

Temperature Variations in Biological Tissues Due to Spatial Dependent Blood Perfusion During Microwave Heating

E.A. Adebile, B.N. Akintewe and J.K. Ogunmoyela

Department of Mathematical Sciences, Federal University of Technology, Akure

Abstract: The spatial dependent blood perfusion is modelled into the Pennes bio-heat equation and solved in conjunction with the Maxwell equation. The non-linear coupled equation is solved using a converging series expansion method. The converging series is invoked by the location of the electric field shock. Appropriate matching condition was introduced at selected matching point. The result agrees with literature on the role of blood perfusion. As blood perfusion increases in some location, the temperature increase while in some others it decreases. The effects of other tissue parameters in relation to the blood perfusion under the effect of the electric field were discussed. Adequate knowledge of the tissue location, physiological and thermo physical properties are to be known for an effective therapy to be performed.

Key words: Blood perfusion, microwave heating, biological tissue, series expansion, spatial dependence

INTRODUCTION

The studies in the literature have shown that more interest and research have been going on investigating the effect of microwave irradiation in biological tissues. Microwaves have been found useful for many industrial applications such as sintering, smelting, drying, communication and navigation equipment, microwave ovens and even as medical devices (Cheveland, 1994; Hill and Pincombe, 1992; Kathy *et al.*, 2000; WHO, 1993; El-dabe *et al.*, 2003; Adebile 1997). In the recent times hardware systems capable of generating microwaves have been developed and are increasingly being used for so many applications such as satellite communication, military radar, smart weapon, high speed data communication, automotive ant collision devices, weapon detection and medical devices (Kathy *et al.*, 2000; Ozen *et al.*, 2000).

Exposure to microwaves radiation at sufficiently high intensities produces increase in the tissue temperature. This has been a means for the study of hyperthermia. At a higher temperature, cancerous cells are destroyed at a given time, similarly at an increased temperature over a given time period more diseased cells are destroyed. Despite the great potential of microwave it is becoming ever more important to identify the limit of safe exposure with respect to thermal hazards. The destruction of the cancerous tissues and sparing of the surrounding normal tissue is the goal of the researches into hyperthermia. Some of the works investigated in the literature include (Foster *et al.*, 1978) which studied the effect of surface cooling and blood flow in the temperature distribution of tissue during microwave heating.

A number of studies have been done by Adebile (1997), Adebile *et al.* (2005), Ogunmoyela (2005) which reveal the effects of model parameters, heating devices, the location of maximum temperature which consequently will guide in the therapy for cancerous tissues and the sparing of normal tissues. In the recent time (Foster *et al.*, 1998; 1999) discussed the thermal analysis of thermal effects of RF fields with model studies. Riu *et al.* revealed (1996) in their thermal model the effects of human threshold of microwaves. Ozen *et al.* (2000) discussed the heat effect analysis of microwave exposed skin by using a multi-layer Human skin model. They were able to predict critic temperature rise at various location of the skin, the prediction of rise in temperature is near the surface and these are dependent on a number of model parameters such as boundary condition, blood perfusion rate and the thermal conductivity of tissue. Lin (2000) gave a preliminary survey in the mechanism of the ware-like behaviour of heat transfer in living tissues. He introduced a new concept of multi-mode energy coupling.

Marchant and Liu (2001) considered the steady state microwave heating of a finite one-dimensional slab. In the work of Hill and Pincombe (1992) he investigated some similarity temperature profiles for the microwave heating of a half space. The governing equations for their model involve the Maxwell's equation coupled with the heat equation. The invariance of the governing heat equation under simple one-parameter transformation group led to the examination of a number of transient temperatures was obtained but in general the resulting ordinary different equations are solved numerically. El-dabe *et al.* (2003) working after Hill and Pincombe (1992) introduced

the effect of convection due to blood flow into the heat equation in Hill and Pincombe (1992) this modified their heat equation to conform to the latter celebrated Pennes, bio-heat model.

