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Optical Bistability Trimode of Fabry-Perot LSA with Inhomogeneous Broadening

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Abstract: The aim of this study is the elaboration of a mathematic model for describing the action of saturable absorber lasers. The anticipant approach for that purpose takes into consideration the phenomenological manner of essential physical parameters necessary to saturable absorber lasers and their influences on the optical bistability. We are going to examine theoretically the optical bistability in the three mode saturable absorber lasers. We'll focus on the case where the two active and absorber milieus sustain a dominant inhomogeneous broadening for a Fabry-Perot cavity. From an interferometric point of view the bistable behavior due to a nonlinear material inside a Fabry-Perot cavity is studied. Starting from the system of the Rate Equation (RAE), we have solved numerically this type of equation for stationary case. We have elaborated program in Matlab allowing us to determine the effect of this optical bistability and to draw the curves representing the density of the photons in the pumping function of the active milieu in the case if the spontaneous emission is neglected.

Key words: Optical bistability, inhomogenous widening, fabry-pérot cavity

INTRODUCTION

The transmission of information by optical way has known a considerable progress which led the scientists to wonder about the possibilities of creating systems with purely optical memories.

As a sole active optical bistable system, the laser containing Saturable Absorber () has been a standard model in a great number of theoretical studies on absorptive Optical Bistability OB[1]. Made to operate in two stable low and high output levels, may exhibit a hysteresis cycle of photon density versus laser pumping power[2].

This type of optical bistability is absorptive; the nonlinear milieu may be a gas or a liquid. As the milieu has a weak nonlinear coefficient, this requires important luminous energies in order to induce optically the non linearity of the milieu. This is why the optical phenomenon is not observed unless these substances are placed in a resonator.

The lasers containing the saturable absorber easily offer the possibility to observe the absorptive optical bistability[2].

The saturable absorber is a nonlinear resonant absorber whose absorption coefficient can vary in a reversible way under the effect of a sufficiently intense flow of given frequency[2,3].

$$\frac{dn_j}{dt} = -\chi_j n_j + \sum_{\mu} Bg(\omega_\mu - \omega_j)(n_j+1)\left[\int_{z-\frac{1}{2}}^{z+\frac{1}{2}} N_a \sin^2\left(\frac{\pi q_j}{L} z\right)dz - \int_{z-\frac{1}{2}}^{z+\frac{1}{2}} N_b \sin^2\left(\frac{\pi q_j}{L} z\right)dz\right] \quad (1)$$

This study is devoted to investigate the OB behaviour of Fabry-Perot when the inhomogenous atomic linewidth greatly exceeds the homogeneous one in trimode case.

Rate equation: The model under consideration consists of a planar-mirror Fabry-Pérot resonator of length L, containing the amplification and absorption cells with the same length l at the coordinates z-l/2 and z + l/2, respectively. The cavity losses and the pumping rates of the cells are incorporated into the relevant phenomenological parameters of the model. Both amplifier and absorber are considered as ensembles of two-level

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atoms whose atomic linewidths are assumed to be of Lorentzian homogeneously broadened with the same halfwidth Γ. The inhomogeneous gain profile of halfwidth ε centred at ω_0 is composed of a continuous distribution of homogeneous packets at frequencies ω_J. We consider the case where the cavity can sustain only three modes of photons number n_j with circular frequency ω_j at a detuning $\Delta_J = |\omega_j - \omega_0|$[4,5].

For simplicity, we assume the cavity losses $\div_j$ in these modes to be constant. In the rate equation approximation, such a system obeys the following Eq[6,7]:

$$\frac{dN_{\mu a}}{dt} = R_{\mu a} - N^{\mu a}\left[\sum_j^k Bg(\omega_\mu - \omega_j)n_j + \gamma_a\right] \quad (2)$$

$$\frac{dN_{\mu b}}{dt} = R_{\mu b} - N^{\mu b}\left[\sum_j^k Bg(\omega_\mu - \omega_j)n_j + \gamma_b\right] \quad (3)$$

B, here, is the Einstein coefficient and g $(\omega_J - \omega_J)$ which can be usually represented by a function of the form:

$$g(\omega_\mu - \omega j) = \frac{\Gamma^2}{\Gamma^2 + 4(\omega_\mu - \omega_j)^2}$$

$N_{\mu a}$ and $N_{\mu b}$ are the densities of population differences between the atomic upper and lower levels in both media.

