} \quad (8)$$

$$\bar{\sigma}_F = (F_t/\ b.m_n).Y_F.Y_S.Y_\beta.K_A.\bar{K}_V.\bar{K}_{F\beta}.\bar{K}_{F\alpha} \quad (9)$$

And neglecting variation of all but K_V, $K_{H\beta}$, $K_{H\alpha}$, etc. and carrying out the differentiation, we obtain for the standard deviations:

$$\sigma(\sigma_H) = \frac{1}{2}.\bar{\sigma}_H.\sqrt{\left(\frac{\sigma\ (K_V)}{\bar{K}_V}\right)^2 + \left(\frac{\sigma(K_{H\beta})}{\bar{K}_{H\beta}}\right)^2 + \left(\frac{\sigma(K_{H\alpha})}{\bar{K}_{H\alpha}}\right)^2} \quad (10)$$

$$\sigma(\sigma_F) = \bar{\sigma}_F.\sqrt{\left(\frac{\sigma\ (K_V)}{\bar{K}_V}\right)^2 + \left(\frac{\sigma(K_{F\beta})}{\bar{K}_{F\beta}}\right)^2 + \left(\frac{\sigma(K_{F\alpha})}{\bar{K}_{F\alpha}}\right)^2} \quad (11)$$

Both of these can then be calculated using ISO formulae for K_V, $K_{H\beta}$, $K_{H\alpha}$, etc., (1996).

Mean and standard deviation of σ_{hp}, σ_{fp}: Although the variability in the material strength arises from changes in many parameters (ISO/6336, 1996) only the material hardness and the roughness factors (Z_R, Z_{RrelT}) are considered here as manufacturing random variables (since, in the experimental work, it is not possible to measure any other material parameters). However, according to the ISO calculation procedure (ISO/6336, 1996) the parameters Z_V and Z_L also depend on the hardness, but only for through-hardened steels.

The mean and the standard deviations for σ_{HP}, σ_{FP} may then be obtained from (ISO/6336, 1996) as:

$$\bar{\sigma}_{HP} = \frac{\bar{\sigma}_{H\lim}}{S_{H\min}}.Z_N.Z_L.Z_V.\bar{Z}_R.Z_W.Z_X \quad (12)$$

$$\bar{\sigma}_{FP} = \frac{\bar{\sigma}_{F\lim}}{S_{F\min}}.Y_{ST}.Y_{NT}.\bar{Y}_{RrelT}.Y_{\delta relT}.Y_X \quad (13)$$

Finally equations for standard deviations may be written as:

$$\sigma(\sigma_{HP}) = \bar{\sigma}_{HP}.\sqrt{\left(\frac{\sigma(\sigma_{H\lim})}{\bar{\sigma}_{H\lim}}\right)^2 + (0.08)^2.\left(\frac{\sigma\ (R_Z)}{\bar{R}_Z}\right)^2} \quad (14)$$

$$\sigma(\sigma_{FP}) = \bar{\sigma}_{FP}.\sqrt{\left(\frac{\sigma(\sigma_{F\lim})}{\bar{\sigma}_{F\lim}}\right)^2 + \left(\frac{1}{100}\right)^2.\left(\frac{\sigma(R_Z)}{\bar{R}_Z+1}\right)^2 .\left(\frac{\bar{Y}_{ReIT}-5.306}{\bar{Y}_{ReIT}}\right)^2} \quad (15)$$

Statistical analysis by using monte carlo method: The statistics (mean and standard deviation) of the parameters K_V, $K_{H\beta}$, $K_{H\alpha}$, σ_H, σ_F, etc., can also be estimated by using Monte Carlo simulation (Haung, 1980l Al-Shareedan, 1987). The simulation process involves generating a succession of random numbers with uniform distribution over the interval (0.0-1.0). These sets of random values can be then used to generate synthetic values for each manufacturing error (in the prescribed tolerance range). For example, to obtain random values within the tolerance bands of any manufacturing error having a uniform distribution over 0.0 to 1.0 with a mean equal to 0.5, the following equation may be used.