The purposes of this study is to introduce into the bio heat equation the spatial dependent blood perfusion term so we can study the effect of the electromagnetic properties on the temperature profile of the biological tissues. The steady state is considered here

Mathematical formulations: The governing equation for our problem is given below:

$$\frac{\partial H}{\partial x} + \epsilon \frac{\partial E}{\partial t} + \partial E = 0 \tag{1}$$

$$\frac{\partial E}{\partial x} + \Omega \frac{\partial H}{\partial t} = 0 \tag{2}$$

$$\alpha C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) - \omega_b \alpha_b c_b (T - T_b) + Q(T) |E|^2 \tag{3}$$

with the initial and boundary conditions

$$\begin{aligned} T(x, 0) &= \frac{T_c}{L} x; T(0, t) = 0, T(L, t) = T_c \\ E(x, 0) &= \frac{E_0}{L} x; E(0, t) = 0; E(L, t) = E_0 \\ H(x, 0) &= \frac{H_0}{L} x; H(0, t) = 0, H(L, t) = H_0 \end{aligned} \tag{4}$$

El-dabe *et al.* (2003) solved the above mathematical model numerically for a constant blood perfusion with respect to given layer for $m = 1$. Adebile and Ogunmoyela (2005) solved the problem for varying blood perfusion with spatial variable but neglected the electric and magnetic effect of the microwave heating. In this study we seek for solution to the problem analytically and also model the blood perfusion to be dependent on the spatial variable.

Following the assumptions in El-dabe *et al.* (2003) and Adebile and Ogunmoyela (2005) respectively we consider that:

$$Q(T) = T^m \tag{5}$$

$$\omega_b p_b c_b = (p + qx)^n \tag{6}$$

Similarly using the dimensionless variables

$$\tau = \frac{tv}{L^2}, \eta = \frac{x}{L}, \theta = \frac{T}{T_b}, c_1 = \frac{cb}{cp}$$

$$\begin{aligned} \bar{E} &= \frac{E}{E_0}, \bar{H} = \frac{H}{H_0}, \frac{p_b}{p}, \lambda_1 = \frac{\omega \epsilon E_0}{LH_0} \\ \lambda_2 &= \frac{L\sigma E_0}{H_0}, \lambda_3 = \frac{\eta H_0 v}{LE_0}, \lambda = \frac{L^2 T_b^{m-1} |E_0|^2}{\nu \alpha c p} \end{aligned} \tag{7}$$

The dimensionless equations corresponding to Eq. 1-4 are:

$$\frac{\partial H}{\partial \eta} + \lambda_1 \frac{\partial E}{\partial \tau} + \lambda_2 E = 0 \tag{8}$$

$$\frac{\partial E}{\partial \eta} + \lambda_3 \frac{\partial H}{\partial \tau} = 0 \tag{9}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{1}{p} \frac{\partial^2 \theta}{\partial \eta^2} - \alpha(\eta)(\theta - 1) + \lambda Q |E|^2 \tag{10}$$

Subject to the following dimensionless initial and boundary conditions.

$$\begin{aligned} \theta(\eta, 0) &= \frac{T_c}{T_b} \eta; \theta(0, \tau) = 0; \theta(1, \tau) = \frac{T_c}{T_b} \\ E(\eta, 0) &= \eta; E(0, \tau) = 0; E(1, \tau) = 1 \\ H(\eta, 0) &= \eta; H(0, \tau) = 0; H(1, \tau) = 1 \end{aligned} \tag{11}$$

Where

$$\omega_b p_b c_b(x) = \alpha(\eta) = (a + b\eta)^n \tag{12}$$

Method of solution: The governing equations for the steady state are:

$$\frac{\partial H}{\partial \eta} + \lambda_2 E = 0 \tag{13}$$

$$\frac{\partial E}{\partial \eta} = 0 \tag{14}$$

$$\Omega \frac{\partial^2 \theta}{\partial \eta^2} - (a + b\eta)^n (\theta - 1) + \lambda |E|^2 \theta^m = 0 \tag{15}$$

with the boundary conditions

$$\begin{aligned} \theta(0) &= 0, \theta(1) = \Omega_1 \\ E(0) &= 0, E(1) \\ H(0) &= 0, H(1) = 1 \end{aligned} \tag{16}$$

