

## Strategic Modeling of Single Sampling Attribute Plans Based on Quality Loss Function to Ensure Product Quality

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**Abstract:** In any manufacturing process acceptance sampling is essential to accept products from the supplier or from the internal customer (between the processes). Acceptance between the processes is essential because further processing on a bad semi-processing product would be a waste. Attribute single sampling plans psychologically pressurizing the producer to produce quality products and are also easy to use and administer. The quality loss function developed by Taguchi emphasis the need for maintaining the specification mean as the target to reduce loss to the society. The acceptance sampling plans presently followed don't accommodate different quality losses associated with not maintaining the specification mean. The authors propose a method for designing single sampling attribute plans to accommodate this quality loss function while setting the specification limits for evaluating the products. A numerical case example is also presented. New levels of AQL and LTPD are chosen and corresponding changes in the given specifications are made yielding to smaller sample size. The new tightened specification allows the probability of occurrence of the defective in a smaller size of the sample. The parameter ATI calculated shows considerable improvements from the existing sampling plans. Thus the proposed procedure shifts the application of acceptance sampling plan to acceptance control plan to ensure minimum deviations from the desired specification mean to reduce the loss to the firm as well as to the society.

**Key words:** Attribute sampling plan, quality loss function, nominal the best, AQL, LTPD, ATI

### INTRODUCTION

Modern day industrial products require number of process before reaching the customer. Each process has its own quality specifications. Inspection for acceptance in each process is required to accept the products between the processes or from the vendors, or at the end of manufacturing process before shipping. Acceptance sampling is suitable for this purpose and also offers the following advantages:

- When testing is destructive, 100% inspection is not feasible.
- When the cost of inspection is high and will increase the product cost.
- Delay in inspection will affect the delivery schedules and increases inventory.

Acceptance sampling is the process of evaluating a group of products or material in a lot for the purpose of

accepting or rejecting the lot as either conforming or non-conforming to quality specifications. An acceptance sampling plan is a statement of the sample size to be used and acceptance criteria for sentencing the individual lots. In acceptance sampling the products are submitted in lots, 'N' is the number of products in a lot, 'n' is the number of defectives accepted in sample called as acceptance number, 'r' is the rejection number, equal or more number of defectives in the sample will decide the lot to be rejected in single sampling plans  $r = c+1$ .

This out of specification is a common concept to ensure product quality. But it implies that all products that meet specification are good, whereas those that do not are bad. But customers perceived quality as meeting the target rather than meeting the specifications. Therefore, the role of acceptance sampling plan needs to be changed to acceptance control plan by adjusting the procedure to match the changing inspection environment. Hence, it is essential to strategically model the existing sampling plan to this new customer perception of quality.

The entire idea of adjusting the sampling plan design is further motivated by the Genichi Taguchi's Loss Function-Nominal the best which states that, any departure from the desired mean value will incur loss not only to the manufacturing firm but also to the society. Hence, it becomes increasingly important to reduce variability. This strategic sampling plan model exerts psychological pressure on producer to make the products with minimum variability (James and William, 2004).

Many doubts over the role of acceptance sampling in modern quality control were cleared by Stephen and Marcus (1999) as follows. Sampling inspection is not fundamentally about taking remedial measures on items of fixed and known quality. It is instead, about gathering information and making economical determinations about the likely state of quality, in contexts where quality varies/type is unknown.

Many issues like the role of acceptance sampling plans in preventing rather than detecting defectives, extremely low defect rates expected/required today (measured in few parts per million) and legitimate role of acceptance sampling in today's industrial practice were discussed by Samuel *et al.* (2000). They also demonstrated the effectiveness of acceptance sampling plan in accumulating process quality history.