γ_a and γ_b denote the relaxation rates of the upper levels in the amplifying and absorbing atoms, respectively.

The absorber pumping rate $R_{\mu b}$ is assumed to be constant. In the Lorentzian model, the laser pumping rate $R_{\mu a}$ with the constant pumping rate R_0 can be expressed as follows:

$$R_{\mu a}(\omega_\mu) = \frac{R_a \varepsilon^2}{\varepsilon^2 + 4(\omega_\mu - \omega_0)^2}$$

The integral given in the system of Eq. by,

$$\left[\int_{z-\frac{1}{2}}^{z+\frac{1}{2}} N_a \sin^2\left(\frac{\pi q_j}{L} z\right) dz - \int_{z-\frac{1}{2}}^{z+\frac{1}{2}} N_b \sin^2\left(\frac{\pi q_j}{L} z\right) dz\right]$$

indicates the effect of interference produced in the Fabry-Pérot resonator[4].

It can take various values function of the position of the two milieus and absorbing in the optical resonator. We took into consideration the general case where active milieu and absorbing milieu placed at the milieu of the cavity resonant. Populations $N_{\mu a}$, $N_{\mu b}$ are independent of position z and consequently the term between hooks can be written in the following way[3]:

$$(N_a - N_b)\left[\int_{z-\frac{1}{2}}^{z+\frac{1}{2}} \sin^2\left(\frac{\pi q_j}{L} z\right) dz\right] = \frac{A_0}{2}$$

Where the value of A_0 is:

$$A_0 = 1 - \frac{L}{q_j \pi} \sin\left(\frac{\pi q_j 1}{L}\right) \cos\left(\frac{2\pi q_j z}{L}\right)$$

In the general case, $N_{\mu a}$, $N_{\mu b}$ and n_j are functions of time, however one is interested in the stationary behavior of the :

$$\left\{ \frac{dn_j}{dt} = \frac{dN_{\mu a}}{dt} = \frac{dN_{\mu b}}{dt} = 0 \right.$$

While replacing $\sum_j g(\omega_\mu - \omega_j)$ by the integral

$$\frac{1}{\pi \Delta\Omega} \int_{-\infty}^{+\infty} \frac{\Gamma^2}{\Gamma^2 + 4(\omega_\mu - \omega_j)^2} d\omega_j$$

and the sum $\sum_\mu R_\mu(\omega_\mu)$ by the integral $\frac{1}{\pi\varepsilon}\int_{-\infty}^{+\infty} R_\mu d\omega_\mu$ in the Eq. 1, after calculation and transformation, we ended to the following Eq:

$$n_j = \frac{A_0 \Gamma \Delta\Omega}{2\varepsilon}(n_j + 1)\left[\frac{(\sigma_a K - \sigma_b)Q_j \Gamma + 2(\sigma_a K \xi - \sigma_b)\Delta\Omega}{Q_j^2 \Gamma^2 + 2\Delta\Omega\Gamma(\xi + 1)Q_j + 4\xi\Delta\Omega^2}\right] \quad (4)$$

Here pumping $\frac{B}{x_j} \cdot \frac{R_a}{\gamma} = \sigma_a$ of the active milieu parameter and $\frac{B}{x_j} \cdot \frac{R_{\mu b}}{\gamma} = \sigma_b$ pumping of the absorber milieu parameter.

This Eq is the fundamental equation describing the action of the multimode considered in a stationary state, $\Delta\Omega$ being the difference in frequency between two successive modes.

To determine the conditions of existence of the optical bistability in the trimode case (j = 0, j = ±1), the laser intensity or the photons densityis defined by $Q_j = \frac{B}{\gamma} n_j$.

