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Robust Control of a Dam River System with a Modified Smith Predictor

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Abstract: These last years, there is a growing interest for developing tools to the robust control of systems with delays, among these tools there is the Smith predictor the structure of this controller was suggested for the control of the industrial processes with delays, like: Steel factories, transmission lines… etc, but the idea can be generalize for all the processes which have long delays among which: the hydraulic systems with free arias. This study proposes to compare the contribution in the control robustness between the traditional Smith predictor and a modified Smith predictor based in the use of the H8 norm. The satisfaction of the robust and performances condition will be showed and the robustness margins will be compared through an application of the two techniques in the control of a dam river system and also show what the modified Smith predictor can bring in the point of view of stability and performances.

Key words: System with delay, smith predictor, IMC, hinfinity, robustness, pade approximation

INTRODUCTION

The water resources becomes increasingly rare in the world, this is for what it is extremely important to find systems in order to optimize their management, among the techniques used for that, there are the dams upstream the rivers in order to control the use of water, our contribution consists to find a controller who maintain the downstream flow near close to a set point value taking of account the randomized consummation of water from the various users (in particular farmers: indeed it is known that the irrigation is the principal rivers water consumer).

Description of the dam river system: For our study, we will consider a simplified system represented in Fig. 1 with a dam upstream the river and a measuring station to measure the water flow downstream and a pumping station just upstream, the consumers (farmers) pumping stations are distributed along the river[1].

To simplify, it is supposed that the pumping stations can be gathered in only one station at the end of the right part of the river, the rate of flow (Qout) represents the request of the farmers, it is not measurable (the farmers can pump the quantity of water which they want as they want: on-demand system of management) and thus considered as a disturbance which must be reject.

The controlled variable is the rate of flow downstream at the end of the river (Qend), this rate must be kept in an interval defined for ecological and hygienic reasons instead of farmers consummation:

$$0.5\,m3/s < Qend < 5\,m3/S \tag{1}$$

Thus our objective is to synthesize a controller to keep the flow downstream close to a set point flow instead of the not measured water consummation of the users, in other words the role of the controller is to act on the flow upstream “Qup” with an aim of maintaining the flow downstream “Qend” as constant as possible in the considered interval, which means that the controller must reject or attenuate the immeasurable disturbances “Qout”.

The system is modelled by a transfer function of second order plus a delay:[1]

$$G_0(s) = \frac{\exp(-s\tau)}{(1+sK_1)(1+sK_2)} \tag{2}$$

With: K_1 = 9995.1+3310.5j; K_2=9995.1-3310.5j; τ = 24463 sec.

After the numerical application we obtain:

$$G_0(s) = \frac{\exp(-24463s)}{(1+1.999.10^4.s+1.109.10^8.s^2)} \tag{3}$$

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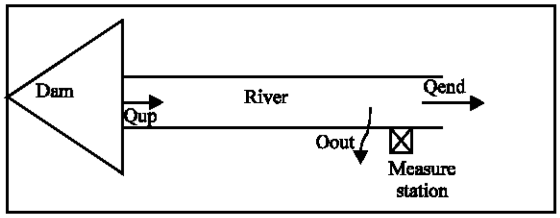

Fig. 1: the simplified "dam river" system

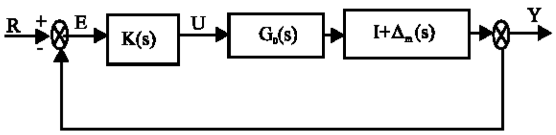

Fig. 2: The perturbed closed loop system

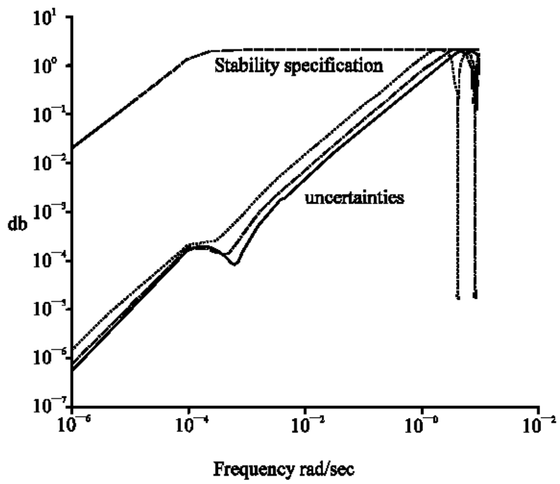

Fig. 3: Frequency domain representation of the norms of the stability specification and the output multiplicative uncertainties

Description of the uncertainties acting on the system: The system to control is defined by a nominal model and a field of uncertain model, in our system uncertainties are due to the various rates of flow (Qout) which represent the request of the farmers: Indeed they can pump water when they want with the quantity that they want. To represent these uncertainties we use the output multiplicative form: Fig. 2.

