Journal of Engineering and Applied Sciences 2 (5): 948-958, 2007

Dynamic Analysis of Prestressed Rayleigh Beam Carrying an Added Mass and Traversed By Uniform Partially Distributed Moving Loads

[1]I.A. Adetunde, [2]F.O. Akinpelu and [3]J.A. Gbadeyan
[1]Department of Applied Mathematics, University for Development Studies, Navrongo, Ghana
[2]Department of Pure and Applied Mathematics, Ladoke Akintola University of Technology, Ogbomoso, Nigeria
[3]Department of Mathematics, University of Ilorin, Ilorin, Nigeria

Abstract: An analytical-numerical method is presented that can be used to determine the dynamic behavior of pre-stressed Rayleigh been carrying an added mass and traversed by uniform partially distributed moving loads.This study demonstrates the transformation of a familiar governing equation into a new solvable coupled partial differential equations, been solved using Finite Difference Method. Furthermore, the study show that the response of structures due to moving mass which has often been neglected in the past, must be properly taken into account because it often differs significantly form the moving force model.

Key words: Dynamic, prestressed, transformation, traversed, loads

INTRODUCTION

As pointed out by Esmailzadeh and Ghorash (1994, 1995) the study of analysis of structures carrying moving loads apparently started in the middle of the last century, when railway construction began. Now there are many scientific research papers and even a few books devoted to this field (Esmailzadeh and Ghorash, 1994, 1995; Akin and Mofid, 1989; Fryba, 1971; Adetunde, 2003; Akinpelu, 2003; Clough and Penzien, 1993). Since the middle of the last century, when railway construction began, much of the research has focused on the effect of various physical phenomena on concentrated/distributed moving masses of Euler Bernoulli, Rayleigh and Timoshenko beam.

In spite of all the published work, there seems to be very little literature concerned with the pre stressed beams (beams which do experience compression when no external load is applied i.e. artificial creation of stresses in structure before loading) of any type. This problem has some practical applications: They are commonly incorporated in the design of aero planes. Advances in technology have accelerated the utilization of such pre- stressed structural elements. In general an aircraft is subjected to a wide range of temperature variation during flight which may cause considerable tensile or compressive pre-stressed in the beams when they are fixed in the plane direction. It is therefore, of technological interest to investigate to what extent the dynamic response of the beam is affected by the moving loads.

The main objectives of this study are to:

- Present a very simple technique to analyze the governing differential equation of a pre-stressed Rayleigh beam.
- Present a very simple technique to determine the response of simple supported pre-stressed Rayleigh beam, carrying an added mass and traversed by uniform partially distributed moving loads.
- Determine the response of amplitude of the deflection of the simply supported pre-stressed Rayleigh beam. Carrying an added mass and traversed by uniform partially distributed moving loads.
- Determine the variation in the lateral displacement of the simply supported pre -stressed Rayleigh beam carrying an added mass and traversed by uniform partially distributed moving loads.

MATHEMATICAL MODEL

We consider the case of a partially distributed load M which assumed to strike a finite Rayleigh beam of length L, initially at time $t = 0$ and advancing uniformly along the beam with a constant velocity, V.The beam is assumed to be simply supported at the left hand end of the beam (Fig. 1) while the beam has an attached mass at the other end $x = L$.

Corresponding Author: Adetunde Isaac, Department of Applied Mathematics and Computer Science, Faculty of Applied Sciences, University for Development Studies, P.O.Box 24, Navrongo, Ghana

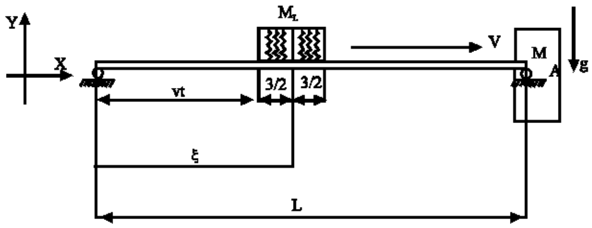


Fig. 1: Mathematical model

PROBLEM FORMULATION

The partial differential equation describing the traversed displacement of a pre-stressed Rayleigh beam carrying an added mass at one of its ends and traversed by uniform partially distributed load is given as

$$\frac{EI\partial^4 W(x,t)}{\partial x^4} + \frac{M\partial^2 W(x,t)}{\partial t^2} - \frac{Mb^2\ {}^4 W(x,t)}{\partial x^2 t^2} - \frac{N\partial^2 W(x,t)}{\partial x^2} = G(x,t)$$

Where

E = The modulus of elasticity

I = The second moment of area of the beam's cross-section.

M = The mass per unit length of the beam

W(x,t) = The deflection of the beam

b^2 = The radius of gyration

x = The spatial coordinate

t = Time

N = Is the pre stressed constant.

G (x,t) = The resulted concentrated force which can be defined as

$$G(x,t) = \frac{1}{\in}[-M_L g - M_L\left[\frac{\partial^2 W(x,t)}{\partial t^2} + \frac{2v\partial^2 W(x,t)}{\partial x \partial t} + \frac{v^2\partial^2 W(x,t)}{\partial x^2}\right]$$
$$\left[H(x-\xi+\in/2)-H(x-\xi-\in/2)\right]$$

Where

ζ = A particular distance along the length of the beam

$\in$ = The length of the load

M_L = The constant mass of the load which is assumed to be constant with the beam during the course of the motion.

g = Acceleration due to gravity.

H = Heaviside unit function, which is defined as

$$H(x) = \begin{cases} 1 \\ 0 \end{cases} \quad (3)$$

Here the first term in the first square bracket on the R.H.S of Eq. 2 describes the constant gravitational force, while the second term accounts for the effect of acceleration in the direction of the transverse deflection W(x,t) the third term is for complementary acceleration and the fourth term for the centripetal acceleration. The second square bracket describes the Heaviside unit function.

BOUNDARY CONDITIONS

Equation 1 and 2 is subject to the following end supports. At the end x = 0, one of the following holds.

$$\left.\begin{aligned} W(x,t) = 0 = W'(x,t) \\ W(x,t) = 0 = W''(x,t) \\ W'''(x,t) = 0 = W''(x,t) \\ W'(x,t) = 0 = W''(x,t) \end{aligned}\right\} \quad (4)$$

These conditions are sometimes called classical boundary conditions.