Study of the central mode j = 0: In the general case we have $K = \varepsilon^2/(\varepsilon^2+4m^2\Delta\Omega^2)$ and $\omega_j - \omega_0 = m\,\Delta\Omega$ where m is the number of intervals between the successive modes considered. For the central mode m = 0, K is equal to 1.

So, neglecting the spontaneous emission 4 becomes:

$$Q_0^2 + \left[\frac{2(1+\xi)}{\Gamma} - \frac{A_0}{2\varepsilon}(\sigma_a K - \sigma_b)\right]\Delta\Omega Q_0 + \left[\frac{4\xi}{\Gamma^2} - \frac{A_0}{\varepsilon\Gamma}(K\xi\sigma_a - \sigma_b)\right]\Delta\Omega^2 = 0 \quad (5)$$

- At the saturation $\xi = 1$, the characteristic equation of the photons density is given by:

$$Q_0^2 + \left[\frac{4}{\Gamma} - \frac{A_0}{2\varepsilon}(\sigma_a K - \sigma_b)\right]\Delta\Omega Q_0 + \left[\frac{4}{\Gamma^2} - \frac{A_0}{\varepsilon\Gamma}(K\sigma_a - \sigma_b)\right]\Delta\Omega^2 = 0 \quad (6)$$

In order to find the two positive solutions Q_{01}, Q_{02}. and knowing that Δ is descriminant of the equation, S being the sum of the two solutions and P the product, we have to check at the same time the following conditions:

$$\Delta > 0$$
$$S = Q_{01} + Q_{02} > 0 \quad (7)$$
$$P = Q_{01} + Q_{02} > 0$$

What gives us: $\Delta = \left(\frac{A_0\Delta\Omega}{2\varepsilon}(\sigma_a K - \sigma_b)\right)^2$ and

$$S = Q01 + Q02 = \left[\frac{A_0}{2\varepsilon}(\sigma_a K - \sigma_b) - \frac{4}{\Gamma}\right]\Delta\Omega > 0$$

That leads to: $\sigma_a > \frac{1}{K}\left[\sigma_b + \frac{8\varepsilon}{A_0\Gamma}\right] = \sigma_{a1}$

$$P = Q01 \times Q02 = \left[\frac{4}{\Gamma^2} - \frac{A_0}{\varepsilon\Gamma}(\sigma_a K - \sigma_b)\right]\Delta\Omega^2 > 0$$

And then to : $\sigma_a < \frac{1}{K}\left[\sigma_b + \frac{4\varepsilon}{A_0\Gamma}\right] = \sigma_{a2}$

With saturation, when ξ = 1, there isn't optical bistability when the three conditions (7) previously enumerated are not checked at the same time.

- For $\xi \neq 1$ $(0 < \xi < 1)$ the characteristic equation of the photons densityis given by:

$$Q_0^2 + \left[\frac{2(1+\xi)}{\Gamma} - \frac{A_0}{2\varepsilon}(\sigma_a K - \sigma_b)\right]\Delta\Omega Q_0 + \left[\frac{4\xi}{\Gamma^2} - \frac{A_0}{\varepsilon\Gamma}(K\xi\sigma_a - \sigma_b)\right]\Delta\Omega^2 = 0 \quad (8)$$

The checking of the conditions (7) gives us:

$$\delta = 4K^2\Delta\Omega^4\left[\left(\frac{A_0}{\varepsilon}\right)^3\frac{(1-\xi)}{\Gamma}\sigma_b\right]$$

$$\sigma_{a1} = \frac{1}{K}\left[\sigma_b + \frac{4\varepsilon}{A_0\Gamma}(1-\xi) + 4\sqrt{\frac{\sigma_b\varepsilon(1-\xi)}{A_0\Gamma}}\right]$$

$$\sigma_{a2} = \frac{1}{K}\left[\sigma_b + \frac{4\varepsilon}{A_0\Gamma}(1-\xi) - 4\sqrt{\frac{\sigma_b\varepsilon(1-\xi)}{A_0\Gamma}}\right]$$

