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Recognising Fatigue Damaging Features in Variable Amplitude Loadings Using the *S*-Transform Approach

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Abstract: This study describes the use of the *S*-Transform to identify fatigue features in variable amplitude loadings. In normal situation, experimental fatigue loadings exhibit variable amplitude and nonstationary loading pattern, for which the traditional frequency domain analysis cannot provide accurate results. However, the time-localisation transform provides a promising solution. Since the *S*-transform if the simplification of the advanced time-frequency localization method, i.e., the wavelet transform, it is a good idea to study this transform in order to identify these fatigue features. The results obtained from this study showed that the high amplitude events were detected in the variable amplitude loading based on the difference pattern of the time-frequency localisation. These results were also compared to the plot of moving-damage using the Morrow's strain-life fatigue damage model. Based on the promising outcomes, finally, it is proposed that further study of the *S*-transform should be carried out in broader scope of fatigue life assessment.

Key words: Fatigue, Fatigue damaging features, moving-damage, *S*-transform, variable amplitude loading

INTRODUCTION

Many experimental fatigue loadings exhibit time-varying and random pattern, or nonstationary characteristics, which provide a challenge in signal analysis. One of the traditional analysis approach for this kind of loadings is related to their application in the frequency domain by means of the Fourier transform. However, this kind of analysis is not suitable for nonstationary signals, as it cannot provide any information of the spectrum changes with respect to time (Newland, 1993).

Realising the limitation of the Fourier transform using nonstationary signals, thus, the time-frequency transform is more suitable method. Recently, a new time-frequency domain analysis has also been introduced for analysing random pattern signal i.e., the *S*-transform (Stockwell *et al*., 1996). This transform is an extension of the ideas of the Continuous Wavelet Transforms (CWT). As continuity to the CWT development, the *S*-transform was introduced in this study in order to simplify the analysis of variable amplitude fatigue loadings.

The objective of this study is to observe the applicability of the *S*-transform in analysing variable amplitude fatigue strain loadings. In addition, this study can also show the potential of *S*-transform to the process in determining the occurrence of fatigue damaging features in the time-varying fatigue loadings.

MATERIALS AND METHODS

At initial stage of this study, it is a good idea to have a literature background of the signal analysis. Thus, information related to the Fourier transform, the Short-Time Fourier Transform (STFT) the wavelet transform and the S-transform is necessary. In addition, the background of related fatigue damage models is also needed.

Signal analysis of random loadings: In the current signal analysis technique, frequency analysis is performed in order to convert a time domain signal into the frequency domain. The results of a frequency analysis are most commonly presented by means of graph having frequency on the *x*-axis and amplitude on the *y*-axis. The algorithm that is used to split the time history into its constituent sinusoidal components is the Fourier transform. This transform was expressed as the summation of sinusoidal waves of varying frequency, amplitude and phase. The most common algorithm used for the Fourier transform is the Fast Fourier Transform (FFT) algorithm which was introduced in order to have a faster discrete Fourier transform calculation of the time series (Smith, 1999).

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Various FFT algorithms were developed and the algorithm which was introduced by Cooley and Tukey (1965) is the most commonly used because of its simplicity and fast computing time.

Using the Fourier transform the frequency components of an entire signal can be analysed, but it is not possible to locate at what point in time that a frequency component occurred or its duration. This is not problematic when a stationary signal is analysed. However, Fourier analysis is not suitable for nonstationary signals. If there is a time localisation due to a particular feature in a signal such as impulse, this will only contribute to the overall mean valued frequency distribution and feature location on the time axis is lost (Newland, 1993). Thus, the STFT was later developed in order to solve the irregularities behaviour of FFT using nonstationary signals.