$$e_i = \frac{e_{max} + e_{min}}{2} + (R_i - 0.5) \quad (16)$$
$$(e_{max} - e_{min}) = \bar{e} + (R_i - 0.5).T_e$$

Where e_i is the required random value of the error (e), e_{max} and e_{min} are the maximum and minimum (tolerance) limits of the specific manufacturing error (e), ē is the mean value of the error and R_i is the random number from a standardised uniform rectangular distribution described by: $\bar{R} = 0.5$ and $\sigma(R) = 1/\ (2.\sqrt{3})$

For a normal distribution of , the equation (18) becomes:

$$e_i = \sigma(e).Z_i + \bar{e} \quad (17)$$

Where ē and σ(e) are the required mean and standard deviation of the normal error (e) and Z_i is a random value from a standardised normal distribution described by statistics $\bar{Z}$ = 0 and σ(Z) = 1. Generating a succession of many values for the error (e_i from Eq. 18 or 19 will simulate exactly what happens during actual production. The other manufacturing errors e (i = 1,........n) (may be generated in the same or different distributions. The statistics (mean and standard deviation) of a parameter such as σ_H Eq. 1, which depends on the e_i can then be determined by calculating a sufficient number of σ_H, using successive sets of the random values e_i.

A computer program has been written to calculate K_V, $K_{H\beta}$, $K_{H\alpha}$, σ_H, σ_{HP}, etc,. (and the corresponding "bending strength" parameters) from randomly distributed values of the various gear errors generated in this way. The statistics of the performance parameters were computed from 8000 sets of random input data. The results are as shown in Fig. 1 to 4.

Both the Monte Carlo and the analytical methods have been used to estimate the statistics of the various parameters (Table 2). The manufacturing errors that are considered random variables are assumed to have rectangular distributions and their tolerances are taken according to ISO/6336 (ISO 1328, 1995).

To verify the proposed statistical methods, Table 3 shows the characteristics of the gear pair taken as an example. Both gears were manufactured from a case hardening steel having as MQ material quality (ISO/6336, 1996; DIN 3996, 1986).

The numerical results from both methods are shown in Table 3 . Figure 1 to 4 shows the histograms of the 8000 calculated values for each parameter under consideration. As can be seen from Table 3, the results of the exact» analytical method are in good agreement with those calculated by the Monte Carlo method (for which the interaction between K_V, $K_{H\beta}$ and $K_{H\alpha}$ is automatically taken into account). The maximum error of the exact analytical method in estimating σ (σ_H) and σ (σ_F) is of order 10%.

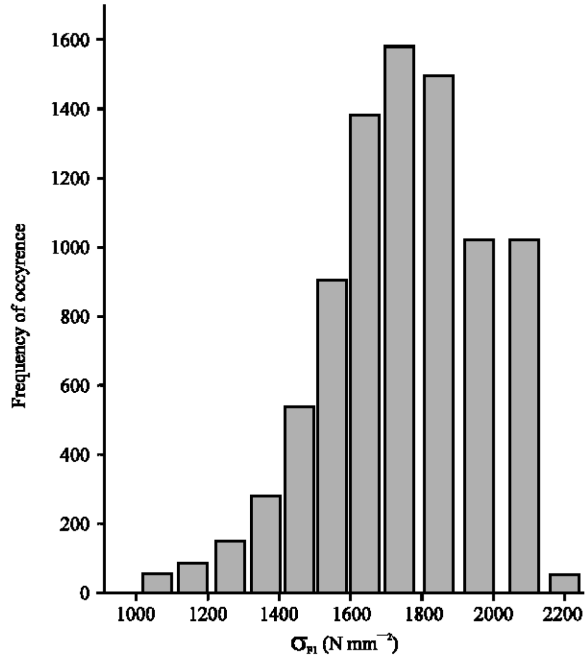

Fig. 1: Distribution of σ_H (Mean=1775.44 N mm^{-2}, STD=189.48 N mm^{-2}, T_1=16.33 N.m)