$$\sigma_a \in \left]-\infty, \sigma_{a2}\right[\cup \left]\sigma_{a1}, +\infty\right[$$

$$P = Q01 \times Q02 = \left[\frac{4}{\Gamma^2} - \frac{A_0}{\varepsilon\Gamma}(\sigma_a K - \sigma_b)\right]\Delta\Omega^2 > 0$$

$$\Rightarrow \sigma_a > \frac{1}{K}\left[\sigma_b + \frac{4\varepsilon(1+\xi)}{A_0\Gamma}\right] = \sigma_{a3}$$

$$P = Q_{01} \times Q_{02} = \frac{4\xi}{\Gamma^2} - \frac{A_0}{\varepsilon\Gamma}(K\xi\omega_a - \sigma_b) > 0$$

$$\Rightarrow \sigma_a < \frac{1}{K}\left[\frac{\sigma_b}{\xi} + \frac{4\varepsilon}{A_0\Gamma}\right] = \sigma_{a4}$$

To sum up, in order to find two positive solutions $Q_{01,02} > 0$, i.e., the optical bistability effect, the value of σ_a must satisfy at the same time the following conditions:

$$\left.\begin{aligned} &\sigma_a > \frac{1}{K}\left[\sigma_b + \frac{4\varepsilon}{A_0\Gamma}(1-\xi) + 4\sqrt{\frac{\sigma_b\varepsilon(1-\xi)}{A_0\Gamma}}\right] = \sigma_{a1} \\ &\sigma_a < \frac{1}{K}\left[\sigma_b + \frac{4\varepsilon}{A_0\Gamma}(1-\xi) - 4\sqrt{\frac{\sigma_b\varepsilon(1-\xi)}{A_0\Gamma}}\right] = \sigma_{a2} \\ &\sigma_a > \frac{1}{K}\left[\sigma_b + \frac{4\varepsilon(1+\xi)}{A_0\Gamma}\right] = \sigma_{a3} \end{aligned}\right\} \quad (9)$$

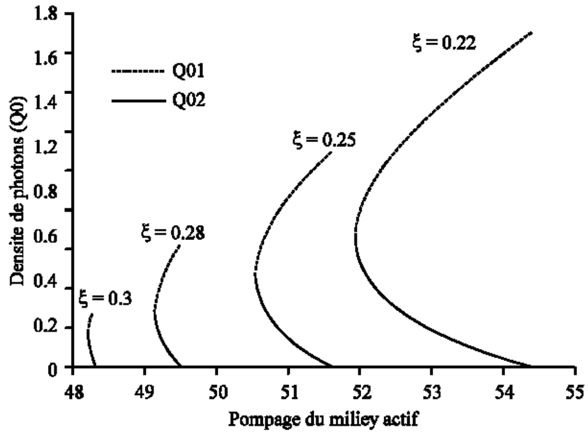

Fig. 1: Curves of the photons density Q_0 according to the pumping of the active milieu (σ_a) where ξ varies

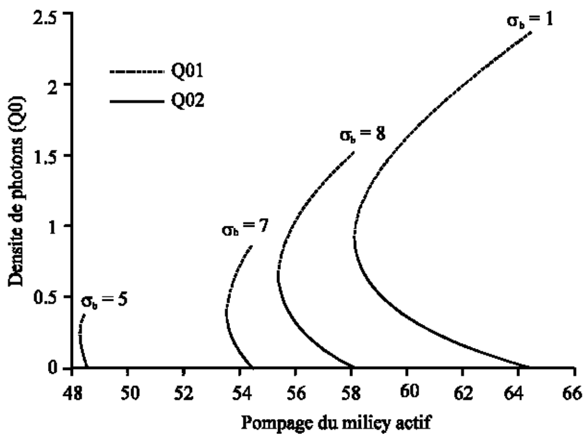

Fig. 2: Curves of the photons density Q_0 according to the pumping of the active milieu (σ_a) where σ_b varies