The STFT is one of the methods of time-frequency analysis, which it aims to produce frequency information with a localisation in time. It provides information about when and at what frequencies a signal event occurs (Matlab User's Giude, 1998). The STFT approach assumes that if a time-varying signal is divided into several segments, each can be assumed stationary for analysis purposes. The Fourier transform is applied to each of the segments using a window function, which is typically nonzero in the analysed segment and is set to zero outside (Patacas, 2000). The most important parameter in the analysis is the window length, which is chosen to isolate the signal in time without any distortions.

The STFT was developed from the Fourier transform, and it is defined as

$$X(\tau,\omega)=\int_{-\infty}^{\infty}\omega(t-\tau)e^{-i\omega t}h(t)dt \tag{1}$$

where h (t) $e^{-i\omega t}$ is the Fourier transform of the windowed signal, $\boldsymbol{\omega}$ is the frequency and τ is the time position of the window (Chui, 1991). The result of this transformation is a number of spectra, each localised in a windowed segment.

While a useful tool, the STFT has a resolution problem, i.e. short windows provide good time resolution but poor frequency resolution. On the other hand, long windows provide good frequency resolution, but poor time resolution. Thus, the wavelet transform is one of the most recent solutions to overcome the shortcomings of STFT (Grossmann and Morlet, 1984).

A wavelet is a small wave with a signal energy concentrated in time, on the condition of admissibility condition. The wavelet transform is defined in the time-scale domain and is a significant tool for analysing time-localised features of a signal. It represents a windowing technique with variable-sized region. The harmonic form of the wavelet transform can be derived from the Fourier transform, which gave

$$X(\omega)=\int_{-\infty}^{\infty}h(t)\sin\left(\frac{t-d}{\tau}\right)dt \tag{2}$$

where *a* is a scale parameter which controls the frequency by dilating or scaling the time ***t***. The parameter *d* translates the basic sine wave up and down the time axis and it is known as the translation parameter. A wavelet transform can be classified as either a CWT or a Discrete Wavelet Transform (DWT) depending on the discretisation of the scale parameter of the analysing wavelet. The CWT is given by

$$W(\tau,d)=\int_{-\infty}^{\infty}h(t)\omega\left(t-\tau,d\right)dt \tag{3}$$

As continuity to the CWT development, the *S*-transform was introduced for the simplification to the data analysis. The *S*-transform is a time–frequency representation which an analysing function is the product of a fixed Fourier sinusoid with a scalable, translatable window (Pinnegar, 2006). It combines elements of the wavelet transform and the windowed Fourier transform. The *S*-transform can also be generalized to include windows that have frequency-dependent functional form, and frequency-dependent complex phase modulation, essentially giving the phase-shifted wavelets.

The *S*-transform, which is introduced in this correspondence, is based on a moving and scalable localising Gaussian window. The *S*-transform is unique which provides frequency-dependent resolution while maintaining a direct relationship with the Fourier spectrum. These advantages of the *S*-transform are due to the fact that the modulating sinusoids are fixed with respect to the time axis, whereas the localizing scalable Gaussian window dilates and translates.

Accordingly (Stockwell *et al*., 1996) the *S*-transform of a function h(t) in Eq. 3 is defined as the CWT with a specific mother wavelet multiplied by the phase factor, i.e.,

$$S(\tau,d)=e^{i2\pi f\tau}\,W(\tau,d) \tag{4}$$

where the mother wavelet is defined as

$$\omega(t,f)=\frac{|f|}{\sqrt{2\pi}}e^{-\frac{t^2f^2}{2}}e^{-i2\pi ft} \tag{5}$$

and f is the frequency of the samples. Written out explicitly, the *S*-transform is mathematically defined as

$$S(\tau,d)=\int_{-\infty}^{\infty}h(t)\frac{|f|}{\sqrt{2\pi}}e^{-\frac{(\tau-t)^2f^2}{2}}e^{-i2\pi ft}dt \tag{6}$$