**Principle:** A sampling plan is designed for a given acceptable quality level (AQL) and average outgoing limit (AOQL), the sample size and the acceptance number are the important parameters of the sampling plan. Using these parameters an operating characteristic curve (OC curve) Fig. 1, can be drawn to show the variation of the probability of acceptance for the variation in fraction defective ( $p_1$ ) for a given producer's risk ( $\alpha$ ). It is the risk taken by the producer, when the lot is submitted for inspection (i.e., probability of good lot being rejected). Lot tolerance percent defective (LTPD) is the fraction defective ( $p_2$ ) for a given consumer's risk ( $\beta$ ), it is the risk taken by the consumers, when the lot is accepted after inspection. It is the probability of accepting the bad lot (Richard and Byron, 1992).

In acceptance sampling the lots are submitted for evaluation. The samples (sample size 'n') are taken from each lot and inspected, if the number of defectives in a sample is less than or equal to the acceptance number the lot will be accepted, otherwise lot will be rejected. If the fraction defective in the lot is less than AQL, more than 95% of the lots will be accepted, or a lot with a fraction defective less than AQL is submitted for acceptance 100 times, it will be accepted more than 95 times, the probability of acceptance.

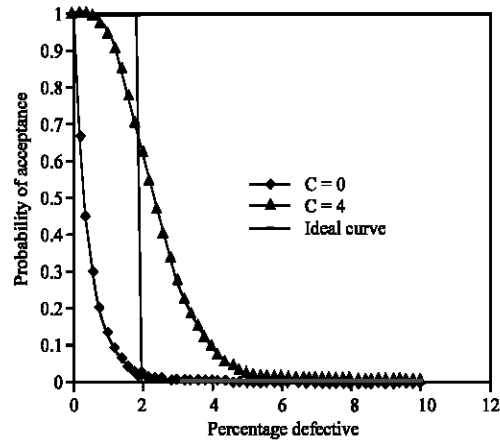


Fig. 1: Operating characteristic curve

If the fraction defective of the submitted lots is more, then the probability of acceptance will be less, when it is equal to or more than LTPD very few lots will be accepted and the consumer is taking the risk of accepting the bad lots. A sampling plan will be good when its OC curve is sensitive enough to detect any increase in fraction defectives; for this  $p_1$  should be closer to  $p_2$ , when  $p_1$  becomes equal to  $p_2$  OC curve becomes ideal. Sampling plan parameters 'n' and 'c' are designed such that OC curve passes through co-ordinates (AQL,  $1-\alpha$ ) and (LTPD,  $\beta$ ).

Whittingham proposed a method of choosing sampling plan for the given AQL and the LTPD, the sample size 'n' and the allowable number of defectives 'c' can be calculated for various values of AQLs and LTPDs and their associated values of  $\alpha$  and  $\beta$ . However, as 'n' and 'c', must be integers, a sampling plan that approximately passes through required identifying points can be obtained from standard table.

The need for an unstructured approach to acceptance sampling is highlighted by Victor *et al.* (1990). They demonstrated that, although, acceptance sampling is a structured procedure in general, unstructured approaches is suitable in make use of total resources. They have a developed a compromise plan based on costs and quality, which gives immediate impact in dealing with competing quality objectives, system constraints and system parameters. This sampling plan also proves to be optimal from an economic, statistical, or behavioral perspective.

**The problem:** By acceptance sampling the products are evaluated as conforming or non-conforming to the required quality specifications. Therefore, the producer is more focused in meeting the specification by sorting out the non-conformance products by inspection. But it leads

to higher amount of scarp, rework, repair costs and ultimately poor product quality. Also difficulties related to variability are often unnoticed or unattended. However, from the customer perception, the product that barely meets the specification is as good (or bad) as the product that is barely out of specification. Therefore, it is essential to develop an evaluation system by combining cost, target and variation from the target.

In this study, a single sampling attribute plan model is presented to address this problem of satisfying both the customer and producer needs.

**MATERIALS AND METHODS**

The proposed methodology is motivated by the following example Kacker (1985). Sony was manufacturing television tubes in 2 plants, Sony-USA and Sony-Japan. One of the quality characteristics is color density of the tube with a tolerance of  $m \pm 5$ . Sony-USA able to produce all the color tubes within this specification limits. The distribution is uniform over the specification range. Whereas the Sony-Japan followed the Taguchi Quality loss function-Nominal the Best and the resulting distribution is normal. Some parts outside the specification limits and are scrapped as shown in Fig. 2. However, later it was discovered that Sony-Japan television sets considered as better quality sets than their USA counter parts. Therefore, it is essential to control variations from the mean as well within the specification limits.