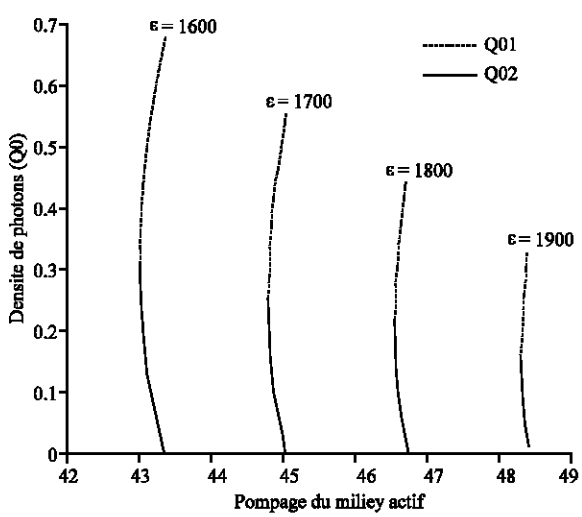

Fig. 3: Curves of the photons density Q_0 according to the pumping of the active milieu (σ_a) where ε varies

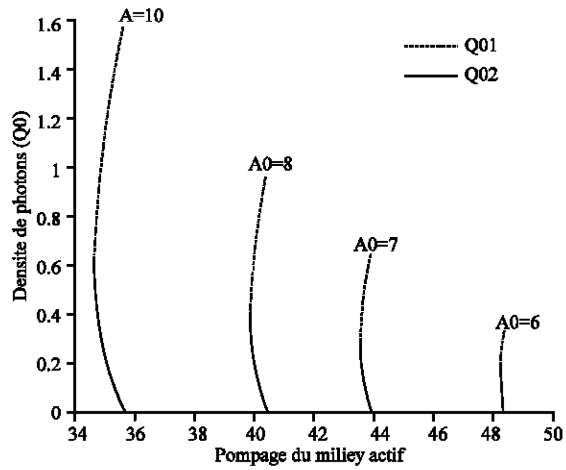

Fig. 4: Curves of the photons density Q_0 according to the pumping of the active milieu (σ_a) where A_0 varies

The previous conditions (9) are satisfied, the equation has two positive roots and thus, the effect of optical bistability exists.

The curves of hysteresis which follow Fig. 1-4 illustrate the representation of the photons density Q_0 according to pumping of the active milieu and consequently the optical bistability effect for the physical parameters (σ_b, ξ, ε, A_0).

Study of the mode j = 1: By symmetry of modes, compared to the central mode, it is easier, for mathematical calculation to consider the mode j = +1 or j = -1.

The number of modes considered is equal to 2, so:

$$m = 1 \Rightarrow K = \frac{\varepsilon^2}{\varepsilon^2 + 4\Delta\Omega^2}$$

Thus, after transformations, the Eq 4, will neglecting spontaneous emission, becomes an equation of the second order function of Q_1:

$$Q_1^2 + \left[\frac{2(1+\xi)}{\Gamma} - \frac{A_0}{2\varepsilon}(\sigma_a K - \sigma_b)\right]\Delta\Omega Q_1 + \left[\frac{4\xi}{\Gamma^2} - \frac{A_0}{\varepsilon\Gamma}(K\xi\sigma_a - \sigma_b)\right]\Delta\Omega^2 = 0$$

- With saturation, $\xi = 1$ the characteristic Eq of the photons densityis given by:

$$Q_1^2 + \left[\frac{4}{\Gamma} - \frac{A_0}{2\varepsilon}(\sigma_a K - \sigma_b)\right]\Delta\Omega Q_1 + \left[\frac{4}{\Gamma^2} - \frac{A_0}{\varepsilon\Gamma}(K\sigma_a - \sigma_b)\right]\Delta\Omega^2 = 0$$