For the attached mass at the other end (x = L) we have (i.e., the non-classical boundary condition)

$$\begin{aligned} EI\,W''(L,t) - \omega^2 J\,W(L,t) = 0 \\ EI\,W'''(L,t) + \omega^2 M_L(L,t) = 0 \end{aligned} \quad (5)$$

Where, J is the mass moment of inertia at the end of the beam, ω^2 is the circular frequency and M_A is the attached mass at the end x = L.

The corresponding initial conditions are:

$$W(x,0) = 0 = \frac{\partial w}{\partial t}(x,0) \quad (6)$$

SOLUTION TECHNIQUE

In order to solve the I.B.V. problem described by Eq. 1-6, we first substitute Eq. 2 into Eq. 1 so that the governing differential equation becomes

$$\frac{EI\partial^4 W(x,t)}{\partial x^4} + \frac{M\partial^2 W(x,t)}{\partial t^2} - \frac{Mb^2\partial^4 W(x,t)}{\partial x^2 \partial t^2} - \frac{N\partial^2 W(x,t)}{\partial x^2}$$
$$= \frac{1}{\in}[-M_L g - M_L[\frac{\partial^2 W}{\partial t^2} + \frac{2v\partial^2 w}{\partial x \partial t} + \frac{V^2\partial^2 w}{\partial x^2}]$$
$$[H(x-\xi+\in/2)-H(x-\xi-\in/2)] \quad (7)$$

We assume that the transverse displacement of the beam, W(x, t) can be expressed in the form

$$W(x,t)\sum_{i=1}^{\infty}\phi_i(t)X_i(x) \quad (8)$$

Where $\phi_i(x)$'s = Are the unknown function of time
$X_i(x)$'s = Are the known eigen function of free vibration of the beam.

We remark that $\phi_i(t)$ and $\Psi_i(t)$ are the unknown functions of time t and $X_i(x)$ are the normalized deflection curves. Substituting Eq. (8) into Eq. (7) we have

$$\begin{aligned}&EI\sum_{i=1}^{\infty}\phi_i(t)X^{iv}(x)+m\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x)-mb^2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X''_i(x)-N\sum_{i=1}^{\infty}\phi_i(t)X_i''(x)=\\&\frac{1}{\in}\left[M_Lg-M_L\left[\sum_{i=1}^{\infty}X_i(x)\ddot{\phi}_i(t)+2V\sum_{i=1}^{\infty}\phi_i(t)X_i(x)+V^2\sum_{i=1}^{\infty}\phi_i(t)X_i''(x)\right]\right]\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right]\end{aligned} \tag{9}$$

We further assume that the load function can be expressed as

$$F(x,t)=\sum_{i=1}^{\infty}\psi_i(t)X_i(x) \tag{10}$$

Where $\iota_i(t)$ are unknown functions of time and $X_i(x)$ as said earlier. Multiplying both sides of the r.h.s of Eq. (9) by $X_j(x)$ and taking the definite integrals of both sides along the length L of the beam w.r.t. x, we have

$$\begin{aligned}&\frac{M_Lg}{\in}\int_0^L X_j(x)\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right]dx-\frac{M_L}{\in}\left(\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\int_0^L X_i(x)X_j(x)\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right]dx\right)-\\&\frac{2M_LV}{\in}\left(\sum_{i=1}^{\infty}\dot{\phi}_i(t)\int_0^L X_i'(x)X_j(x)\left[H\left(x-\xi+\frac{\in}{2}\right)-H\left(x-\xi-\frac{\in}{2}\right)\right]dx\right)-\frac{M_LV}{\in}\sum_{i=1}^{\infty}\dot{\phi}_i(t)\int_0^L X_i''(x)X_j(x)\left[H\left(x-\xi+\frac{\in}{2}\right)-\right.\\&\left.H\left(x-\xi-\frac{\in}{2}\right)\right]dx\Bigg)=\psi_i(t)\int_0^L X_i(x)X_j(x)\end{aligned} \tag{11}$$

Evaluating the improper integrals in the l.h.s of equation (11) terms by terms by integral by parts and using the following two properties of Dirac delta functions (see Appendix)

$$\int_{x_0}^{X^2} X_j(x)\ \delta\ (x\text{-}x_1)dx = X_j(x).\ \text{Provided}\ x_0 < x_1 < x_2 \tag{12}$$

$$\frac{d}{dx}\ H\ (x - x_1) = \delta\ (x - x1) \tag{13}$$

We finally obtained

$$\begin{aligned}&-M_Lg\left[X_i(\xi)+\frac{\in^2}{24}X_i''(\xi)\right]-M_L\sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left\{X_i(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i''(\xi)X_j(\xi)+2X_i'(\xi)X_j'(\xi)+\right.\right.\\&\left.\left.X_i(\xi)X_j''(\xi)\right]\right\}-2M_LV\sum_{i=1}^{\infty}\dot{\phi}_i(t)\left\{X_i'(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i'''(\xi)X_j(\xi)+2X_i''(\xi)X_j'(\xi)+X_i'(\xi)X_j''(\xi)\right]\right\}\\&-M_LV^2\sum_{i=1}^{\infty}\phi_i(t)\left\{X_i''(\xi)X_j(\xi)+\frac{\in^2}{24}\left[X_i^{iv}(\xi)X_j(\xi)+2X_i'''(\xi)X_j'(\xi)+X_i''(\xi)X_j''(\xi)\right]\right\}=\psi_i(t)\end{aligned} \tag{14}$$

Note

- In the r.h.s of Eq. 11 we have made use of the orthonormal principle.
- On noting that Eq. 10 is the applied force F(x,t), Eq. 9 now becomes

APPENDIX

In order to derive Eq. 13 the function F (x, t) is assumed to be expressible as

$$F(x,t) = \sum_{i=1}^{\infty} \psi_i(t) X_i(x) \qquad (A1)$$

Where the $\Psi_i(t)$'s are unknown functions of time. By substituting for W (x, t) from Eq. 8, multiplying both sides of the r.h.s. of Eq. 10 by $X_j(x)$ and taking the definite integral of both sides along the length of the beam with respect to X we obtain Eq. 14.

$$-\frac{M_L}{\in}\int_0^L X_j(x)\,[H(x-\xi+)-H(x-\xi-)]dx - \frac{M_L}{\in}\Big[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_j(x)\,X_i(x)[H(x-\xi+\tfrac{\in}{2})-H(x-\xi-\tfrac{\in}{2})]dx\Big]\frac{-2M_L V}{\in}$$
$$\Big[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_j(x)\,X_i(x)[H(x-\xi+\tfrac{\in}{2})-H(x-\xi-\tfrac{\in}{2})]dx\Big]$$
$$\frac{-M_L V^2}{\in}\Big[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_i''(x)X_j(x)[H(x-\xi+\in)-$$
$$H(x-\xi-\tfrac{\in}{2})]\,dx\Big] = \sum_{i=1}^{\infty}\psi_i(t)\int_0^L X_j(x)X_i(x)dx \qquad (A2)$$

