Journal of Engineering and Applied Sciences 2 (5): 904-908, 2007

Tridimentional Analyze of the Maximum Load of a Superficial Foundation with Theory of the Analysis Limit

[1]M. Meksaouine, [2]G. Boukhatem and [3]M.L. Samai
[1]Département d'Hydraulique, Université Badji Mokhtar, Annaba, Algeria
[2]Département de Génie Civil, Université El hadaek, Skikda
[3]Département de Génie Civil, Université Mentouri, Constantine

Abstract: This study is about a problem of interaction soil-structures concerning the determination of the structural capacity (or load limit) of superficial foundation by the method of the limit analysis; and more precisely to develop the approach kinematics in the case of the COULOMB criteria. The used theoretical tool is the kinematics' theorem of the theory of the limit analysis and more exactly a superior bound of the solution in the case of the criterion of COULOMB and the Mathcad software is used to obtain numeric results or these last ones indicate that the presentations made by the limit analysis it 3D supply significant limitations on the fields of speeds (a minimum exists).

Key words: Foundation, mechanics of soils, limit analysis, coulomb's criteria

INTRODUCTION

We suggest contributing modestly to the enrichment of the scientific and technological knowledge which results from several centuries on the superficial foundations, benefiting from a tool which up to here in summer little exploited (run) in this domain; the theory of the analysis limits using the kinematics' theorem which can deal with very complex problems in the space of a massif of ground.

This research is dedicated to a brief presentation of the method of the calculation in the break (or analysis limits) and in the calculation of the maximum load of the superficial foundations.

We propose a model in three dimensions with the use of the kinematics' theorem of the analysis limit by considering that the foundation puts the ground in abutment (Léonard, 1967).

The analysis of the equations and the results is treated (handled) by the software Mathcad.

CALCULATION IN THE BREAK

The calculation has the break has for object to encircle a domain of potentially bearable loads (responsibilities) for a structure the load of which depends on a number finished by parameter: It does not enter its research applications to know the state of the constraints and the deformations of the ground about a given level of requests. Because it aim only at framing (supervising) a domain, it uses a criterion of rupture and not a law of behavior. This is of a big convenience seen the difficulty elaborating such a law enough realistic.

The stability of a configuration can be established by the calculation in the break only for austere hypotheses on the material (the principle of Hill's normality) which we shall not make (Hill, 1950).

In the space of loads the domain of these loads is convex and contains the load anybody. All load situated except the domain will entail the ruin of the system. But a load in the domain can be considered only as potentially bearable.

For the frame of the edges of the domain, both methods approach by the inside and approach around are additional.

The first one, said static approach, is made in the sense of the security. It consists in showing a statically acceptable field of constraints and in expressing that its satisfies the criterion of rupture. Some number of studies was made on this subject both for the purely coherent circles (Davis *et al*., 1980) and for the frottant circles (Mulhauss, 1985). The sharpness of the approach is connected to the intuition of the researcher and to its capacity to realize the best compromise between a realistic field and practicable analytical calculations. The works previously quoted were the object of experimental validation and showed that it approached suitably the reality.

The second approach is said kinematics. Among the others, the previous authors applied her to the cavities of

Corresponding Author: M. Meksaouine, Département d'Hydraulique, Université Badji Mokhtar, Annaba, Algeria

the purely coherent grounds. Indeed, the criterion of break in simplified case (Criterion of COULOMB), because only intervenes the cohesion and the angle of friction and because the rise of the loose power is explicit about is the envisaged field of speed. In what concerns the frottant circles, we were able to notice the absence of works.

We thus suggest developing such an approach and seeing in which measure it is exploitable.

STATEMENT OF THE METHOD AND THE CHOICE OF THE CONFIGURATION

Kinematic approach for a coherent rubbing environment (middle): The kinematics' approach (superior border) of the calculation in the break is based on the conception of virtual mechanisms of rupture. Inspired by the forms of collapse observed on reality. It allows building an approach by excess of the domain of stability. The necessary condition of stability is obtained for every mechanism by writing that the power *Pext* of the outside efforts in the considered mechanisms cannot exceed the maximal resistant power corresponding P_{max}^{res}.

$$P_{ext} \leq P_{max}^{res}$$

The quantity P_{max}^{res} is a functional of the envisaged virtual mechanism and it depends only on the material. It represents physically the maximal power which the massif is capable of developing in the envisaged mechanism, because of its capacities of resistances.

Mechanisms of break ***According to coulomb*** **3D:** The velocity is tilted by an angle φ (Fig. 1).