The solutions to (13) and (14) subject to (16) b. c. are:

$$E(\eta) = H(\eta - a) = \begin{cases} 0, & \eta < a \\ 1, & \eta \geq a \end{cases} \quad (17)$$

$$H(\eta) = \eta H(\eta - a) = \begin{cases} 0, & \eta < a \\ \eta, & \eta \geq a \end{cases} \quad (18)$$

The energy Eq. in 15 becomes:

$$\Omega \frac{d^2\theta}{d\eta^2} - (a + b\eta)^\eta (\theta - 1) + \lambda \theta^m |H(\eta - a)|^2 = 0 \quad (19)$$

$$\theta(0) = 0, \theta(1) = \Omega_1 \quad (20)$$

Where

$$\Omega = P_r^{-1}, \Omega_1 = T_c T_b^{-1}$$

$H(\eta - a)$ is the Heaverside function

The solution to the Electric field provokes that solution for the temperature field be sought in two regions (Region, I and II) and these accompanied with adequate matching conditions

The relevant equation for Regions I and II are:

$$\Omega \frac{\partial^2 \theta_1}{\partial \eta^2} - (a + b\eta)^\eta (\theta_1 - 1) = 0 \quad (21)$$

$$\theta_1(0) = 0, \theta_1(a) = (H), 0 \leq \eta \leq a \quad (22)$$

for region I;
and

$$\Omega \frac{\partial^2 \theta_2}{\partial \eta^2} - (a + b\eta)^\eta (\theta_2 - 1) + \lambda \theta_2^m = 0 \quad (23)$$

$$\theta_2(a) = (H), \theta_2(1) = \Omega_1 \quad (24)$$

for region II

we seek for solution to the Eq. in (21) and (22) by using the power series method:

$$\theta_1^i = \sum_{i=0}^{\alpha} p_i \eta^i; a \leq \eta \leq a \quad (25)$$

and

$$\theta_2^0 = \sum_{i=0}^{\alpha} q_i (1 - \eta)^i; a \leq \eta \leq 1 \quad (26)$$

using the series in (12) in Eq. 21 and collecting the coefficient of η^0, η^1, η^2 we have, respectively

$$\begin{aligned} 2\Omega P_2 + f(P_0 - 1) &= 0 \\ 6\Omega P_3 + (f P_1 + g(p_0 - 1)) &= 0 \\ 12\Omega P_4 + (f p_2 + g p_1 + h(p_0 - 1)) &= 0 \end{aligned} \quad (27)$$

which gives

$$\begin{aligned} P_2 &= -f \frac{(p_0 - 1)}{2\Omega} \equiv m_1 \\ P_3 &= Q_1 P_1 + Q_2 \\ P_4 &= N_2 P_1 + N_3 \end{aligned} \quad (28)$$

$$\text{where } Q_1 = -\frac{f}{6\Omega}, Q_2 = -\frac{g(p_0 - 1)}{6\Omega}$$

$$N_2 = -\frac{g}{12\Omega}, N_3 = -\frac{(f m_1 + h(p_0 - 1))}{12\Omega}$$

$$f = \alpha a^\eta$$

$$g = \alpha n a^{n-1} b$$

$$h = \frac{\alpha n(n-1)a^{n-2}b^2}{2}$$

using the series in (16) in Eq. 21 we have after collecting the coefficient of η^0, η^1, η^2 , respectively:

$$\begin{aligned} 2\Omega q_2 - (A - q_0 A) + \lambda q_0^m &= 0 \\ 6\Omega q_3 + (D - q_0 D + q_1 A) + \lambda m q_0^{m-1} q_1 &= 0 \\ 12\Omega q_4 + (A q_2 - D q_1 + h(q_0 - 1)) + \lambda m q_0^{m-1} q_2 \\ + \lambda m(m-1) q_0^{m-2} q_1^2 &= 0 \end{aligned} \quad (29)$$