The above integrations can be convienently carried out term by term by defining them as follows:

$$K = -\frac{M_L g}{\in}\int_0^L X_j(x)\,[H(x-\xi+\tfrac{\in}{2})-H(x-\xi-\tfrac{\in}{2})]dx$$

$$L = -\frac{M_L}{\in}\Big[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_j(x)\,X_i(x)[H(x-\xi+\tfrac{\in}{2})-$$
$$H(x-\xi-\tfrac{\in}{2})]dx\Big]$$

$$M = \frac{-2M_L V}{\in}\Big[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_j(x)\,X_i(x)[H(x-\xi+\tfrac{\in}{2})-$$
$$H(x-\xi-\tfrac{\in}{2})]dx\Big]$$

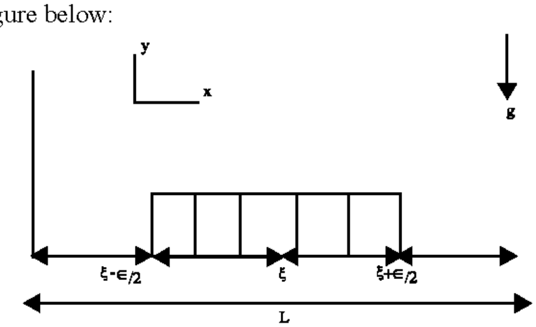
$$N = \frac{-M_L V^2}{\in}\Big[\sum_{i=1}^{\infty}\phi_i(t)\int_0^L X_i''(x)X_j(x)[H(x-\xi+\in)-$$
$$H(x-\xi-\tfrac{\in}{2})]dx\Big]$$

$$O = \sum_{i=1}^{\infty}\psi_i(t)\int_0^L X_j(x)X_i(x)dx \qquad (A3)$$

Consider the first term that is K, using integration by parts,

$$K = -\frac{M_L g}{\in}$$
$$\Big\{\Big[\Big[H(x-\xi+\tfrac{\in}{2})-H(x-\xi-\tfrac{\in}{2})\Big]-\int_0^L X_j(x)dx\Big|_0^L\Big]$$
$$\int_0^L\Big[\int_0^L X_j(x)dx\,[H'(x-\xi+\tfrac{\in}{2})-H'(x-\xi-\tfrac{\in}{2})]\Big]\Big\} \qquad (A4)$$

Using the property of Heaviside function and the figure below:



$$K = \frac{-M_L g}{\in}\int_0^L\int_0^L X_j(x)\delta(x-\xi+\tfrac{\in}{2})dx -$$
$$\int_0^L\int_0^L X_j(x)\delta(x-\xi-\tfrac{\in}{2})dx \qquad (A5)$$

Now setting $D(x) = \int_0^L X_j(x)dx$, equation K becomes;

$$K = \frac{-M_L g}{\in}\left[\int_0^L D(x)\delta(x-\xi+\tfrac{\in}{2})dx - \int_0^L D(x)\delta(x-\xi-\tfrac{\in}{2})dx\right] \qquad (A6)$$

Thus by the property of Dirac delta function, we have

$$K = \frac{-M_L g}{\in}\left[D(\xi + \frac{\in}{2}) - D(\xi + \frac{\in}{2})\right] \quad (A7)$$

$$K = \frac{-M_L g}{\in}\left[\int_0^L X_j(\xi + \frac{\in}{2})d\xi - \int_0^L X_j(\xi - \frac{\in}{2})d\xi\right] \quad (A8)$$

Using Taylor's series expansion, we obtain

$$K = \frac{-M_L g}{\in}\left[\int_0^L \left[X_j(\xi) - \left(\frac{\in}{2}\right)X'\ (\xi) + \frac{\left(\frac{\in}{2}\right)^2 X''(\xi)}{2!} + \frac{\left(\frac{\in}{2}\right)^3 X'''(\xi)}{3!} + \frac{\left(\frac{\in}{2}\right)^4 X^{iv}(\xi)}{4!} + \ldots - X_j(\xi) + \left(\frac{\in}{2}\right) X'(\xi) - \frac{\left(\frac{\in}{2}\right)^2 X''(\xi)}{2!} + \frac{\left(\frac{\in}{2}\right)^3 X'''(\xi)}{3!} - \frac{\left(\frac{\in}{2}\right)^4 X^{iv}(\xi)}{4!} \ldots\ \right]dx\right]$$

Hence

$$K = -M_l g\ [X_i(\zeta) + \frac{\in^2}{24} X''_i(\zeta)] + 0(\varepsilon^3) \quad (A10)$$

Following similar argument, the second improper integral

$$L = -M_L \sum_{i=1}^{\infty} \ddot{\phi}_i(t) \int_0^L \left[X_i(\xi + \frac{\in}{2})X_j(\xi + \frac{\in}{2}) - X_i(\xi - \frac{\in}{2})X_j(\xi - \frac{\in}{2})\right]dx \quad (A11)$$

Which reduces to

$$L = -M_L \sum_{i=1}^{\infty} \ddot{\phi}_i\ (t)\ \{X_i(\zeta)X_j(\zeta) + [X''_i(\zeta)\ X_j(\zeta) + 2\ X'_i(\zeta)\ X'_j(\zeta) + X_i(\zeta)\ X''_j(\zeta)] \quad (A12)$$

Also the third and the fourth improper integrals M and N becomes

$$M = -2M_L V \sum_{i=1}^{\infty} \dot{\phi}_i(t) \int_0^L \left[X_i'(\xi + \frac{\in}{2})X_j(\xi + \frac{\in}{2}) - X_i'(\xi - \frac{\in}{2})X_j(\xi - \frac{\in}{2})\right]dx \quad (A13)$$

$$N = -M_L V^2 \sum_{i=1}^{\infty} \phi_i(t) \int_0^L \left[X_i''(\xi + \frac{\in}{2})X_j(\xi + \frac{\in}{2}) - X_i''(\xi - \frac{\in}{2})X_j(\xi - \frac{\in}{2})\right]dx \quad (A14)$$