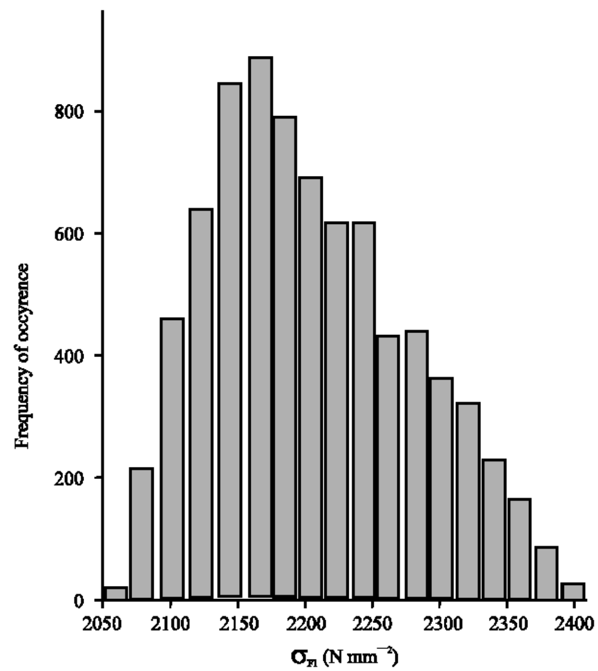

Fig. 2: Distribution of σ_{HP1} (Mean = 2203.73 N mm^{-2}, STD = 74.31 N mm^{-2})

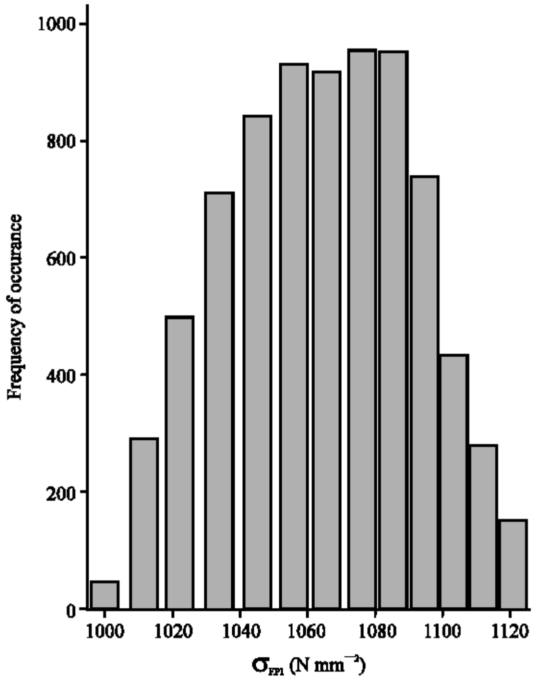

Fig. 3: Distribution of σ_{F1} (Mean=1245.30 N mm^{-2}, STD=226.61 N mm^{-2}, T_1 =16.23N.m)