If the *S*-transform is a representation of the local spectrum, a simple operation of averaging the local spectra over time can be used to give the Fourier spectrum, i.e.,

$$\int_{-\infty}^{\infty}S(\tau,f)d\tau=H(f) \tag{7}$$

Strain-life fatigue damage models: It is common that the service loads acquired on components of machines, vehicles, and structures are analysed for fatigue life using crack growth approaches. This approach is suitable for high capital valued structures. On the other hand, the ability to inspect for cracks and monitor their growth until a maximum allowable defect size is reached, enables a component or structure useful life to be extended beyond the original design life. However, it is not generally feasible for applying the crack inspection process for the inexpensive components that are made in large numbers because of the costs restriction. Examples of components which fall in this category are automobile engine, steering and suspension parts (Conle and Landgraf, 1983). For these components, the life prediction based on crack initiation is important in order to avoid the fatigue failure. Therefore, a fatigue life estimation based on the related strain-based approach is usually used in these cases (Dowling, 2006).

Current industrial practice for fatigue life prediction is to use the Palmgren-Miner linear damage rule (Palmgren, 1924; Miner, 1945). This linear damage rule is mathematically defined as

$$D=\sum_{i=1}^{n}\frac{N_i}{N_{fi}} \tag{8}$$

where D is the fatigue damage, n is the number of loading blocks, N_i is the number of applied cycles and N_{fi} is the number of constant amplitude cycles to failure. The component will fail when the value of D is more than unity or 1.0.

For the strain-based approach, this rule is normally with the related strain-life fatigue damage model. The first strain-life model which has been introduced for the life prediction method is the Coffin-Manson relationship (1954, 1965). This relationship is mathematically defined by

$$\varepsilon_a=\frac{\sigma'_f}{E}(2N_f)^b+\varepsilon'_f(2N_f)^c \tag{9}$$

where E is the material modulus of elasticity, ϵ_a is the true strain amplitude, $2N_f$ is the number of reversals to failure, σ'_f is the fatigue strength coefficient, ϵ' is the fatigue ductility coefficient, b is the fatigue strength exponent and c is the fatigue ductility exponent.

In some realistic cases, the situation of fatigue spectrum loading involves non-zero mean stresses or strain. The mean stress effect model which is applicable to be used for this study is the Morrow's strain-life model (1968). Mathematically, this model is defined as

$$\varepsilon_a=\frac{\sigma'_f}{E}\left(1-\frac{\sigma_m}{\sigma'_f}\right)(2N_f)^b+\varepsilon'_f(2N_f)^c \tag{10}$$

where σ_m is the mean stress of a particular cycle.

Despite of the fact that these models are widely used for fatigue life prediction, several limitations were also found in the analysis using variable amplitude loadings, which it may lead to the erroneous prediction. The fatigue damage is accurately calculated for Constant Amplitude (CA) loadings when using the Palmgren-Miner rule with these two models, especially for the research and industrial applications (Fatemi and Yang, 1998).

RESULTS AND DISCUSSION

Signals selection for the analysis: The accuracy of analysing variable amplitude fatigue strain loadings was evaluated by the application to two loading types.

The first category consists of an artificial loading which was defined to test the possible behaviour of the fatigue failure. The basic statistical properties of this loading was summarised in Table 1, while the time history and the frequency spectrum based on the Power Spectral Density (PSD) distribution were shown in Fig. 1. For this case, the PSD plot shows the vibrational energy distribution of a signal in the frequency domain.

The T1 signal, as illustrated in Fig. 1a, was defined to have 16,000 data points which were sampled at 400 Hz. The logic of producing T1 was to verify the *S*-transform ability to identify the sections of fatigue damage occurrence, especially with a signal containing large transients in a small amplitude background. T1 consists of a combination of sinusoidal and random segments of different amplitude or frequency. This loading was intentionally defined to be a mixture of both high amplitude bump events and low amplitude harmonic backgrounds.