Respectively which finally reduces to

$$M = -2M_L V \sum_{i=1}^{\infty} \dot{\phi}_i\ (t)[X_i'(\zeta)X_j(\zeta) + \frac{\in^2}{24}\ [X'''_i(\zeta)\ X_j(\zeta) + 2\ X''_i(\zeta)\ X_j(\zeta) + X'(\zeta)\ X_j''(\zeta)]] + 0(\varepsilon^3) \quad (A15)$$

And

$$N = -M_L V^2 \sum_{i=1}^{\infty} \phi_i\ (t)\ )[Xi(\zeta)X''_j(\zeta) + \frac{\in^2}{24}\ [X^{iv}_i\zeta\ X_j(\zeta) + 2\ X'''_i(\zeta)\ X'_j(\zeta) + X''_i(\in)\ X''_j(\zeta)]] + 0(\varepsilon^3) \quad (A16)$$

Respectively,
By using orthogonality relation

$$0 = \sum_{i=1}^{\infty} \psi_i(t) \int_0^L X_j(x) X_i(x) dx \text{ becomes } \Psi_i(t). \quad \text{(A17)}$$

SIMPLY SUPPORTED PRESTRESSED RAYLEIGH BEAM WITH AN ATTACHED MASS

The dynamic response of the system under consideration (a beam carrying a mass at the end x = L and traversed by partially distributed moving load) having a simply supported boundary conditions is considered. (In particular the beam under consideration is simply supported at x = 0 while carrying a mass at the end x = L).

The end conditions are as prescribed in Eq. (4- 6) and corresponding kernel can be easily shown as

$$X_i(x) = \text{Sin} \frac{q_i}{L} x + \beta_i \text{Sinh} \frac{q_i}{L} x \quad (16)$$

Where

$$\beta_i = \frac{EI\, q_i \,\sin q_i\, x + \omega^2 JL \cos qi}{EI\, q_i\, \sin q_i\, x - \omega^2 JL \cosh q_i\, x} \quad (17)$$

and qi is the roots of the associated transcendental frequency equation given as

$$\cos q_i \,\sinh q_i - \sin qi \cosh q_i - \frac{2\omega^2 MA \sin qi \sinh qi}{EI_{qi}{}^2} - \frac{2\;2\; JL \cos qi \cosh qi}{EI_{qi}{}^2}$$

$$\frac{+\omega^2 M_A J}{(EI)^2 q_i{}^2}[\sin q_i \cosh q_i - \cos q_i \sinh q_i] = 0 \quad (18)$$

The transcendental frequency Eq. 18 is solved, using Newton Raphson's method.

The governing differential equation for vibration of the beam, for the particular case under consideration could be obtained thus by deriving exact governing equations by employing Eq. 16 and evaluating the exact values of the integral in Eq. 11. After along lengthy simplification, we finally have

$$m\sum_{i=1}^{\infty}\omega^2\phi_i(t)X_i(x) + m\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i(x) - mb^2\sum_{i=1}^{\infty}\ddot{\phi}_i(t)X_i''(x) = \sum_{i=1}^{\infty}\left[\sin\frac{q_i}{L}\left(\zeta+\frac{\in}{2}\right) + B_i\sinh\frac{q_i}{L}\left(\zeta+\frac{\in}{2}\right)\right]$$

$$\left[\left\{-\frac{M_L g}{\in}\left[2\sin\frac{q_i}{L}\xi\sin\frac{q_i}{2L}(\in + B_i)\sinh\frac{q_i}{L}\xi\sinh\frac{q_i\in}{2L}\right] - \sum_{i=1}^{\infty}\ddot{\phi}_i(t)\left\{\frac{LM_L}{(q_i-q_j)}\left[\sin\frac{\in}{2L}(q_i-q_j)\cos\frac{\xi}{L}(q_i-q_j)\right] - \right.\right.\right.$$

$$\frac{LM_L}{\in(q_i+q_j)}\left[\sin\frac{\in}{2L}(q_i+q_j)\cos\frac{\xi}{L}(q_i+q_j)\right] + \frac{LB_iM_L}{\in(q_i^2-q_j^2)}\left[q_i\sin\frac{(-\in)}{2L}(1-q_i)\cos\frac{\xi}{L}(1-q_i) + \sin\frac{(-\in)}{2L}(1+q_i)\right.$$

$$\left.\cos\frac{\xi}{L}(1+q_i) + q_j(\sin\frac{\in}{2L}(q_i-q_j)\xi\cos\frac{\xi}{L}(q_i-q_j) + \sin\frac{\in}{2L}(q_i+q_j)\cos\frac{\xi}{L}(q_i+q_j))\right] +$$

$$+\frac{M_L B_i^2 M_L}{\in(q_i+q_j)}\left[\cosh\frac{\xi}{L}(q_i+q_j)\sin\frac{\in}{2L}(q_i+q_j)\right] - \frac{M_L B_i^2 L}{\in(q_i-q_j)}\left[\cosh\frac{\xi}{L}(q_i+q_j)\sinh\frac{\in}{2L}(q_i-q_j)\right]$$

$$--2M_L V\sum_{i=1}^{\infty}q_i\dot{\phi}\left\{\frac{1}{(q_i+q_j)}\right\}\left[\sin\frac{\xi}{L}(q_i+q_j)\sin\frac{\in}{2L}(q_i+q_j)\right] + \frac{1}{\in(q_i-q_j)}\left[\cosh\frac{\xi}{L}(q_i-q_j)\right.$$

$$\sin\frac{\in}{2l}(q_i - q_j)\Bigg] + \frac{LB_i}{\in(q_i^2 + q_j^2)}\Big[q_i(\sin\frac{\in}{2L}(1+q_i)\sin\frac{\xi}{L}(1+q_i) + \sin\frac{\in}{2L}(1+q_i)\sin\frac{\xi}{L}(1+q_i) +$$

$$q_i(\sin\frac{\in}{2L}(1-q_i)\cos\frac{\xi}{L}(1-q_j) + \sin\frac{\in}{2L}(1+q_j)\cos\frac{\xi}{L}(1+q_j)\Bigg] + \frac{B_i^2}{\in(q_i+q_l)}\Bigg[\sinh\frac{\xi}{L}(q_i+q_j)$$

$$\sinh\frac{\in}{2L}(q_i+q_j)\Bigg] + \frac{B_i^2}{\in(q_i+q_j)}\Bigg[\sinh\frac{\xi}{L}(q_i-q_j)\sinh\frac{\in}{2L}(q_i-q_j)\Bigg)\Bigg]\Bigg\} + M_L V^2\sum_{i=1}^{\infty}\phi_i(t)$$