Velocity: From similar triangle we shall have:

$$V_2 = V_0 \cdot \frac{\sin\left(\frac{\pi}{2} - \alpha + \varphi\right)}{\sin(\alpha - 2\varphi)}; \quad V_{12} = V_0 \cdot \frac{\sin\left(\frac{\pi}{2} + \varphi\right)}{\sin(\alpha - 2\varphi)}$$

$$V_3 = V_0 \cdot \frac{\sin\left(\frac{\pi}{2} - \alpha + \varphi\right)}{\sin(\alpha - 2\varphi)} \cdot \frac{\sin(\beta + 2\varphi)}{\sin\left[\pi - (\Omega + \beta + 2\varphi)\right]};$$

$$V_{23} = V_0 \cdot \frac{\sin\left(\frac{\pi}{2} - \alpha + \varphi\right)}{\sin(\alpha - 2\varphi)} \cdot \frac{\sin\Omega}{\sin\left[\pi - (\Omega + \beta + 2\varphi)\right]}$$

Lateral surfaces:

$$S_I = \pi \cdot \frac{b^2}{2} \cdot \frac{\sin\alpha}{\sin 2\alpha}; \quad \text{base}S_{II} = \pi \cdot \left(\frac{b \cdot \sin\alpha \cdot \sin(\alpha + \beta)}{\sin 2\alpha \cdot \sin\beta}\right)^2$$

$$S_{II} = \pi \cdot b^2 . \left[\frac{\sin\alpha \cdot \sin(\alpha + \beta)}{\sin 2\alpha \cdot \sin\beta}\right] \cdot \left[\frac{\sin^2\alpha}{\sin 2\alpha \cdot \sin\beta} + \frac{1}{2\cos\beta}\right] - \pi \cdot \frac{b^2}{4\cos\beta} - \pi \frac{b^2 \sin\alpha}{2\sin 2\alpha};$$

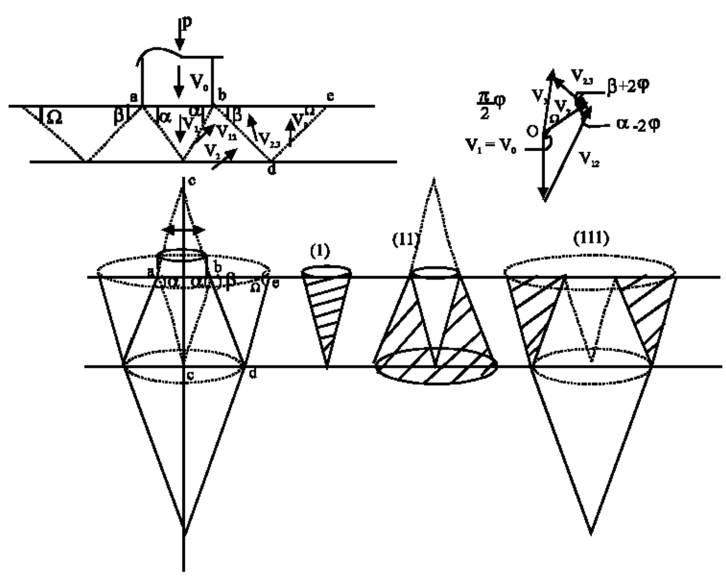

Fig. 1: Mécanisme of rupture of soil under superficial foundation and associated diagram of velocity, in 3D according to Coulomb

$$S_{III} = \pi \cdot b^2\left(\frac{1}{2}+\frac{\sin^2\alpha}{\sin 2\alpha}\cdot\left(\frac{1}{tg\beta}+\frac{1}{tg\Omega}\right)\right)\cdot$$
$$\left(\frac{\sin^2\alpha}{\sin 2\alpha\cdot\sin\Omega}+\frac{\sin\alpha\cdot\sin(\alpha+\beta)}{\sin 2\alpha\cdot\sin\beta\cdot\cos\Omega}\right)-$$
$$\pi.b^2\left[\frac{\sin\alpha\cdot\sin(\alpha+\beta)}{\sin 2\alpha\cdot\sin\beta}\right]^2.\frac{1}{\cos\Omega}-S_I-S_{II}$$

Equation of the work: P, W1, W2,W3 are outside forces

$$\begin{cases}\text{Work of p: } P\cdot V_0\cdot\cos 0 = P\cdot V_0\\ \text{Work of W1: } W_1\cdot V_1\\ \text{Work of W2: } 2W_2\cdot V_2\cdot\sin\varphi\\ \text{Work of W3: } 2W_3\cdot V_3\cdot\sin(\Omega+\varphi)\end{cases}$$

Dissipation of energy interns due in the cohesion:

$$\begin{cases}c\cdot S_I\cdot V_{12}\cdot\cos(2\pi-\varphi)\\ c\cdot baseS_{II}\cdot V_2\cos\varphi\\ c\cdot S_{II}\cdot V_{23}\cdot\cos(2\pi-\varphi)\\ c\cdot S_{111}V_3\cdot\cos\varphi\end{cases}$$