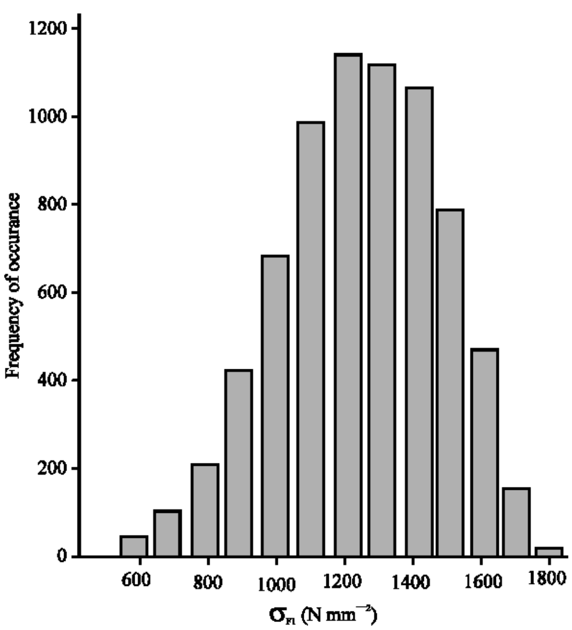

Fig. 4 : Distribution of σ_{FP1} (Mean = 1061.34 N mm^{-2}, STD = 27.27 N mm^{-2})

However, $\sigma K_{H\beta}$ ($\sigma K_{F\alpha}$) has only been estimated with an error of order 24% (because the variation of the individual errors was not small. Clearly, however, this error has little effect on the statistics of the parameters of interest σ_H, σ_F due to the fact that $\sigma(\sigma_H)$ and $\sigma(\sigma_F)$ are mainly defined by $\sigma K_{H\beta}$ and $\sigma K_{F\beta}$ ($\sigma K_{H\alpha}$ and $\sigma K_{F\alpha}$, are always small).

Table 2: Specifications of test gears

Test gears specifications	Pinion/Wheel
Normal module (mm)	0.847
ISO quality	8 to 10
Number of teeth	35/75
Face width (mm)	12.25/12.25
Pitch diameter (mm)	30.876/66.163
Normal pressure angle (α_n)	20.0000°
Helix angle (β)	16.2333°
Applied torque (Name)	16.3/34.9
Application factor	1.000
Endurance limite $\sigma_{H\,limit}$ (N mm^{-2})	1444.0/1444.0
Endurance limit $\sigma_{F\,limit}$ (N mm^{-2})	401.5/401.5

Table 3: Results of the statistical analysis for the gears of the experimental investigation (T_1=16.23 N.m and N_L = 3×10^4)

ISO Prameter name (Mean, STD)	A	B	C	D
f_{ma}	- 4.508	-4.862	-4.508	-4.862
σf_{ma}	32.34	32.197	32.335	32.197
$f_\beta Y$	- 32.414	-32.80	-32.414	-32.80
$\sigma f_{\beta Y}$	13.605	13.684	13.605	13.684
K_V	1.085	1.086	1.085	1.086
σK_V	0.023	0.024	0.023	0.024
$K_{H\beta}$	4.263	4.383	4.247	4.383
$\sigma K_{H\beta}$	0.99	0.914	0.985	0.916
$K_{F\beta}$	3.405	3.499	3.394	3.499
$\sigma K_{F\beta}$	0.683	0.619	0.680	0.620
$K_{H\alpha}$	1.362	1.346	1.388	1.346
$\sigma K_{h\alpha}$	0.173	0.168	0.238	0.188
σ_H	1771.17	1801.85	1775.44	1801.85
$\sigma(\sigma_H)$	257.81	219.72	189.480	172.37
σ_{F1}	1246.98	1263.50	1245.30	1263.50
$\sigma(\sigma_{F1})$	313.72	273.62	226.61	215.68
σ_{F2}	1281.91	1298.89	1280.18	1298.89
$\sigma(\sigma_{F2})$	322.51	281.29	232.95	221.72
σ_{HP1}	2203.73	2190.09	2203.73	2190.09
$\sigma(\sigma_{HP1})$	74.31	67.27	74.31	67.29
σ_{HP2}	2211.67	2201.93	2211.67	2201.93
$\sigma(\sigma_{HP2})$	64.88	60.86	64.88	60.86
σFP1	1060.34	1059.15	1061.34	1059.15
$\sigma(\sigma_{FP1})$	27.27	28.22	27.27	28.22
σ_{FP2}	1153.18	1151.64	1153.18	1151.64
$\sigma(\sigma_{FP2})$	28.47	29.59	28.47	29.59