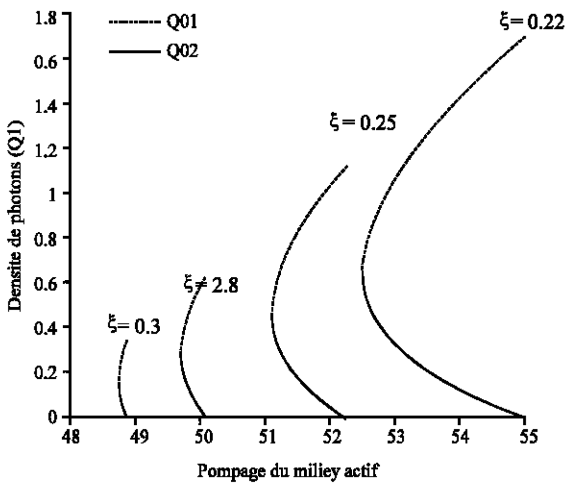

Fig. 5: Curves of the photons density Q_1 according to the pumping of the active milieu (σ_a) where ξ varies

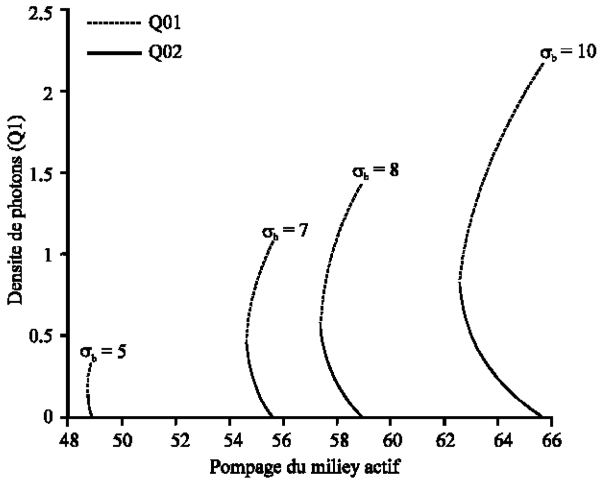

Fig. 6: Curves of the photons density Q_1 according to the pumping of the active milieu (σ_a) where σ_b varies

Just as for the mode central j=0, there isn't optical bistability in the case where the spontaneous emission is neglected with a saturation coefficient equal to 1.

- For $\xi \neq 1$ $(0 < \xi < 1)$ the characteristic equation of the photons densityis given by:

$$Q_1^2+\left[\frac{2(1+\xi)}{\Gamma}-\frac{A_0}{2\varepsilon}(\sigma_a K-\sigma_b)\right]\Delta\Omega Q_1$$
$$+\left[\frac{4\xi}{\Gamma^2}-\frac{A_0}{\varepsilon\Gamma}(K\xi\sigma_a-\sigma_b)\right]\Delta\Omega^2=0$$

The following curves of hysteresis Fig. 5-8 illustrate the representation of the photons density according to pumping of the active milieu and

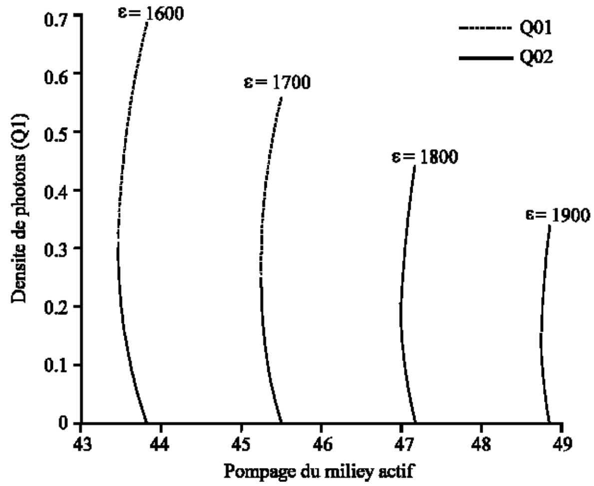

Fig. 7: Curves of the photons density Q_1 according to the pumping of the active milieu (σ_a) where ε varies