$$\Bigg\{q_i^2\Bigg(\frac{-1}{(q_i-q_j)}\Bigg[\sin\frac{\in}{2L}(q_i-q_j)\cos\frac{\xi}{L}(q_i-q_j) + \frac{1}{(q_i+q_j)}\Bigg[\sin\frac{\in}{2L}(q_i+q_j)\cos\frac{\xi}{L}(q_i+q_j)\Bigg] - \tag{19}$$

$$-\frac{B_iL}{(q_i^2+q_j^2)}\Bigg[q_j\sin\frac{\in}{2L}(1-q_i)\cos\frac{\xi}{L}(1-q_i) + \sin\frac{\in}{2L}(1+q_i)\cos\frac{\xi}{L}(1+q_i)\Bigg] +$$

$$q_i\Bigg(\sin\frac{\in}{2L}(1-q_i)\cos\frac{\xi}{L}(1-q_i) + \sin\frac{\in}{2L}(1+q_i)\cos\frac{\xi}{L}(1+q_i)\Bigg)\Bigg] + \frac{B_i^2}{(q_i+q_j)}\Bigg[\cosh\frac{\xi}{L}(q_i+q_j)$$

$$\sinh\frac{\in}{2L}(q_i+q_j)\Bigg] - \frac{B_i^2}{(q_i+q_j)}\Bigg[\cosh\frac{\xi}{L}(q_i-q_j)\sinh\frac{\in}{2L}(q_i-q_j)\Bigg]\Bigg\}\Bigg\}$$

Equation 19 is the desired exact differential equation describing the behaviour of a Prestressed Rayleigh beam carrying an added mass at one of it's ends but traversed by a distributed moving load. The highly coupled equation is solved numerically.

Note: For the case of $q_i = q_j$, we replace the expression involving $1/(q_i - q_j)$ by $q_i\in/2L$.

NUMERICAL ANALYSIS

To solve Eq. 19, recourse can be made to a numerical method; but 2 interesting cases are to be tackled.

Case 1: The moving force Prestressed Rayleigh Beam problem :- A moving force problem is one in which the inertia effects of the moving load are neglected and only the right hand side of the later except the first term in the first curly bracket. (i.e., by neglecting all the terms apart from the first term on the r.h.s of Eq. 19).

Case II: The moving mass Prestressed Rayleigh Beam problem is one in which both the inertia effects and the force effects are retained. i.e., the whole Eq. 19 is the mass problem. To obtain results given in this study, approximate central difference formulas have been utilized for the derivatives in Eq. 19 for both cases (Cases I and II). Thus, for N model shapes Eq. 19 are transformed to a set of N linear algebraic equations, which are to be solved for each interval of time. Regarding the degree of approximations involved, in order to ensure the stability and convergence of the solution, sufficiently small time steps have been utilized.

Computer program was developed and the following numerical data which are the same as those in reference (Gorashi and Esmailzadeh, 1995; Akin and Mofid, 1989) were used, for the purpose of comparisms:

$E = 2.07 \times 10^{11} N/m^2 . I = 1.04 \times 10^{-6} m^4$, $V = 12 km/h$,
$M = 70 kg$, $g = 9.8 m/s^2$, $m = 7.04 kg$,
$L = 10 m$, $t = 0.5 s, 1.0 s, 1 sb = 0.05 m \in = 0.1 m, 1 m$

RESULTS AND DISCUSSION

The numerical method of 2 distinct dynamic problems are discussed. These problems have to do with two cases talked about in the numerical analysis, viz; determining the response of a prestressed Rayleigh beam with an attached mass to uniform partially distributed moving force. Determining the response of a prestressed Rayleigh beam with an attached mass to uniform partially distributed moving mass.

Figure 2 with respect to Table 1, shows the variation of the deflection or W_F (x, t) of the moving force prestressed simply supported Rayleigh beam carrying a lumped mass at its end x = L at t = 0.5s, ε = 0.1m for various values of M_L. It was observed that the amplitude deflection W_F (x, t) increases as M_L increases.

Similar analysis was carried out for Fig. 3 with respect to Table 2, but t = 1.0s, ε = 0.1m, for various values of M_L. It was observed also that the amplitude deflection W_F (x, t) increase as M_L increases.

Figure 4 with respect to Table 3 contain analysis which shows the variety of the deflection W_F (x, t) of the moving force prestressed simply supported Rayleigh

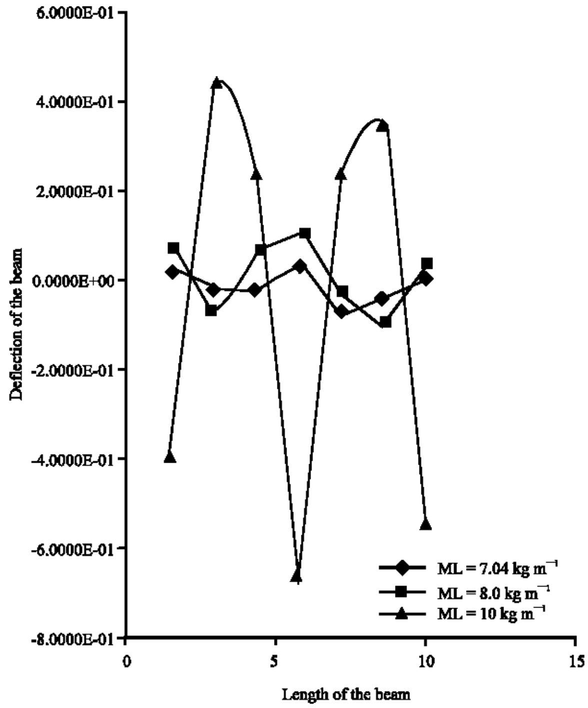


Fig. 2: The variation of the deflection or W_F (x, t) of the moving

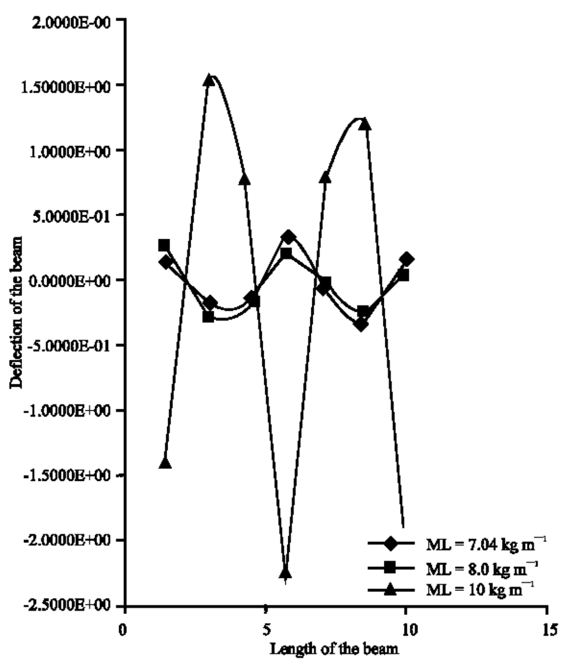


Fig. 3: Shows the variety of the deflection W_F (x, t) of the moving

beam carrying a lumped mass at its end x = L at t = 1s, ε = 1.0 m and for various values of M_L. These we found that the maximum deflection decreases as ε increases for fixed value of M_L and time t.