Stiff foundation no movement lateral, the dissipation of energy internal due in the internal friction is invalid (useless) at the level of the interface ground-foundation. Thus, the equation of the work spells:

$$(P+W_1)\cdot V_0-2W_2\cdot V_2\cdot\sin\varphi-2W_3\cdot V_3\cdot\sin(\Omega+\varphi)=$$
$$C\cdot\cos\varphi\cdot(S_I\cdot V_{12}+baseS_{II}\cdot V_2+S_{II}\cdot V_{23}+S_{III}\cdot V_3)$$

By substituting the velocity and the lengths in the equation of the work we shall have:

$$2P=-W_1+2\frac{\sin\left(\frac{\pi}{2}-\alpha+\varphi\right)}{\sin(\alpha-2\varphi)}\cdot\left[2W_2\cdot\sin\varphi+C.\cos\varphi.\pi.\left(\frac{b.\sin\alpha.\sin(\alpha+\beta)}{\sin 2\alpha.\sin\beta}\right)^2\right]$$
$$\frac{2.\sin\left(\frac{\pi}{2}-\alpha+\varphi\right).\sin(\beta+2\varphi)}{\sin(\alpha-2\varphi).\sin\left[\pi-(\Omega+\beta+2\varphi)\right]}...2W_3.\sin(\Omega+\varphi)+C.\cos\varphi.\pi.b^2.$$

$$\left\{\left(\frac{1}{2}+\frac{\sin^2\alpha}{\sin 2\alpha}\cdot\left(\frac{1}{tg\beta}+\frac{1}{tg\Omega}\right)\right)\cdot\left(\frac{\sin^2\alpha}{\sin 2\alpha.\sin\Omega}+\frac{\sin\alpha.\sin(\alpha+\beta)}{\sin 2\alpha.\sin\beta.\cos\Omega}\right)-\left(\frac{\sin\alpha.\sin(\alpha+\beta)}{\sin 2\alpha.\sin\beta}\right)^2\cdot\frac{1}{\cos\Omega}-\right.+$$
$$\left.\frac{\sin\alpha.\sin(\alpha+\beta)}{\sin 2\alpha.\sin\beta}\cdot\left(\frac{\sin^2\alpha}{\sin 2\alpha.\sin\beta}+\frac{1}{2.\cos\beta}\right)+\frac{1}{4.\cos\beta}\right\}$$
$$+\frac{2.\sin\left(\frac{\pi}{2}+\varphi\right)}{\sin(\alpha-2\varphi)}\cdot\left(C.\cos\varphi\cdot\frac{\pi.b^2.\sin\alpha}{2.\sin 2\alpha}\right)+\frac{2.\sin\left(\frac{\pi}{2}-\alpha+\varphi\right)}{\sin(\alpha-2\varphi).\sin\left[\pi-(\Omega+\beta+2\varphi)\right]}\times$$
$$\left[C.\cos\varphi.\pi.b^2.\left(\frac{\pi.b^2.\sin\alpha.\sin(\alpha+\beta)}{\sin 2\alpha.\sin\beta}\cdot\left(\frac{\sin^2\alpha}{\sin 2\alpha.\sin\beta}+\frac{1}{2.\cos\beta}\right)-\frac{1}{4.\cos\beta}-\frac{\sin\alpha}{2.\sin 2\alpha}\right)\right]$$

With:

$$W_1=\frac{\gamma\cdot b^2\sin^2\alpha}{2\sin 2\alpha}\qquad W_2=\frac{\gamma b^2\sin^3\alpha\sin(\alpha+\beta)}{2\sin^2 2\alpha\sin\beta}\qquad W_3=\frac{\gamma b^2\sin^4\alpha}{\sin^2 2\alpha}\cdot\left(\frac{1}{tg\beta}+\frac{1}{tg\Omega}\right)$$

Modèle n°2: Modèle de Fondation superficielles 3D selon coulomb
Données:

$$C:=0.5\frac{kg}{m^2}\ \ b:=In\ \phi:=30deg\ \ V0:=In\ \ \gamma:=1800\frac{kg}{m^3}$$

Velocity:

$$V12(\alpha) := \frac{V0}{\sin(\alpha)} \quad V23(\alpha,\beta,\Omega) := V0\frac{\sin(\Omega)}{\sin(\beta+\Omega)\cdot\tan(\alpha)} \quad V2(\alpha) := \frac{V0}{\tan(\alpha)} \quad V3(\alpha,\beta,\Omega) := V0\frac{\sin(\beta)}{\sin(\beta+\Omega)\cdot\tan(\alpha)}$$