which gives

$$\begin{aligned} q_2 &= \frac{(A - q_0 A) - \lambda q_0^m}{2\Omega} \equiv L_1 \\ q_3 &= -\frac{(D - q_0 D + A q_1) - \lambda m q_0^{m-1} q_1}{6\Omega} \\ &= G_1 q_1 + G_2 \\ q_4 &= 1(A q_2 - D q_1 + h(q_0 - 1)) - \lambda m q_0^{m-1} \\ &\quad - \frac{h m(m-1) q_0^{m-2} q_1^2}{12\Omega} \\ &= S_1 q_1^2 + S_2 q_1 + S_3 \end{aligned} \quad (30)$$

where

$$\begin{aligned} G_1 &= -\frac{(A + \lambda m q_0^{m-1})}{6\Omega} \\ G_2 &= \frac{-(D - q_0 D)}{6\Omega} \\ S_1 &= \frac{-h m(m-1) q_0^2}{12\Omega} \\ S_2 &= \frac{D}{12\Omega} \\ S_3 &= \frac{-(A L_1 + h(q_0 - 1) + \lambda q_0^{m-1} L_1)}{12\Omega} \end{aligned} \quad (31)$$

using the boundary conditions in (22) and (24) we here

$$P_1 = \frac{\Theta - m_1 a^2 - Q_2 a^3 - N_3 a^4}{(a + Q_1 a^3 + N_2 a^4)} \quad (32)$$

$$\partial_1 q_1^2 + \partial_2 q_1 + \partial_3 = 0 \quad (33)$$

where

$$\begin{aligned} \partial_1 &= S_1(1-a)^4 \\ \partial_2 &= \{(1-a) + G_1(1-a)^3 + S_2(1-a)^4\} \\ \partial_3 &= \{-\Theta - q_0 - L_1(1-a) - G_2(1-a)^3 - S_3(1-a)^4\} \end{aligned} \quad (34)$$

then

$$q_1 = -\frac{-\partial_2 \pm \sqrt{\partial_2^2 - 4\partial_1\partial_3}}{2\partial_1} \quad (35)$$

RESULTS AND DISCUSSION

The graphs displayed in Fig. 1-6 give results that are very significant to the medical experts and multidisciplinary fields for new advances in medical technology. These graphs are for the case $m = 1$.

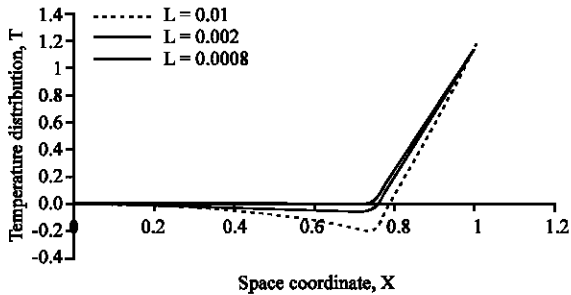


Fig. 1a: Temperature distribution against the Space coordinate for spatial dependent blood perfusion, $w = (a+bX)^n$ where $a = .75$, $n = -1$, $E_0 = 2$, $w_b = .00125$, $g_b = 1060$, $g = 1050$, $cb = 3770$, $cp = 3590$

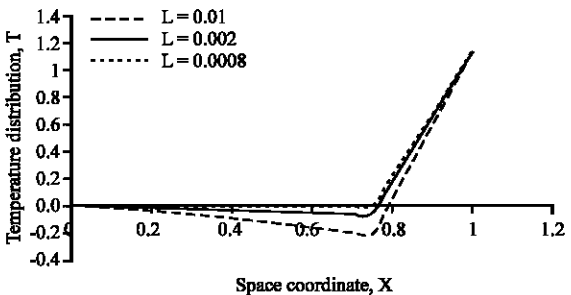


Fig. 1b: Temperature distribution against the Space coordinate for spatial dependent blood perfusion, $w = (a+bX)^n$ for different values of L ; where $a = .75$, $n = 0$, $E_0 = 2$, $w_b = .00125$, $g_b = 1060$, $g = 1050$, $cb = 3770$ and $cp = 3590$