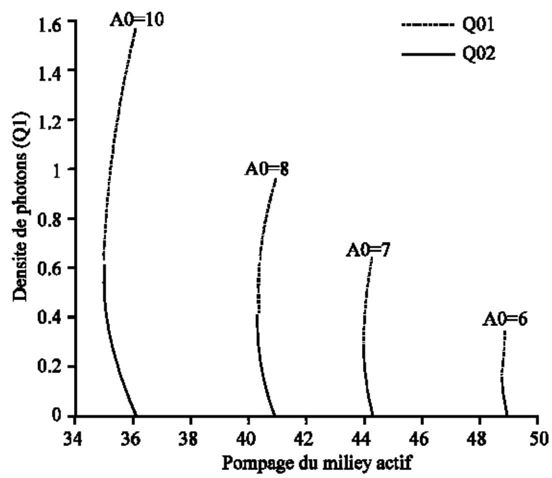

Fig. 8: Curves of the photons density Q_1 according to the pumping of the active milieu (σ_a) where A_0 varies

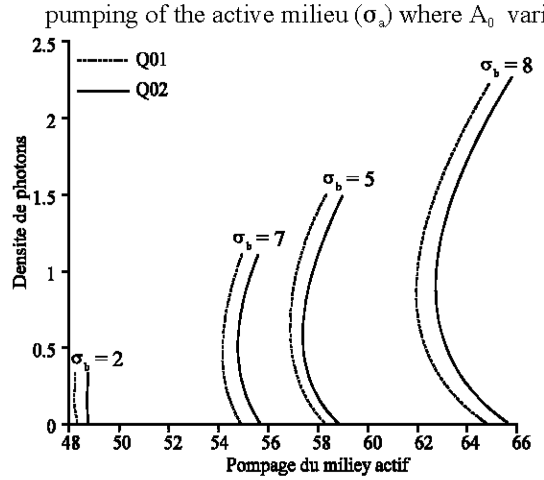

Fig. 9: Comparison between curves of the photons density Q_0 and Q_1 according to the pumping of the active milieu (σ_a) where A_0 vaies

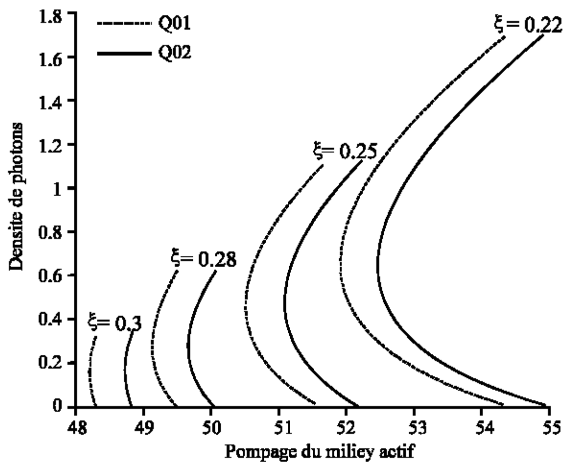

Fig. 10: Comparison between curves of the photons density Q_0 and Q_1 according to the pumping of the active milieu (σ_a) where σ_b varies

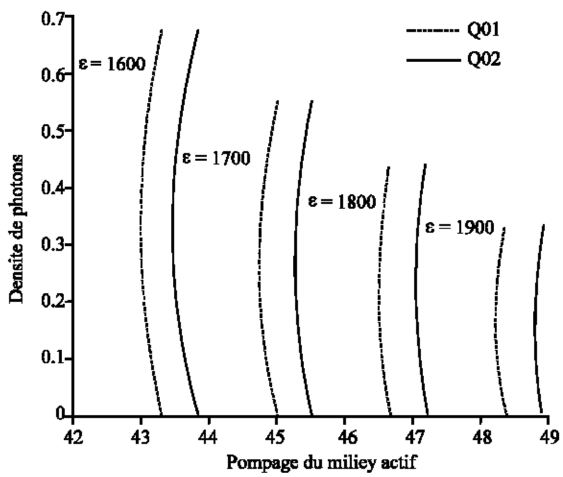

Fig. 11: Comparison between curves of the photons density Q_0 and Q_1 according to the pumping of the active milieu (σ_a) where A_0 varies

consequently the effect of optical bistability for the physical parameters (σ_b , ξ, ε, A_0).