Lateral surfaces:

$$baseS2(\alpha,\beta) := \pi\left(\frac{b}{\sin(2\alpha).\sin(\beta)}.\sin(\alpha).\sin(\alpha+\beta)\right)^2 \quad SI(\alpha) := \pi.b^2\frac{\sin(\alpha)}{2.\sin(\alpha)}$$

$$S2(\alpha,\beta) := \pi b^2.\left(\frac{\sin(\alpha).\sin(\alpha+\beta)}{\sin(2\alpha).\sin(\beta)}\right).\left(\frac{\sin(\alpha)^2}{\sin(2\alpha).\sin(\beta)}+\frac{1}{2.\cos(\beta)}\right)-\frac{\pi b^2}{4.\cos(\beta)}-\frac{\pi.b^2\sin(\alpha)}{2\sin(2\alpha)}$$

$$S3(\alpha,\beta,\Omega) := \pi.b^2.\left[\frac{1}{2}\frac{\sin(\alpha)^2}{\sin(2\alpha)}.\left(\frac{1}{\tan(\beta)}+\frac{1}{\tan\Omega}\right)\right].\left(\frac{\sin(\alpha)^2}{\sin(2\alpha).\sin(\Omega)}+\left(\frac{\sin(\alpha).\sin(\alpha+\beta)}{\sin(2\alpha).\sin(\beta).\cos(\Omega)}\right)\right)-$$

$$\pi.b^2.\left(\frac{\sin(\alpha).\sin(\alpha+\beta)}{\sin(2\alpha).\sin(\beta)}\right)^2\frac{1}{\cos(\Omega)}-SI(\alpha)-S2(\alpha,\beta)$$

Weight:

$$W1(\alpha) := \left(\frac{\gamma.b^2\sin(\alpha)^2}{2.\sin(2\alpha)}\right).I\pi \quad W2(\alpha,\beta) := \left(\frac{\gamma.b^2.\sin(\alpha)^3.\sin(\alpha+\beta)}{2.\sin(2\alpha)^2.\sin(\beta)}\right).Im$$

$$W3(\alpha,\beta,\Omega) := \left[\frac{\gamma.b^2.\sin(\alpha)^4}{\sin(2\alpha)^2}.\left(\frac{1}{\tan(\beta)}+\frac{1}{\tan\Omega}\right)\right].Im$$

Effort:

$$P(\alpha,\beta,\Omega) := \frac{\begin{array}{c}C.\cos(\phi).(SI(\alpha)+base\ S2(\alpha,\beta).\ V2(\alpha).S3(\alpha,\beta,\Omega).V3(\alpha,\beta,\Omega)+\\ 2.W3(\alpha,\beta,\Omega).V3(\alpha,\beta,\Omega).\sin(\Omega,\phi)+2.W2(\alpha,\beta).V2(\alpha).\sin(\phi)-W1(\alpha).V0\end{array}}{V0}$$

Minimization:

$$\alpha := 0.1 \quad \beta := 0.1 \quad \Omega := 0.1$$

Given

$$0.1<\alpha<\frac{\pi}{2} \quad 0.1<\beta<\frac{\pi}{2} \quad 0.1<\Omega<\frac{\pi}{2} \quad 0<\beta+\Omega<\pi-2.\phi$$

F:= Minimize(P, α, β, Ω)

$$F\begin{pmatrix}0.1\\0.765\\1.33\end{pmatrix}$$

$P(F_0, F_1, F_2) = 300.042$ kg

CONCLUSION

- We proposed above mechanisms which normally give us in plan and in space, superior borders of real values, if experimental attempts existed. It is not the case here for reasons of unavailability of the adequate material for such an experiment.
- We limited ourselves to models with lines of horizontal breaks while normally they should be directed downward.
- This study proposes all the same a sketch of solution of this problem, in the case of ground coherent and rubbing, never realized with this method which can be resumed by the other researchers.
- We notice that the maximum load P function of 2 variables, to develop in a graph in the software Mathcad, is concave, in the form of basin, in files data above (Mathcad). What shows well the existence of a minimum, thus then an abutment.

REFERENCES

Davis, E.H., M.J. Gunn, R.J. Mair and H.N. Seniviratne, 1980. The stability of shallow tunnels and underground openings in cohesive material, Geotechnique, 30: 397-419.

Hill, R., 1950. The mathematical theory of plasticity, Oxford: Clarendon Press, pp: 356.

Léonard, 1967. Les fondations. Dunod.

Muelhaus, H., 1985. lower bound solutions for circutar tunnels in 2 or 3 dimensions, Rock. Mech. et Rock Eng.., 18: 37-52.