Table 1:Global statistical parameters of the loading used for this study

Loading name	Global statistical parameters No. of data points	Loading length [s]	Mean [$\mu\epsilon$]	r.m.s.[$\mu\epsilon$]	Skewness	Kurtosis	Crest factor
T1	16000	40.0	0.0	1.5	0.1	7.4	3.9
T2	23000	46.0	15.0	16.7	-0.1	3.4	3.4

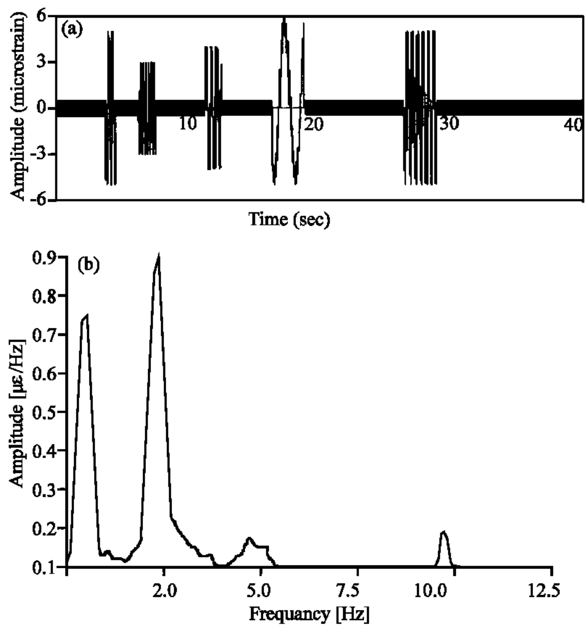


Fig. 1: The plots of T1: (a) Time histories, (b) Frequency spectrum of PSD

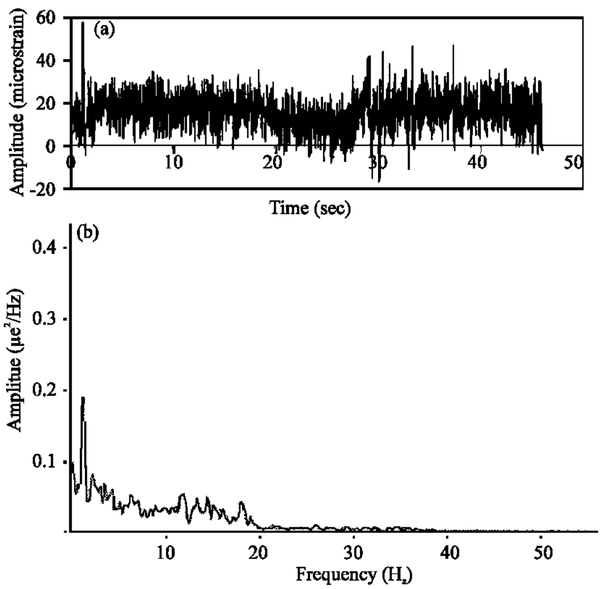


Fig. 2: The plots of T2: (a) Time histories, (b) Frequency spectrum of PSD

The second loading type contains the time histories which was measured on the suspension components of an automobile. The global statistical properties of the loadings are presented in Table 1, while both the time histories and the frequency spectrum were presented in Fig. 2. T2 (Fig. 2a) is the strain loading which was measured on a lower suspension arm of an automobile travelling at 34 km h^{-1} over a pavé test track. It was sampled at 500 Hz for a total of 23,000 data points, producing a record length of 46 sec and it has a tensile mean loading of 15.0 microstrain. Figure 3a and 3b showed a section of the pavé test track used to record this signal and the strain-gauge location on the lower suspension arm during the test, respectively.

Global signal statistical analysis: From the global statistical parameters in Table 1, the kurtosis values for both signals indicate an interesting discussion towards the behaviour of random pattern signals. Kurtosis, which is the signal 4th statistical moment, is the global signal statistic which is highly sensitive to the spikiness of the data. For discrete data sets the kurtosis value is defined as

$$K = \frac{1}{n(\text{r.m.s.})^4} \sum_{j=1}^{n} \left(x_j - \bar{x}\right)^4 \qquad (11)$$

where is the instantaneous value, is the mean value of a signal, r.m.s. is the root-mean-square value (represents the amount of the time-domain vibrational energy of a signal) and n is the number of values in the sampled sequence. For a Gaussian distribution the kurtosis value is approximately 3.0. Higher kurtosis values indicate the presence of more extreme values in a Gaussian distribution, showing the behaviour of a nonstationary signal. The kurtosis value is used in engineering for detection of fault symptoms because of its sensitivity to high amplitude events (Qu and He, 1986).