With: R: set point; E : error; K (S): controller; $\Delta_m(s)$: output multiplicative uncertainty, Y: it correspond to the rate of the downstream flow Qend; U: control variable: it correspond to the rate of the upstream flow Qup.

The considered uncertainty include also the delay variations and the dynamic variations, which are due to the system nonlinearities.

For a reference flow: $Q_0 \in [Q\ min, Q\ max]$ with: Q min = 0.5 m^3/s and Q max = 5 m^3/s, the transfer function of the real system G(s) is then:

$$G(s) = [1 + ?\,m(s)]\, G_0(s) \tag{4}$$

Thus:

$$|\Delta m(s)| = \left| \frac{G(s).G_0(s)}{G_0(s)} \right| \tag{5}$$

With: G0 (S): transfer function of the nominal system; G (S): transfer function of the real system.

Quantification of the uncertainties: Uncertainties acting on the system must be raised or limited by the stability specification (W1 (S)) according to the following sformula[2]:

$$|\Delta m(s)| \le W1(jw), \forall w \tag{6}$$

With: W1 (S) (also called the stability specification): is the weighting filter, it defines the upper limit of the uncertainties acting in the system.

It means that the norm of W1 (S) must be higher than the norm of Δm (S), this enables us to guarantee the stability of all uncertain systems who's those uncertainty norm is lower than the stability specification norm.

Determination of the stability specification: We can approximate the maximum of the uncertainties acting on the system by a weighting filter (stability specification) W1 (S) given by the following function[1]:

$$W1(s) = \frac{2s}{(1000s + 1)} \tag{7}$$

Figure 3 represents the norm of the stability specification with the norms of various uncertainties (this one can result in nominal system parameters variations), it is noted that the norm of the stability specification W1 (S) is greater than the norms of all the uncertainties caused by the other disturbed modes.

Classical smith predictor controller: The idea of the Smith Predictor is to synthesize a controller for the process without the pure delay and in the second time, to calculate a corrector adapted to the process with delay, starting from the controller calculated before.

Thus, calculations are much simpler because the pure delay can introduce a significant number of poles in the origin and thus raise the order of the nominal transfer function[3].

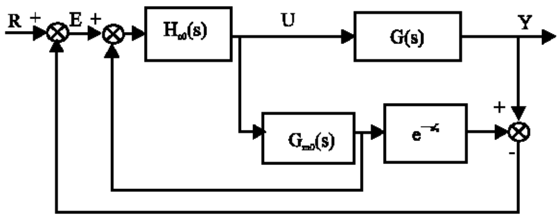

Fig. 4a: Classical Smith Predictor control

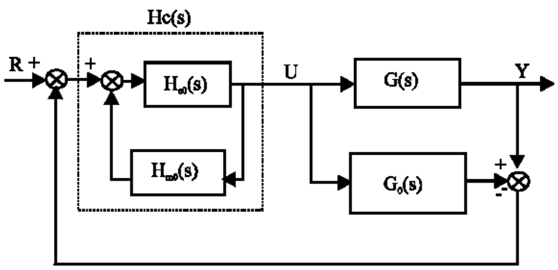

Fig. 4b: System in the "IMC" form

The main advantage of the Smith predictor controller is that the intrinsic delay (that we cannot compensate) occurs out of the loop Fig. 4a, in other hand the Smith predictor method is an easy way to synthesize a correctors for the systems with high pure delays (frequent case in industry)[4], it is applied to the systems put in the IMC (internal model control) form Fig. 4b.

With: H_{c0} (S): the Smith predictor controller, G_0(S): nominal system transfer function, G_{m0} (S): nominal system without delay transfer function, Hc (S): closed loop controller, G(S): real system transfer function.

To design the Smith Predictor Controller for a system with delay, we splits the transfer function of the nominal system G_0 (S) in two terms: [5]

$$G_0(S) = G_{a0}(S).G_{mo}(S) \tag{8}$$

With: G_{a0} (S): pure delay, G_{m0} (S): nominal transfer function without delay.