A = Monte Carlo Approximation; B = Taylor Approximation (withoutinteraction) .(withoutinteraction). C = Monte Carlo Approximation;D = Taylor Approximation (with interaction) . (with interaction)

On the other hand, the simplified analytical method described above shows a maximum error of order 32%. Approximately the same values are however, also found by the Monte Carlo method if the interaction between K_V, $K_{H\beta}$ and $K_{H\alpha}$ is removed (columns A, B, Table 3). The additional error caused by simplified analytical method is thus clearly a consequence of neglecting the interactions, not a shortcoming of the analytical procedure it self. This simplified method has, however, been shown to be quite accurate (less than 15% error) for gears of higher accuracy grades (ISO 7 and better), when the assumption of "small" variability can be more easily justified.

Prediction of life using analytical and numerical methods: Since these methods deal with distributions, it is worthwhile to introduce the gear reliability as a basis for comparing the results.

As explained previously, it is practically impossible to find the probability density function of σ_H, σ_{HP}. The only alternative is to estimate them by fitting the σ_H, σ_{HP} frequency distributions obtained from the Monte Carlo analysis to standard distributions whose density functions are known. For the gears under consideration, these frequencies distributions are as shown in Fig. 1 to 4. If we assume them to be individually normally distributed as shown in Fig. 5 Then the probability that the strength σ_{HP} (σ_{FP}) is greater than the stress σ_H (σ_F) for all possible values over the range of σ_H (σ_F) is given by:

$$R_H = \frac{1}{\sqrt{2\pi}} \int_Z^\infty e^{-Z_H^2/2} = 1 - \phi(Z) \qquad (18)$$

Thus the unreliability (the probability of failure) is defined as:

$$P_H = \phi(Z) \qquad (19)$$

Where,
$\Phi(Z)$ is the standard cumulative normal distribution function.
Z_H is the standard normal variate given by:

$$Z_H = \frac{\overline{\sigma_{HP}} - \overline{\sigma_H}}{\sqrt{(\sigma(\sigma_H))^2 + (\sigma(\sigma_{HP})^2}} \qquad (20)$$

Similar equations can be obtained for R_F and P_F.

The reliability clearly depends on the lower limits of the integral and hence the statistics of σ_H, σ_{HP}.

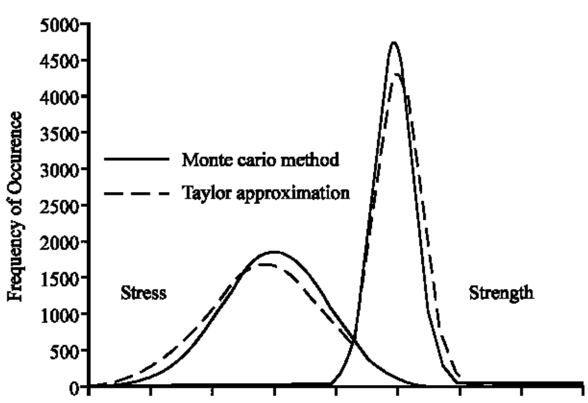

Fig. 5: Contact stress strength interference (T_1= 16.23 N.m at $N_L = 3\times10^4$)

A higher value of reliability can therefore be obtained by:

- Increasing the mean strength $\overline{\sigma_{HP}}$ or reducing the mean stress $\overline{\sigma_H}$.
- Reducing the spread of the two distributions $\sigma(\sigma_{HP})$, $\sigma(\sigma_H)$.

In order to compare the analytical with Monte Carlo analysis, Fig. 5 Shows contact stress-strength interference density functions obtained from the analytical and the Monte Carlo methods for gears at lives of 3×10^4. As can be seen both methods are in good agreement.

Figure 6 shows the reliability that can be expected for different torque ratings for a life of 5×10^5 cycles. As can be seen the gears should be rated to only about 0.78 KW for 99% reliability.