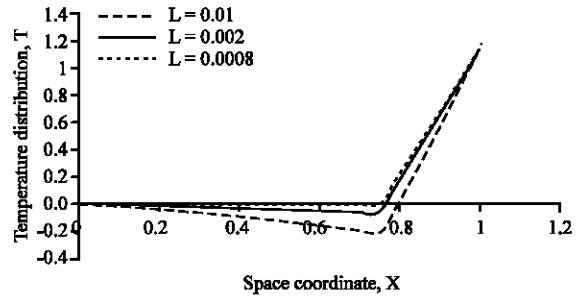


Fig. 1c: Temperature distribution against the Space coordinate for spatial dependent blood perfusion, $w = (a+bX)^n$ considering different values of L ; where $a = .75$, $n = 1$, $E_0 = 2$, $g_b = 1060$, $g = 1050$, $cb = 3770$, $cp = 3590$

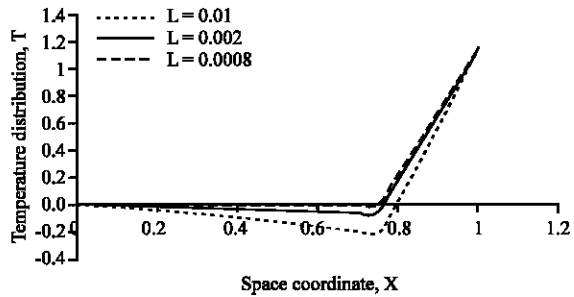


Fig. 1d: Temperature distribution against the Space coordinate for spatial dependent blood perfusion, $w = (a+bX)^n$ for different values of L ; where $a = 0.75$, $k = 0.24$, $n = 2$, $E_0 = 2$, $g_b = 1060$, $g = 1050$, $cb = 3770$, $cp = 3590$

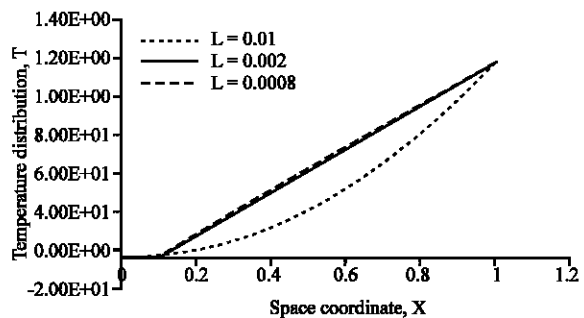


Fig. 1e: Temperature distribution against the Space coordinate for spatial dependent blood perfusion, $w = (a+bX)^n$ at different skin tissue thickness, L ; where $a_2 = 0.1$, $a = 1$, $b = 1$, $n = 1$, $E_0 = 2$, $g_b = 1060$, $g = 1050$, $cb = 3770$, $cp = 3590$

In Fig. 1, it is clear that as L increases the temperature in both regions of our model decreases for every n -a perfusion dependent variable. It is important to note that

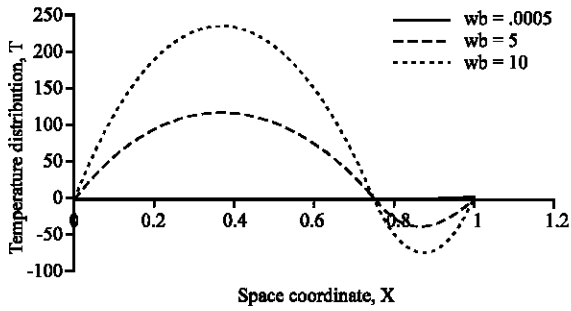


Fig. 2a: Temperature distribution against the space coordinate for spatial dependent blood perfusion, $w = (a+bX)^n$ at different values of blood perfusion; where $a = .75, L = 0.01, n = 0, E_0 = 2, k = 0.24, gb = 1060, g = 1050, cb = 3770, cp = 3590$