Single sampling attribute plans has the important advantage that it puts psychological pressure on the part of producer to produce within the specification limits. On the other hand Taguchi’s Quality Loss function emphasize on the meeting the target rather than simply satisfying the specification limits. Therefore, authors proposed a strategic modeling of single sampling attribute sampling plan to include these 2 features for evaluation of product quality.

Juran (1997) says the producer’s risk as Type-I error or level of significance denoted by ‘ $\alpha$ ’. Figure 3 shows the acceptance region in a normal distribution. The area of acceptance region is 95% and the remaining 5% truncated equally on either side of the normal distribution is the producer’s risk.

The standard normal deviate enables the AQL and LTPD chosen for the design specifications, these normal deviate is altered for the purpose of reducing variability of the process. The altered tolerances associated with them will be referred to us QLF tolerances, the products offered for evaluation would be treated as conforming

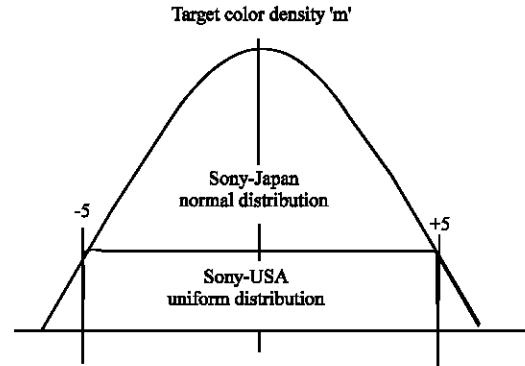


Fig. 2: Customer perception of quality

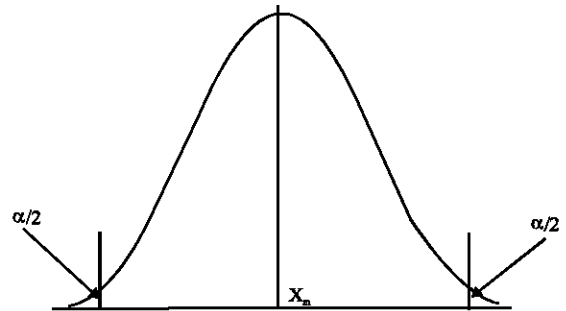


Fig. 3: Type-I error in a normal distribution

or non-conforming to the QLF tolerances, which are subjected to an attribute type inspection (go and no-go).

**Numerical example:** William *et al.* (1999) pointed out the difficulty of publishing papers on acceptance sampling topics without establishing a strong link to the practical problem. Therefore, data for this study is collected from an analysis of a firm that produces a large volume of zip fasteners.

Now, consider one of the important quality characteristic of the zip fastener with a minimum requirement of 50 mm with a tolerance of  $\pm 0.010$  mm. The fraction defective AQL is 0.010 at the ‘ $\alpha$ ’ value of 5% and the fraction defective LTPD is 0.065 at the ‘ $\beta$ ’ value of 10%.

The design of sampling plan is as follows,  
Quality characteristic requirements

- Fraction defective AQL = 0.010
- The fraction defective LTPD = 0.065
- The producer’s risk  $\alpha$  = 0.05
- The consumer’s risk  $\beta$  = 0.10

AAQL Calculation;