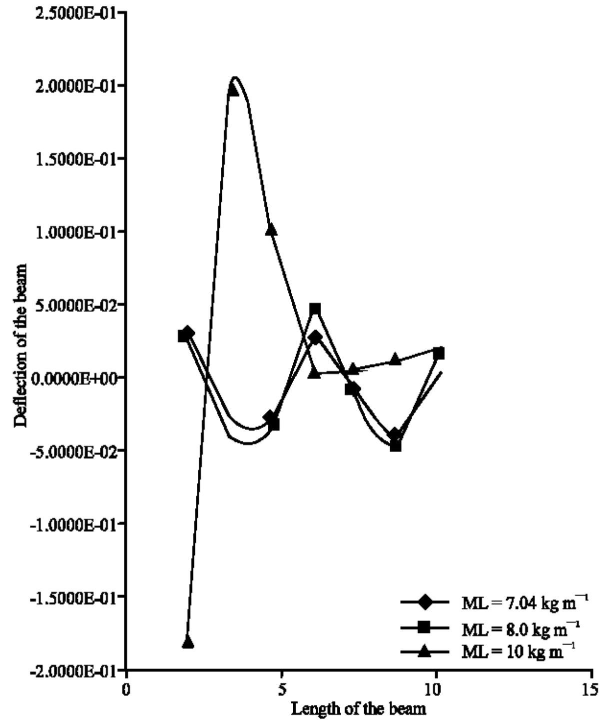


Fig. 4: Deflection W_F (x, t) of the moving force prestressed simply supported Rayleigh beam carrying a lumped

Table 1: Variation of the deflection W_F (x, t) of the moving force prestressed simply supported Rayleigh beam carrying a lumped mass at its end x = L at t = 0.5s, ε = 0.1 m and for various values of M_L

Length of the beam X(m)	W_F (x,t) for M_L = 7.04 kg m^{-1}	W_F (x, t) for M_L = 8.0 kg m^{-1}	W_F (x, t) for M_L = 10 kg m^{-1}
1.469	2.3200E-02	6.5099E-02	-3.9440E-01
2.888	-2.5761E-02	-7.3362E-02	4.3260E-01
4.307	-2.7458E-02	6.5063E-02	2.3767E-01
5.726	3.0172E-02	9.4489E-02	-6.6627E-01
7.145	-7.7132E-02	-2.9304E-02	2.3501E-01
8.564	-3.7150E-02	-9.2205E-02	3.4316E-01
9.983	4.2145E-03	3.9614E-02	-5.4443E-01

Table 2: Variation of the deflection W_F (x, t) of the moving force prestressed simply supported Rayleigh beam carrying a lumped mass at its end x = L at t = 1.0s, ε = 0.1m and for various values of M_L

Length of the beam X(m)	W_F (x,t) for M_L = 7.04 kg m^{-1}	W_F (x, t) for M_L = 8.0 kg m^{-1}	W_F (x, t) for M_L = 10 kg m^{-1}
1.469	1.6241E-01	2.2785E-01	-1.4000E+00
2.888	-1.8032E-01	-2.5677E-01	1.5000E+00
4.307	-1.9221E-01	-2.2772E-01	8.3000E-01
5.726	2.1120E-01	3.3073E-01	-2.3000E+00
7.145	-5.3994E-02	-8.2383E-02	8.2000E-01
8.564	-2.6010E-01	-3.2000E-01	1.2000E+00
9.971	2.9502E-02	1.4000E-01	-1.9000E+00

The next analysis involved a prestressed Rayleigh beam simply supported at x = 0, but an attached mass x = L. The beam was traversed by a uniform partially distributed mass as opposed to a distributed force Fig. 5. with respect to Table 4 contains the various values of the

Table 3: Variation of the deflection W_F (x, t) of the moving force prestressed simply supported Rayleigh beam carrying a lumped mass at its end x = L at t = 1s, ε = 0.1m and for various values of M_L

Length of the beam X(m)	W_F (x,t) for M_L = 7.04 kg m^{-1}	W_F (x, t) for M_L = 8.0 kg m^{-1}	W_F (x, t) for M_L = 10 kg m^{-1}
1.853	0.023068	3.2395E-02	-1.7874E-01
3.206	-0.025649	-3.6486E-02	1.9585E-01
4.559	-0.027373	-3.2431E-02	1.0680E-01
5.912	0.030031	4.6926E-02	1.0655E-02
7.265	-0.0076902	-1.4518E-02	1.0655E-02
8.618	-0.037092	-4.5905E-02	1.5368E-02
9.971	0.0041507	1.9512E-02	2.4770E-02

Table 4: The deflection of the prestressed Rayleigh beam to a moving mass at ε = 0.1m, t = 0.5s, at various values of m (i.e. m = 8 and 10 kg m^{-1}).