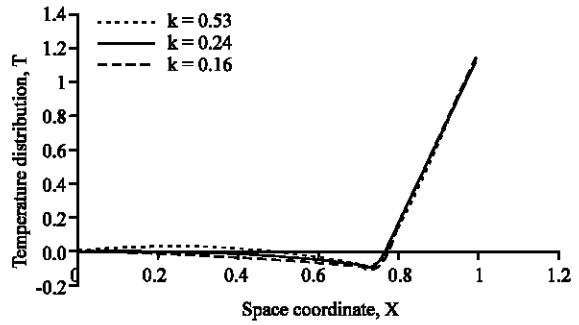


Fig. 3: Temperature distribution plotted against the space coordinate for different thermal conductivities, k ; in $w = (a+bX)^n$, where $a = .75, H = -0.1, wb = .00125, g = 1050, gb = 1060, cb = 3770, cp = 3590, a = 1, b = 1, L = 0.01$ and $n = 1$

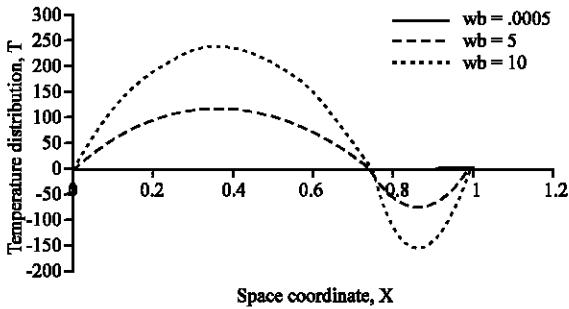


Fig. 2b: Temperature distribution against the Space coordinate for spatial dependent blood perfusion, $w = (a+bX)^n$ at different values of blood perfusion; where $a = .75, L = 0.01, n = 1, k = 0.24, E_0 = 2, gb = 1060, g = 1050, cb = 3770$ and $cp = 3590$

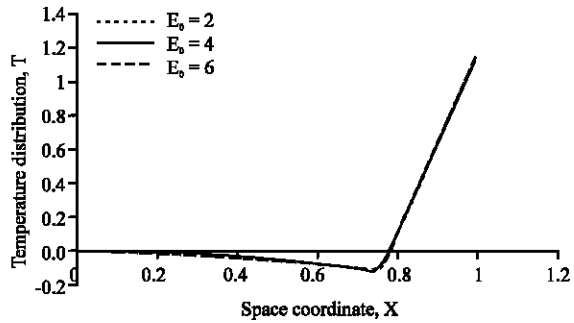


Fig. 4: Temperature distribution plotted against the space coordinate for different values of E_0 , in $w = (a+bX)^n$, where $a = .75, wb = .00125, k = 0.24, g = 1050, gb = 1060, cb = 3770, cp = 3590, a = 1, b = 1, n = 1$ and $L = 0.01$

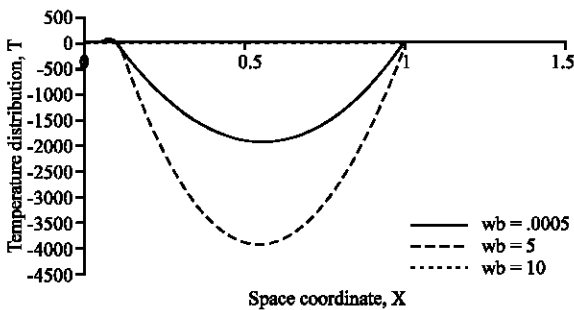


Fig. 2c: Temperature distribution against the Space coordinate for spatial dependent blood perfusion, $w = (a+bX)^n$ at different values of blood perfusion; where $a = 0.1, L = 0.01, n = 2, k = 0.24, E_0 = 2, gb = 1060, g = 1050, cb = 3770$ and $cp = 3590$

as n increases, that is as the blood perfusion increases the temperature profiles decreases in the region 2 where the electric field is on, but in Region 1 the temperature profile are nearly same for all n .