An interesting feature witch may be obtained from these theoretical is the fact that, once the statistics f stress and strength for a specific gear assembly have been calculated at a fixed life, the results can then be adjusted to give the reliability at any number load cycles in the finite life region. All that changes as the life is varied are the mean strength $\overline{\sigma_{HP}}$ witch varies in accordance with the life factor Z_N, so that:

$$\left(\overline{\sigma_{HP}}\right)_0 = \left(\frac{Z_{N0}}{Z_N}\right) \cdot \overline{\sigma_{HP}} \qquad (21)$$

If the coefficient of variation $(\sigma(\sigma_{HP})/(\overline{\sigma_{HP}})$ is assumed to be independent of life (giving parallel (S.N.P)

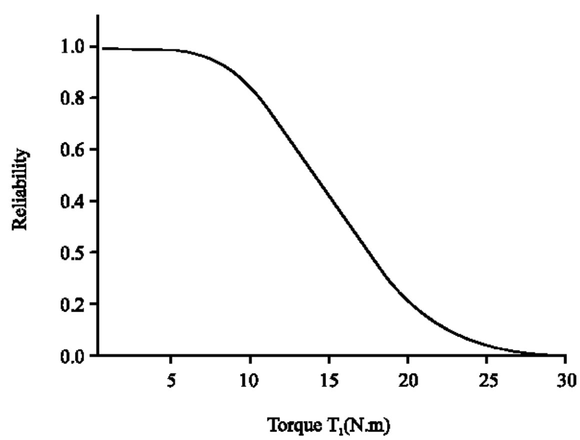

Fig. 6: Reliability vs pinion torque of the gears at $N_L = 5\times10^5$

curve on a log-log plot as shown in Shareedah and Alawi (1987). Then all we need to do is recalculate the standard variate as:

$$Z_H = \frac{\overline{\sigma_{HP}} \cdot \left(Z_N / Z_{N0} \right) - \overline{\sigma_H}}{\sqrt{\left(\sigma(\sigma_{HP})\right)^2 \cdot \left(Z_N / Z_{N0} \right)^2 + \left(\sigma(\sigma_H)\right)^2}} \quad (22)$$

Where Z_N is the new life factor and Z_{N0} is the life factor for witch the statistics of σ_{HP} were first calculated.

Another important quantity that may be determined, once the statistics of stress and strength are known, is the safety factor. For the special case where σ_H (σ_F) and σ_{HP} (σ_{FP}) are normally distributed, the safety factors S_H (S_F) may be extracted from Eq.22 as:

$$S_H = 1 + Z_H \frac{\sqrt{\left(\sigma(\sigma_H)\right)^2 + \left(\sigma(\sigma_{HP})\right)^2}}{\overline{\sigma_H}} \quad (23)$$

It is clear that the safety factor defined above is directly related t reliability (via Z_H) as well as t the quality control of both the material and gear geometric accuracy.

CONCLUSION

The main objective of this work was to develop simple practical methods for predicting the effect of random manufacturing tolerances and material variation on the performance of fine pitch gears. Two theoretical methods were developed in order to predict gear life and reliability.

The following general conclusions were obtained:

For similar gears of the same material and the same stress level, fine pitch gears have higher face load distribution factors than coarse gears (where the elastic deflection is small compared to manufacturing error). For equivalent performance, fine pitch gears must be of higher quality.

The analytical method developed in this work and the Monte Carlo method in substantial agreement in predicting the statistics of the applied stresses and strengths as well as the reliability of gear pairs.

Using the ISO method C analysis with misalignment errors estimated from simplified formulae or, without allowing for all manufacturing and deflection misalignment components, can grossly overestimate the performance of fine pitch gear units. Gear case (bearing bore) alignment tolerances and mesh misalignment due to bearing and wheel, shaft deflections were found to be significant in the gearbox studied.

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