Length of the beam X(m)	W_F (x, t) for M_L = 8.0 kg m^{-1}	W_F (x, t) for M_L = 10 kg m^{-1}
1.469	1.6800E-04	1.6900E-04
2.88	5.6400E-04	5.7000E-04
4.306	1.0300E-03	1.0500E-03
5.726	1.4200E-03	1.4400E-03
7.145	1.6000E-03	1.6200E-03
8.564	1.4500E-03	1.4600E-03
9.983	1.1200E-03	1.1400E-03

Table 5: The lateral transverse deflection of the prestressed rayleigh beam at ε = 0.1m, t = 1s, at various values of the moving mass m (i.e., m = 7.04, 8, 10 and 15 kg m^{-1})

Length of the beam X(m)	W_F (x,t) for M_L = 7.04 kg m^{-1}	W_F (x, t) for M_L = 8.0 kg m^{-1}	W_F (x, t) for M_L = 10 kg m^{-1}	W_F (x, t) for M_L = 15 kg m^{-1}
1.469	2.8900E-04	2.8900E-04	2.8900E-04	2.91E-04
2.88	9.6900E-04	9.6900E-04	9.7100E-04	9.76E-04
4.306	1.7800E-03	1.7800E-03	1.7800E-03	1.79E-03
5.726	2.4500E-03	2.4500E-03	2.4500E-03	2.46E-03
7.145	2.7500E-03	2.4480E-03	2.5720E-03	2.76E-03
8.564	2.4900E-03	2.4900E-03	2.5000E-03	2.51E-03
9.983	1.9400E-03	1.9400E-03	1.9500E-03	1.96E-03

Table 6: The lateral transverse deflection of the prestressed rayleigh beam at ε = 1m, t = 0.5s, for various values of m (i.e. m = 7.04, 8 and 10 kg m^{-1})

Length of the beam X(m)	W_F (x,t) for M_L = 7.04 kg m^{-1}	W_F (x, t) for M_L = 8.0 kg m^{-1}	W_F (x, t) for M_L = 10 kg m^{-1}
1.853	8.3500E-04	8.4200E-04	8.60E-04
3.206	-8.2600E-04	-8.3300E-04	-8.47E-04
4.559	-8.0400E-04	-8.0900E-04	-8.17E-04
5.912	-4.1200E-04	-4.1200E-04	-4.08E-04
7.265	4.2400E-05	4.2600E-05	5.69E-05
8.618	2.7900E-04	2.8600E-04	3.01E-04
9.971	2.3600E-04	2.4100E-04	2.50E-04

Table 7: Deflection of the prestressed rayleigh beam at ε = 1m, t = 1s, for various values of m (i.e. m = 7.04, 8 and 10 kg m^{-1})

Length of the beam X(m)	W_F (x,t) for M_L = 7.04 kg m^{-1}	W_F (x, t) for M_L = 8.0 kg m^{-1}	W_F (x, t) for M_L = 10 kg m^{-1}
1.853	9.1200E-04	1.4000E-03	1.41E-03
3.206	-1.0200E-03	-9.7200E-04	-1.41E-03
4.559	-1.2100E-03	-1.3800E-03	-1.39E-03
5.912	-9.4000E-04	-7.5000E-04	-7.45E-04
7.265	-4.5000E-04	3.4400E-07	1.34E-05
8.618	-2.0300E-05	4.2300E-04	4.38E-04
9.971	1.3800E-04	3.7800E-04	3.86E-04

lateral deflection, W_{MLP} (x, t) for the prestressd Rayleigh beam at various values of x: However ε = 0.1m and t = 0.5s Fig. 6-8, with respect to Table 5-7, respectively contain similar values o W_{MLP} (x, t) but for (i) ε = 0.1 m and t = 1.0s.

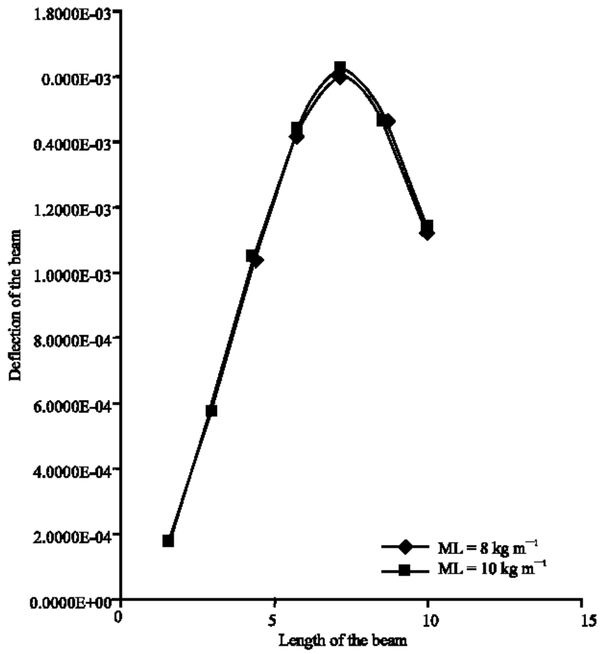


Fig. 5: The prestressed Rayleigh beam to a moving mass at ε = 0.1m, t = 0.5s, at various values

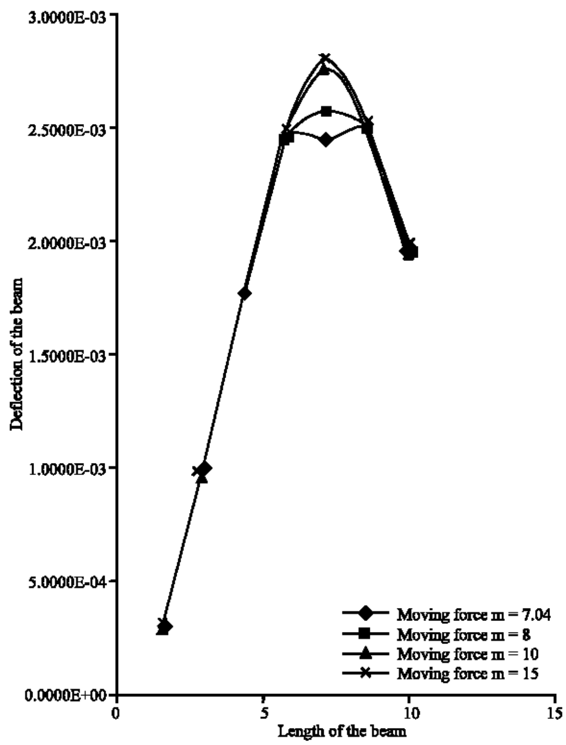


Fig. 6: Various values of m (i.e., m = 7.04, 8 and 10 kg m^{-1}

(ii) ε = 1.0m and t = 0.5m. (iii) ε = 1.0 m and t = 1.0s, respectively. It is evident from these four figures and tables that the amplitude deflection increases as M_L