In Fig. 2, the variation of the blood perfusion constraint ω_b affect the temperature in both regions. In region 1 where the electric field is not directly active, as ω_b increases the temperature increases but the same for different n at a fixed L . In region 2 as ω_b increases the temperature decreases, for fixed n and decreases n increase. This effect underscores the effect of the electric field.

From Fig. 3, the effect of the thermal conductivity is presented. The effect of the thermal conductivity is very significant in region 1 where the electric field is not on unlike in region 2. In region 1 as K increases the temperature increases, while in region 2 there is no significant change.

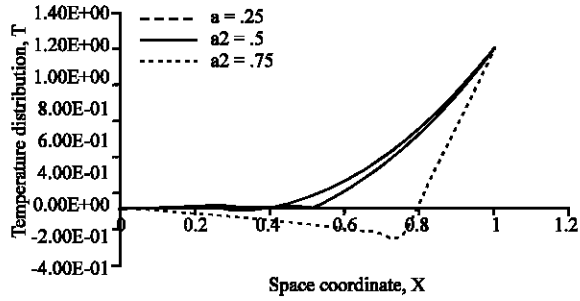


Fig. 5: Temperature distribution against the space coordinate for spacial dependent blood perfusion, $(a+bx)^n$ at different skin depths a_2 , where $n = 2$, $a = 1$, $b = 1$, $L = 0.01$, $w_b = 0.00125$, $k = 0.24$, $g = 1050$, $g_b = 1060$, $cb = 3770$ and $cp = 3590$

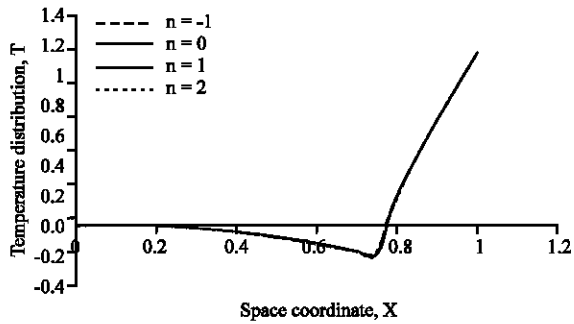


Fig. 6: Temperature distribution against the Space coordinate for spatial dependent blood perfusion, $w = (a+bX)^n$ at different values of n ; where $L = 0.01$, $a = 0.75$, $g = 1050$, $g_b = 1060$, $cb = 3770$, $cp = 3590$ and $E_0 = 2$

In Fig. 4, it is observed that increase in the electric field of the free space, E_0 ; promotes increase in the temperature in region 2 where electric field is on. There is no effect in region 1 since electric field is not on in this region.

In Fig. 5, the effect of the location of the introduction of the electric field is vivid. Higher temperature can be achieved near the tissue surface or anywhere in the tissue as the value of a varies for different L , tissue thickness and for different n but as n increases the temperature at given location decreases.

Furthermore in Fig. 6, increase in n has no effect in region 1 but in region 2, the temperature decreases.

The temperature profiles for $m = 1$ is unique but for $m \geq 1$ multiple solution is advanced as evidenced in equations (30), (32), (34). Never the less heating with Microwave device with $m > 1$ can produce a unique solution if $\vartheta_2^2 = 4\vartheta_1\vartheta_3$.