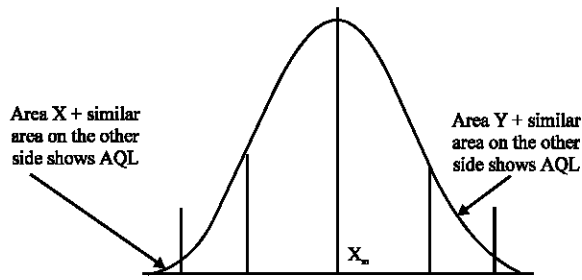


Fig. 4: AQL and AAQL in a normal distribution

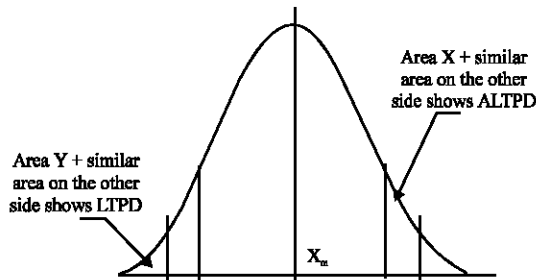


Fig. 5: LTPD and ALTPD in a normal distribution

**Step1:** The standard normal deviate

$$Z = \frac{X - X_m}{\sigma}$$

$$\therefore \sigma = \frac{X - X_m}{Z}$$

Now, the area of % conformities for  $X \pm 0.010$  is as shown in Fig. 4.

$$\text{i.e.,} \quad = \text{Half area of the Normal curve} - (0.010/2)$$

$$= 0.5 - (0.010/2) = 0.495.$$

By using the Normal tables the standard Normal Deviate for 0.495 is 2.58

$$X - X_m = 0.010$$

$\therefore$  The standard deviation  $\sigma$

$$= 0.010 / 2.58 = 0.0038$$

**Step 2:** Select a AAQL say a larger value as 0.065  
Now area for % conformities for this AAQL value is

$$= 0.5 - 0.065/2$$

$$= 0.4675$$

$\therefore$  From the tables, the standard normal deviate for this value is

$$Z = 1.85$$

$\therefore$  To have this value in original units (i.e.,  $Z * \sigma$ )

$$Z * \sigma = 1.85 * 0.0039$$

$$X - X_m = 0.007 \text{ psi}$$

Similarly ALTPD Calculation is as follows,

**Step 1:** Now the area for % conformities represents Fig. 5 for this ALTPD is

$$= 0.5 - 0.065/2 = 0.4675$$

$\therefore$  The standard normal deviate for this value is

$$Z = 1.85$$

$\therefore$  The standard deviation  $\sigma_1$

$$= \frac{X - X_m}{Z}$$

$$= 0.010 / 1.85 = 0.0054$$

**Step 2:** To transform this TQLF tolerances to the normal standard deviate

$$0.007 / \sigma_1 = Z$$

$$0.007 / 0.0054 = 1.3$$

From the normal tables, area outside this statistic,  $Z = 1.3$  is = 0.1936

$\therefore$  The fraction defective ALTPD for TQLF tolerances of  $\pm 0.007$  is 19.36%

By this method we altered the following values

From table for the AAQL and ALTPD  $n = 61$  and  $c = 7$ . And thus, the sample size is reduced without compromising the acceptance sampling parameters, from  $[n = 82, c = 2]$  to  $[n = 61, c = 7]$  with the same producer's and consumer's risk.

From the Fig. 6, it is clear that, the strategic model of the sampling plan pressurizes the producer to make products with lesser deviation from the target value. Thus, the concept of tolerating the defects in the product can be phased out by consistently meeting the target value.

This modified method also moves the role of sampling plan, from sentencing the lot as acceptable/unacceptable to an action that, provide feedback about the quality levels. These quality levels, urges the producers to develop an effective process control to produce close to the target (Edward *et al.*, 1999).