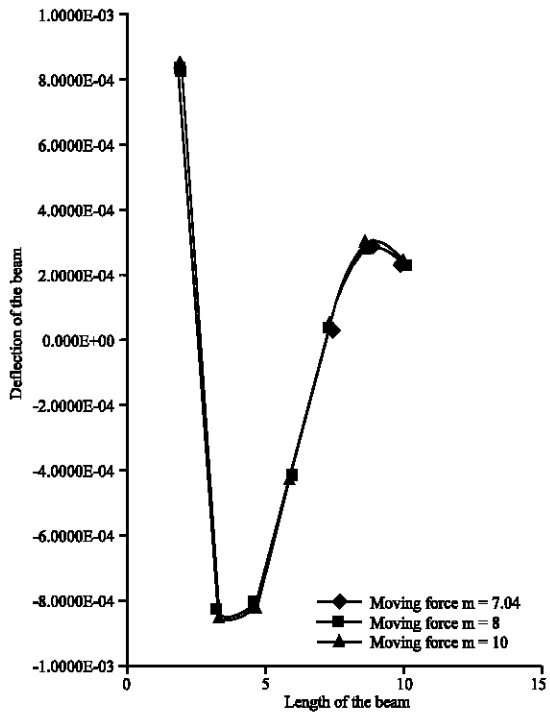


Fig. 7: For various values of m (i.e. m = 7.04, 8 and 10 kg m^{-1}

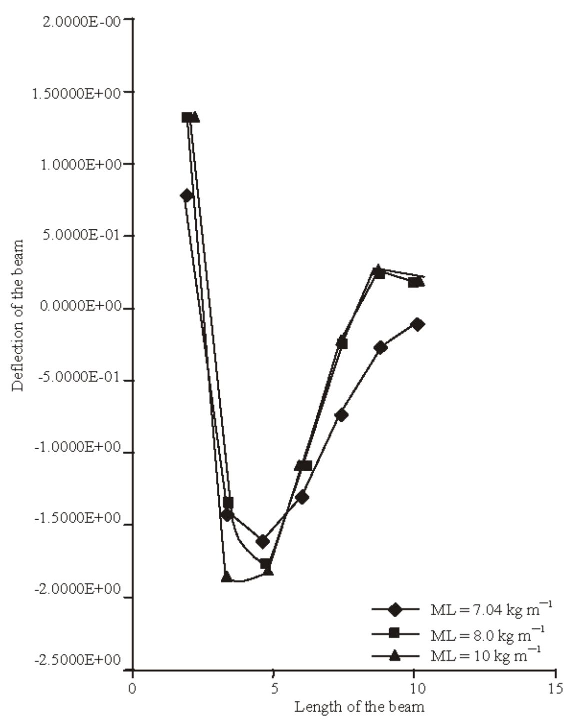


Fig. 8: For various values of m (i.e. m = 7.04, 8 and 10 kg m^{-1}

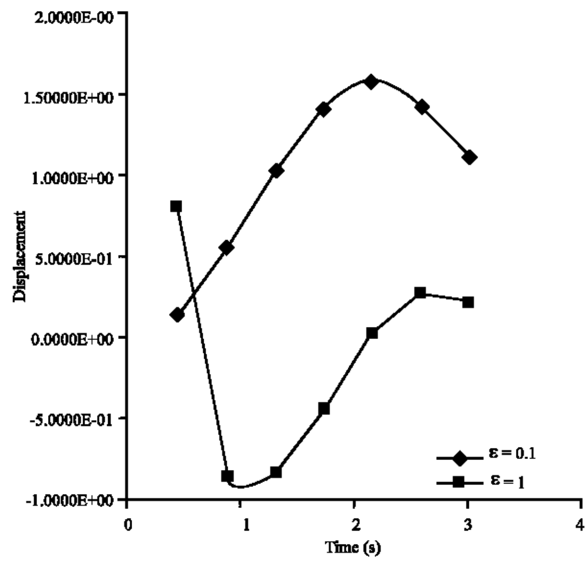


Fig. 9: The variation of deflection W_{MLP} (x, t) against time t

Table 8: The Displacement of the Prestressed Rayleigh beam at different ε(ε = 0.1 and 1.0) at various time t(s)

Length of the beam X(m)	ε = 0.1	ε = 1
0.43	1.6700E-04	8.3500E-04
0.86	5.6100E-04	-8.2600E-04
1.29	1.0300E-03	-8.0400E-04
1.72	1.4200E-03	-4.1400E-04
2.15	1.5900E-03	4.2000E-05
2.58	1.4400E-03	2.7900E-04
3.01	1.1200E-03	2.3600E-04

increases for a particular value of t and ε. Also the amplitude deflection increases as time t increases for various values of M_L and the fixed value of ε.

Figure 9 with respect to Table 8 shows the variation of deflection W_{MLP} (x, t) against time t. It can be seen from this that amplitude deflection decreases as å increases or various values of t and a fixed value of M_L.

CONCLUSION

The problem of investigating the dynamic analysis of prestressed Rayleigh beam carrying an added mass and traversed by uniform partially distributed moving loads is studied. Analytical numerical technique is used to solve the pertinent initial-boundary value problem. It was observed that

- Amplitude deflection of the moving force prestressed Rayleigh beam are greater than those of the moving mass.
- It was also observed that amplitude deflection decreases as ε increases for various values of t and a fixed value of M_L.

Consequently, relying on the result of the moving mass may be misleading because from the comparisms of the moving force and moving mass (Fryba, 1971) results indicates an at least 80% different between the two results and thus shows the importance of including mass in real design conditions where the velocity is high. Finally, the writers believe that the methods will efficiently serve design engineers in real design conditions.

REFERENCES

Akin, J.E. and M. Mofid, 1989. Numerical solution for response of beams with moving masses. J. Struct. Eng., 115: 120-131.

Adetunde, I.A., 2003. Dynamic analysis of elastic Reyleigh beams carrying an added mass and traversed by uniform partially distributed moving loads. Ph.D. Thesis, University of Ilorin, Ilorin. Nigeria.

Akinpelu, F.O., 2003. The response of Euler Bernoulli beams with an attached mass to uniform partially distributed moving loads. Ph.D. Thesis. University of Ilorin, Ilorin. Nigeria.

Clough, R.W. and J. Penzien, 1993. Dynamics of Structures. McGraw-Hill, Inc. (2nd Edn).

Esmailzadeh, E. and M. Ghorashi, 1994. Vibration analysis of beams traversed by uniform partially distributed moving, Masses. J. Sound and Vibration 184: 9-17.

Fryba, L., 1971. Vibration of solids and structures under moving loads. Noordhoff International Publishing, Groningen, the Netherlands.

Gorashhi, M. and E. Esmailzadeh, 1995. Vibration Analysis of beams traversed by a moving, Mass. J. Eng., 8: 213-220.