For both loadings, the kurtosis values were more than 3.0 i.e., 7.4 for T1 and 3.4 for T2. It is suggested that both loadings have a nonstationary behaviour which are suitable to be investigated using the time-frequency signal analysis. Since nonstationary loadings are common in the case of fatigue analysis, signal modelling has often been used in the time-frequency domain (by means of the wavelet transform) (Abdullah *et al.*, 2006, 2007; Abdullah and Zaharm, 2006) due to its efficiency for the purpose of loading simulation. An alternative approach using the

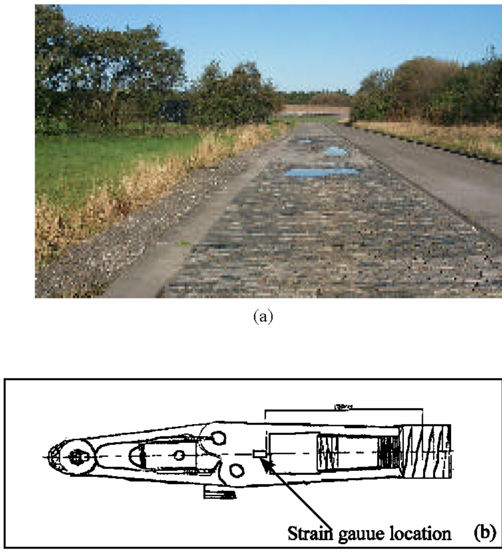


Fig. 3: Road surface and a lower suspension arm used in the measurement process of T2: (a) Pavé test track used for the test, (b) Strain gauge positions on a lower suspension arm for measuring.

S-transform is useful to be performed in order to observe any possible signal variation in fatigue loadings.

Relation between the fatigue damage criterion and the *S*-transform: The results of analysis were presented in two forms of graph, i.e., the moving-damage based on Morrow's strain-life fatigue damage model, and the time-frequency localization mapping by means of the *S*-transform.

Figure 4a shows the pattern of the moving-damage that was calculated using the Morrow's strain-life fatigue model. The purpose of this analysis was to show the loading segment which has high damaging pattern. From this figure, it has been shown that there were 5 sections of fatigue damaging occurrence. Comparing this plot with the original T1 loading (refer to Fig. 1a), the fatigue damage occurrences were located within the same period of high amplitude events of T1.

This T1 loading was then analysed by means of time-frequency localisation using the *S*-transform, as the result were shown in Fig. 4b. This figure shows the pattern of high amplitude events that were detected at the same position of the original T1, for which these events were distributed between 0.5 to 5 Hz. In order to match up this result with the moving-damage analysis, it was shown that the time-frequency localisation mapping of this signal by means of the *S*-transform (simulated in the Matlab® environment) occurred at the similar position to those fatigue damaging events.