The following curves of hysteresis Fig. 9-12 illustrate the representation comparison between the photons density Q_0 and Q_1 according to the pumping of the active milieu (σ_a) and consequently the effect of optical bistability for the physical parameters (σ_b, ξ, ε, A_0).

DISCUSSION

The first observation is that there is no optical bistability, when the coefficient of saturation ξ = 1

It occurs in the opposite case (0<ξ<1); caused by a saturation different from the two active and absorber milieus.

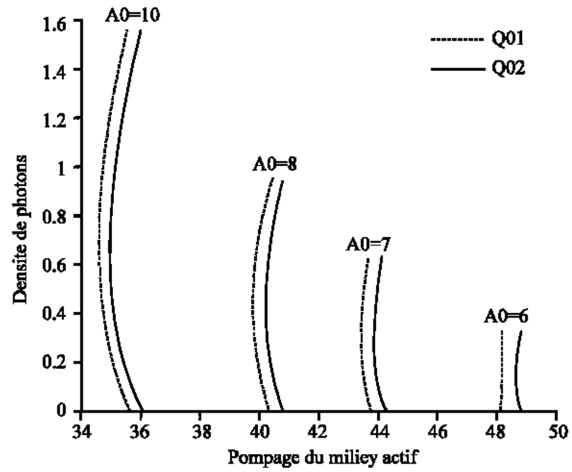

Fig. 12: Comparison between curves of the photons density Q_0 and Q_1 according to the pumping of the active milieu (σ_a) where A_0 varies

From these hysteresis curves and for the three modes considered, the influence of the various physical parameters of (LSA) on the interval of the optical bistability is summarized as follows:

- The reduction in the coefficient of saturation ξ, involves an increase in the interval of the effect of the optical bistability and in the extent of the photons density (Fig. 1 and Fig. 6) for respectively mode j = 0 and j = ±1. This is due to the difference in energy of the absorbing milieu which must correspond to the frequency of transition from the laser in the active milieu. In the initial state, the saturable absorber is in its maximum opacity (the non saturated state). The irradiation of the filter at the frequency of oscillation produces processes of resonance absorption and spontaneous emission. In this case, when the pumping of the absorbing milieu (σ_b) increases, saturation is maximum. The processes of relieving ensure the return at the fundamental state and the absorber returns to its initial non saturated state, once the irradiation ends.
- When the pumping of the absorber milieu σ_b increases, the interval of the optical bistability effect increases as well as the extended of the photons density depicted in Fig. 2 and 6 respectively for mode j = 0 and j = ±1.
- When the value of A_0 increases Fig. 4 and 8, we notice an increase in the interval of the effect of the optical bistability as well as the extent of the photons density. This is probably due to the growth of the interfered waves which generate an increase in the intensity in the cavity.

- The increase in the inhomogenous line broadening ε Fig. 5 and 7, involves a diminution in the interval of the optical bistability and extent of the photons density as well.

We also observe that the effect of the optical bistability appears with the central mode then with the modes j = ±1. The curves are identical for each parameter, but are shifted as for pumpings of the active milieu. This similarity is probably due to the fact that the modes are very close (they are three consecutive modes). The interval of the optical bistability is almost identical for the same physical parameter of the LSA for the j = 0 mode and the other modes j = 1 and j = -1 shown in the Fig. 9-12.

Thus, for the trimode study, there is no influence on the interval of the optical bistability of a mode compared to another.

CONCLUSION

It follows from the above results that in a Fabry-Pérot laser with saturable absorber and dominant inhomogeneous line broadening we can expect an optical bistability to appear under certain conditions. These conditions depend clearly on changes in the inhomogeneous line width and other parameters. A general rule applies here: An increase in the inhomogeneous broadening reduces the mode intensity and the OB range. The OB effects can be maintained in the pump energy is increased in the amplifying part of the laser[6].

These conclusions and the results should provide a particular description of the OB effect and of the operation of a Fabr-Perot trimode laser with a saturable absorber when the spontaneous emission is neglected.

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