We can deduce analytically the controller Hc (S) (Fig. 4b) by minimizing the H2 norm of the error E for all the disturbances W, it is given by the following formula:[5,6].

$$Hc(s) = \frac{1}{(1+\lambda S)^n \, G_{mo}(s)} \tag{9}$$

With: n is an integer selected in order to make Hc (S) proper and $\lambda$is the design parameter, it is selected in such way that the robustness stability condition is satisfied (we will further show how to calculate it).

Modified smith predictor controller: This method is based on the optimal control theory: it consists in designing a PID controller by minimizing the H8 norm of the error for a system presented in the "IMC" form, the interesting characteristic of this method is that the controller is calculated analytically in addition to the popularity of the PID controller who can be allotted particularly to his performances and his simplicity[7].
According to Fig. 4b, we can write:

$$Hc(s) = \frac{Hc0(s)}{1 + Hc0(s).Gm_0(s)} \tag{10}$$

$$H_{c0}(s) = \frac{H_c(s)}{1 - G_{mo}(s).H_c(s)} \tag{11}$$

The main objective of the automatic control is to let the output of the system follow the reference (i.e.: to minimize the error E), let see the Fig. 5:

With: D: disturbance, yr: Reference, y: Output, according to this figure we can write the following relations:

$$y = GK(1+GK)^{-1} y_r + (1+GK)^{-1} d \tag{12}$$

$$e = y_r - y = (1+GK)^{-1}(d - y_r) \tag{13}$$

$$\frac{e}{d - y_r} = (1+GK)^{-1} = S \tag{14}$$

S is called the sensitivity, therefore: minimizing the error tends to minimize the sensitivity S and that will give place to an optimal control problem, suppose that the optimal performances are ensured by the minimization of the H8 norm of the sensitivity, i.e.: min|W3 (S).S (S) $|_\infty$, where W3 (S) is the performances specification[8].

Application in the dam river system: The synthesis of an optimal H8 PID controller for the "dam river" system will be made on a more adapted model, i.e: we will approximate the delay of our model by a simple 1st order rational function, there exists several methods of delays approximation among which: Pade approximation[9].

The Pade method consists of approximate the delays by rational fractions whose development in Taylor series in the vicinity of the origin coincides with that of a given function.

In our study the delay will be approximated according to the first order Pade formula like following:

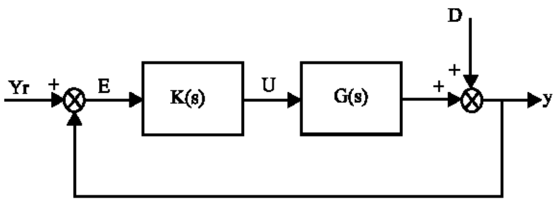

Fig. 5: Closed loop system

$$e^{-\tau s} = \frac{1-\frac{\tau}{2}s}{1+\frac{\tau}{2}s} \quad (15)$$

Then the approximated model becomes:

$$G(s) = \frac{(1-\frac{\tau}{2}s)}{(1+\frac{\tau}{2}s)(1+sK_1)(1+sK_2)} \quad (16)$$

It is what we will consider as the nominal system and it will be used to synthesize the PID H∞ controller. The error introduced by the Pade approximation is included in the output multiplicative uncertainty. An analytical procedure of design is developed for the given system, indeed according to the maximum modulus theorem, there is a fundamental result concerning the complex functions: $|W3(s).S(s)|$ does not reach its maximum values at an interior point of the open right half plane[5].

In addition, G (S) has one zero ($s = \frac{2}{\tau}$) in the open right half plane, therefore, Hc(S) must be selected in such way that the following inequality relation holds:

$$\|W3(s).(1-G(s).Hc(s)\|\infty \geq \left|W3\left(\frac{2}{\tau}\right)\right| \quad (17)$$

Consequently we have:

$$\min\|W3(s).S(s)\|\infty = \min\|W3(s).(1-G(s).Hc(s))\| \quad (18)$$

Let: $W3(s) = \frac{1}{s}$ for a unit step entry yr, thus to compensate for the pole in the origin introduces by W3 (S), a constraint will be imposed in the procedure of design:

$$\lim_{\to\infty}(1-G(s).Hc(s)) = 0 \quad (19)$$

With this constraint, the optimal controller Hcopt(S) is obtained as follows:

$$Hcopt(S) = (1+\frac{\tau}{2}S)(1+k_1 s)(1+K_2 s) \quad (20)$$

Finally, in order to make Hcopt proper, we will use the following low-pass filter:

$$f(s) = \frac{1}{(1+\lambda s)^3} \quad (21)$$

Thus:

$$Hc(s) = H_{copt}(s).f(s) = \frac{(1+K_1 s)(1+K_2 s)(1+\frac{\tau}{2}s)}{(1+\lambda s)^3} \quad (22)$$