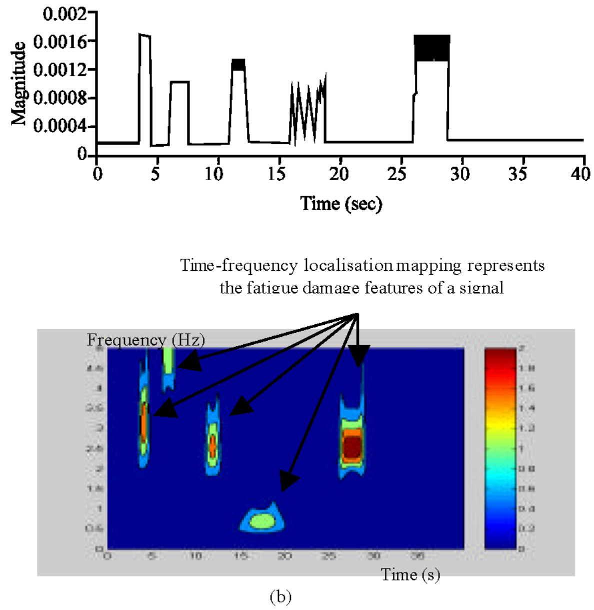
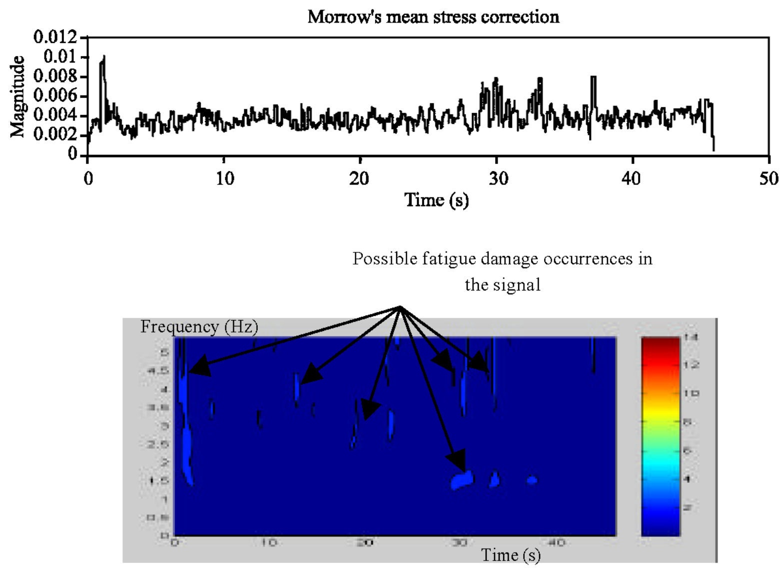


Fig. 4: The pattern of high amplitude events produced by the *S*-transform for T1

Using the experimental data set of T2 (Fig. 5) the moving-damage pattern (Fig. 5a) was observed and compared to the time-frequency localisation mapping (Fig. 5b). In Fig. 5b, several patterns of high-energy localisation mapping were observed in the time-frequency plot. Comparing to the information extracted from the artificial signal of Fig. 4, the experimental measured fatigue loadings (T2) showed several occurrences of fatigue damage features.

Since T2 exhibit almost a random pattern with several transient effects (the loading is nonstationary based on the statistical values in Table 1) the pattern of low-energy localisation mapping has been observed in the most of the time-frequency plot. From the findings of Fig. 5, it is suggested there is a possibility that the fatigue damage features can be accurately detected using the *S*-transform. However, further investigation should be performed in order to determine the change-point detection of where are the starts and end positions of the identified fatigue damaging features in a variable amplitude fatigue loadings.

The results of this study demonstrate the capability of this transform to identify fatigue damaging features in a variable amplitude fatigue strain loading. The simulation result showed the significant high amplitude events have been detected from the colour differences of this time-frequency localisation technique. Finally, it is suggested that the *S*-transform is one of the suitable methods to be



(b)

Fig. 5: The pattern of high amplitude events produced by the *S*-transform for T2

used for the fatigue features detection. Further investigation should also be carried out in order to discover meaningful results.

CONCLUSION

The objective of this study is to investigate the applicability of the *S*-transform to determine fatigue damaging events in a variable amplitude fatigue loading. The simulation results showed that the high amplitude events were detected in the variable amplitude loading based on the difference pattern of the time-frequency localisation. Thus, it is suggested that the *S*-transform is able to detect fatigue damaging events from a random fatigue loading. With the findings of this study, it is suggested that further developments in the S-transform will find applications in a broad range of disciplines, particularly in the fatigue life analysis research.

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