CONCLUSION

The effect of the electric field has been investigated on biological tissue with spatial dependent blood perfusion. The results revealed that:

- Temperature profiles in the two regions are affected by the effect of the electric field ∇E .
- The blood perfusion parameters; n, ω_b, a and b will significantly affect the temperature profiles.
- Increase in thermal conductivity provokes increase in temperature where there is no electric field but these effects are overcome by the electric field.
- The electric field of the free space E_0 , significantly affect the temperature when electric field is on otherwise no clear effect.
- The point of application of electric field can be varied and this affects the contribution to temperature (an increase or decrease) by different parameters.
- Preferential heating can be adequately achieved through adequate monitoring of parameters.
- The possibility of hot spot is revealed, but approach to hinder this phenomena can be achieved with adequate knowledge of tissue parameters and appropriate regulation of designed engine by engineers.
- Insight to the provocation of cold spot is advanced and this will be useful in cryosurgery.
- Stability of solution is guaranteed for $m = 1$ but instability is advanced for $m > 1$ because of the existence of multiple solution.

REFERENCES

- Adebile, E.A., 1997. Prediction of temperature rise in tumours and surrounding normal tissues during microwave heating Ph.D Thesis, Obafemi Awolowo University Ile-Ife, Osun State.
- Adebile, E.A. and R.O. Ayeni and Y.A.S. Aregbesola, 2005. Steady-state temperature in biological tissues undergoing microwave hyperthermic. Int. J. Biol. Phys. Sci. Science Focus, pp: 101-108.
- Adebile, E.A. and J.K. Ogunmoyela, 2005. Thermoregulation in biological tissues during microwave hyperthermia. Part I Spatial: Dependent Blood Perfusion Effect, Abacus.
- Cleveland, R.F. Jr., 1994. Radio frequency Radiation in the Environment: Sources, Exposure Standard and Related Issue. In: Carpenter, D.O.; Ayrapetyan, S., (Eds.), Biological Effects of Electric and Magnetic Fields. New York: Academic Press: 1: 53-81.

- El-dabe Nabuk, T., A.A. Mohamed Mona and F. El-sayed Asthma, 2003. Effects of microwave heating on the thermal states of biological tissues. *Afr. J. Biotechnol.*, pp: 453-459.
- Foster, K.R., H.N. Kritikos and H.P. Schwan, 1978. Effect of surface cooling and blood flow on the microwave heating of tissues. *IEEE Trans. Bioed. Eng.*, pp: 313-316.
- Foster, K.R., L. NozanoLieto and P.J. Riu, 1998. Heating of tissues by microwave: A model analysis. *Bioelectromagnetics*, 19: 420-428.
- Foster, K.R. and S.L. Erdreich, 1999. Thermal model for microwave hazards and their role in standards development, *Bioelectromagnetics*, 20: 526-3.
- Hill, J. and A. Pincombe, 1992. Some similarity temperature profiles for the microwave heating of a half-space. *J. Austral. Math. Soc. Ser. B.*, 33: 290-320.
- Kathy, L., J.A. Ryan, J.R. D'Andrea, Jauchem and P.A. Mason, 2000. Radio Frequency Radiation of Millimeter wave Length: Potential occupational Safety Issues Relating to Surface Heating. *Health Physics*.
- Liu, J., 2000. Preliminary survey on the mechanism of the wave like behaviors of heat transfer in living tissue. *Forschung Im ingenieurwesen Springer-Verlag*, 66: 1-10.
- Marchant, T. and B. Liu, 2001. On the heating of a two-dimensional slab in a microwave cavity: Aprature Effects. *Anziam J.*, 43: 137-148.
- Ozen, S., S. Colekci, M. Merdan, 2000. Overexposure of Electric and Magnetic Field for patient and operators from RF Diathermy Equipment, Isik2000 workshop on Biomedical information Engineering, Isik University, Istanbul, Turkey, pp: 190-192,
- Riu, P.J., K.R. Foster, D.W. Blick, E.R. Adair, 1996. A Thermal model for human thresholds of microwave evoked warmth sensation. *Bioelec-Tromagnetics*, 18: 578-583.
- World Health Organization (WHO), 1993. Environmental Health Criteria 137: Electromagnetic Fields (300Hz-300 GHz). 53, Geneva.