The Optimal H8 PID controller is given analytically by deducing from the Eq. 9 the controller Hc0 (S) and by comparing it with a conventional PID controller given by:

$$Hpid(s) = K_c(1+\frac{1}{Tis}+Tds)\frac{1}{Tis+1} \quad (23)$$

Then we find the new H8 PID controller parameters like following:

$$Tf = \frac{\lambda^2}{2\lambda+\tau}, Ti = K1+K2, Td = \frac{K1.K2}{Ti}, Kc = \frac{Ti}{2\lambda+\tau}$$

Evaluation of the system robustness

Robust stability and performances conditions: Using the "IMC" representation, the robust stability condition will be: "The system represented by Fig. 4b is stable instead of all the output multiplicative uncertainties described in the Eq. 15" if:[4]

- nominal system is stable.

$$|Ty(jw)| < |W1(jw)^{-1}| \;\forall w. \quad (24)$$

With: Ty is the transfer from the reference R to the output Y.
And the robust performances condition is:

$$|S(jw)| < \frac{1}{|W3(jw)|} \;\forall w \quad (25)$$

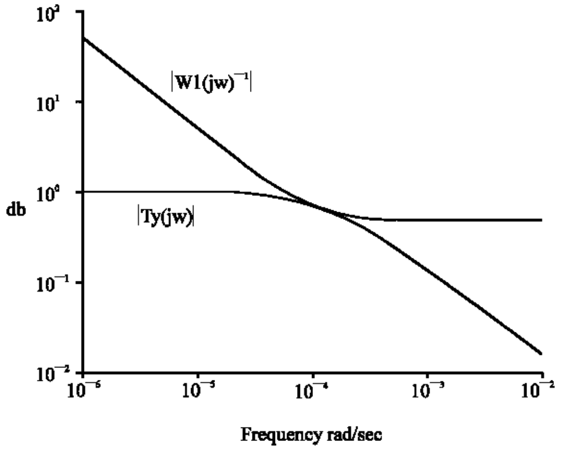

Fig. 6a: Stability and performances robustness condition (Smith predictor)

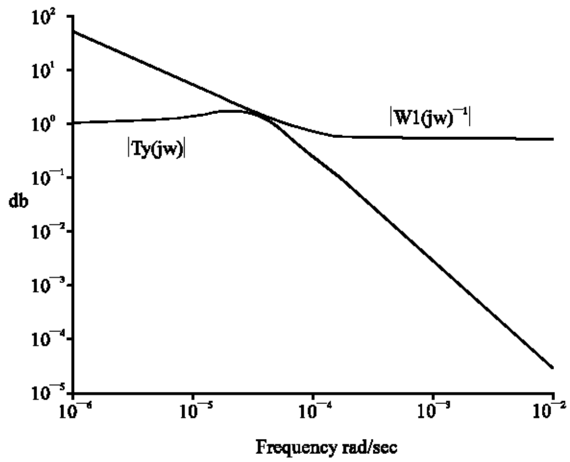

Fig. 6b: Stability and performances robustness condition (Modified Smith predictor)

In let us combine the Eq. 24 and 25, we find the robust stability and performances condition:

$$|Ty_0(jw)|WI(jw)+|[1-Ty_0(jw)]W3(jw)|<1 \quad (26)$$

$$\frac{|Ty_0(jw)|}{1-|[1-Ty_0(jw)]W3(jw)|}<WI(jw)^{-1} \quad (27)$$

Robustness margin: Another way to evaluate the system robustness instead of the uncertainties: is to determine the gain and phase margins (traditional robustness evaluation)[4], these measurements are not well adapted to the robustness evaluation in the case of the delay variations, the delay and modulus margins are more useful, these margins offer a simple method to evaluate the controlled systems robustness within an acceptable variations in the gain, phase and delay[8].