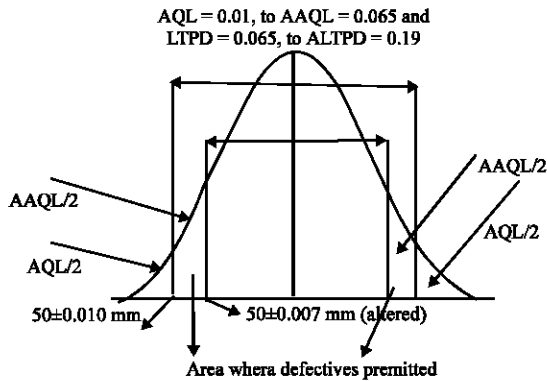


Fig. 6: Working of strategic model of single sampling attribute plan

The above numerical example illustrates how a quality characteristic with bilateral limit is calculated. This will apply in majority of cases. In this example, the mean of the distribution and the nominal value of the quality characteristic are assumed to coincide therefore  $X_m$  is known. In many well-organized plants process capability studies were carried out periodically. If this is so although variations arise by variations in the mean, the standard deviation is nearly constant from lot to lot which is why, if a process is controlled by an X and R chart and found to be stable. If this is not the case, it is a fairly simple exercise to get a sufficiently good estimate of the standard deviation by taking samples from the process and subjected to simple statistical examinations.

### RESULTS AND DISCUSSION

Consider a lot size of 1000 and the fraction defective in submitted lot is 0.02.

For sampling plan based on standard table.

The average number of samples required for assuring the given quality specifications is Average Total Inspected ATI and is calculated as follows

$$\begin{aligned} \text{Sample size } n &= 82 \\ \text{Acceptance number } c &= 2 \end{aligned}$$

The probability of acceptance of the above sampling plan based on Poisson Probability distribution ( $P_a$  (Poisson)) from Table G, is 0.773 (Montgomery, 1991).

$$\begin{aligned} \text{Then ATI} &= n \cdot P_a + N(1 - P_a) \\ &= 291 \end{aligned}$$

For new sampling plan,

$$\begin{aligned} \text{Sample size } n &= 62 \\ \text{Acceptance number } c &= 7 \end{aligned}$$

The probability of acceptance of the above lot ( $P_a$ ) from Table G, is 1

$$\begin{aligned} \text{Then ATI} &= n \cdot P_a + N(1 - P_a) \\ &= 62 \end{aligned}$$

ATI is reduced from 291-62 for the same quality assurance, yielding to lower inspection costs.

However, in testing the sample, if the value is outside the given specification limit (i.e.,  $50 \pm 0.010$  mm) then the distribution of the lot is unknown, so sample size from the standard tables are to be used.

(i.e., If the sample value is 50.015 or 49.08 (outside the  $50 \pm 0.010$  mm), the distribution of the lot is unknown, then the sampling plan extracted from the table,  $n = 200$  and  $c = 0$  should be used).

### CONCLUSION

The entire idea of modeling the sampling plan is motivated by the Genichi Taguchi's Loss Function, which states that, any departure from the desired mean value will incur loss not only to the manufacturing firm but also to the society. Hence, it becomes increasingly important to reduce variability. The proposed strategic modeling of attribute sampling plan exerts psychological pressure on producer to make the products with minimum variability from the mean. The proposed model also reduces the sample size to be used, without violating the statistical conditions. The increase in acceptance number, probability of acceptance is justified by the corresponding reduction in the specification limits. This procedure can also be easily extended to situations with unilateral specification limits. It is obvious that reduction in sample size resulting in lower inspection costs, time, personnel and inventory. This modified method of sampling plan design provides better protection for the consumer as well as producer, accumulation of quality history, feedback for process control and pressurize the producer to improve the process.

### Notations

- $X_m$  = Mean of assumed distribution based on given AQL.
- $X_m$  = Mean of assumed distribution based on given LTPD.
- $s$  = Standard deviation of the assumed distribution.
- AQL = Acceptable Quality Level.
- AAQL = Altered Acceptable Quality Level.

ATI = Average Total Inspected.  
c = Acceptance number.  
LTPD = Lot Tolerance Percent Defective.  
ALTPD = Altered Lot Tolerance Percent Defective  
N = Lot size.  
n = Sample size to be used.  
p = Fraction defective of the submitted lot.  
 $P_a$  = Probability of acceptance.  
 $p_1$  = Fraction defective for a given ( $\alpha$ ).  
 $p_2$  = Fraction defective for a given ( $\beta$ ).  
 $\alpha$  = Producer's risk.  
 $\beta$  = Consumer's risk.

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