Modulus margin: Let the transfer function of the open loop system oh the Fig. 5:

$$L(S)=K(S).G(S) \quad (28)$$

The modulus margin (Mm): is the minimal distance between the function L and the point (- 1,0) in the Nyquist plan[3]

$$M_m=\inf\{|1+L(jw)|, w\in R\} \quad (29)$$

$$M_m=|1+L(jw)|_{\min}=|S_y(jw)^{-1}|=\frac{1}{\|S_y\|_\infty}\} \quad (30)$$

Delay margin: The delay margin is the maximum of time delay τ which allows the closed loop of all the disturbed processes to remain stable[3].

$$Md=\frac{Q}{W_{c\tau}} \quad (31)$$

Where: Q is the phase margin (rad) and Wcτ is the frequency of the intersection of L with the unit circle in the Nyquist plan (rad/sec).

How to calculate λ parameter: The following algorithm allows us to find the optimal λ parameter[3]:

- To check the Eq. 27 for a given λvalue
- If the Eq. 27 was not satisfied, decrease λ.
- Repeat 1 and 2 until: the Eq. 27 is satisfied.

If no value of λleads to the satisfaction of the Eq. 27, the robust performances condition must be lowered.

Application: The dam river system is of 20Km length, the farmer's water consumption is supposed between 0.5 and 5 m^3/sec. If we selected the performances specification W3 such as: $W3(jw)=\frac{1}{MP}$, with MP = 3, i.e: the maximum of the sensitivity modulus |S| remains lower than 3 for all the disturbed models.
The initial value of λ is given by[1]:

$$\lambda_0=\left[\left(\frac{MP+1}{MP-1}\right)^2-1\right]^{1/2} / W' \quad (32)$$

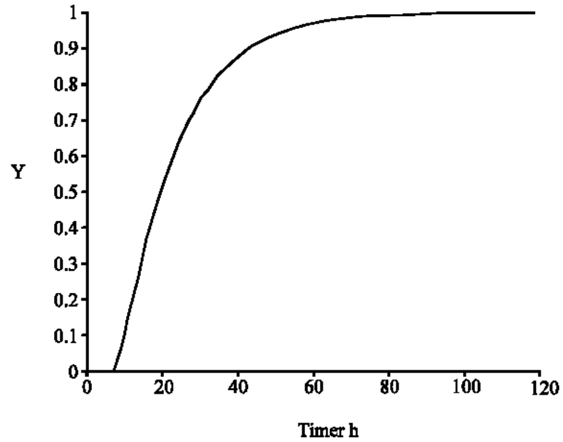

Fig. 7a: Nominal system step response (Smith predictor)

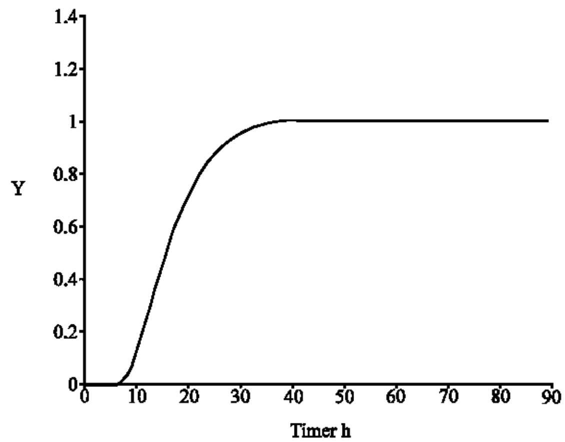

Fig. 7b: Nominal system step response (Modified Smith predictor)

With: W' is the frequency for which W3 (jW') = 1

The obtained optimal value of λis: $\lambda = 18577sec = 0.59\lambda_0$, it's the value for which the robust stability and performances condition is satisfied.

RESULTS

Frequency and time responses: The time and frequency responses of the dam river system controlled by both the traditional and modified Smith predictor are illustrated by Fig. 6(a,b), 7 (a,b): They show the satisfaction of the robust stability and performances condition for the two systems. The step responses of the nominal system (with delay) show no overshoot and a fast time response for the two systems (faster for system controlled by the modified Smith predictor).

Robustness margins: The following comparative table shows the various robustness margins for the system controlled by both the traditional and modified Smith predictor. The robustness margins are overall better for the system controlled by the modified Smith predictor.

CONCLUSION

The Smith Predictor is a powerful tool to control systems with delay. In the stage of the adjustment of the Controller, the delay is not taken into account but at the implementation, the delay effect is eliminated by using the retroaction effect (feedback) of the prediction from the variable to be regulated, this prediction requires a good model of the system to be regulated, the error minimization leads to obtain good performances and stability margins, indeed: the feedback of the error prediction leads to take account of the modelling errors and noises which act on the system.

Another method also based on the same structure of the Smith Predictor uses a model of the system whose delays is approximated by the Pade method, allows improved stability and performances margins.

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