

## A More Accurate Correlation for the Productivity Index of Horizontal Wells

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**Abstract:** Because their great efficiency in producing higher flow rates per unit pressure drawdown, horizontal wells have currently become a popular alternative for the development of hydrocarbon fields around the world. So far, most of the introduced correlations to estimate the productivity index for these wells have shown certain differences among their results. This does not allow us to properly establish which one of them provides the closest value to the actual one, since there is no evidence of a trustable enough reference point. Throughout the years, several investigations for the determination of horizontal-well productivity index have been carried out. These researches have been focused on the determination of steady-state solutions for the above-mentioned parameter, therefore, a diverse number of correlations have been introduced. This study proposes an improved steady-state correlation to calculate productivity index for horizontal wells and evaluates the most commonly used existing correlations to estimate this parameter by using numerical simulation. Besides that, a sensitivity analysis on the influence of the variation of each variable in the existing and proposed models was carried out. The analysis was conducted by generating a synthetic drawdown test by means of a commercial reservoir simulator. Using the pressure derivative curve, a time range where steady-state behavior takes place was defined. Then a simulation was performed with the purpose of determining the pressure distribution in the reservoir within that range of time. This allows us to estimate the horizontal-well productivity index for any drainage radius. There is no way to establish what correlation is the most accurate since the productivity index of a horizontal well cannot be physically measured. Therefore, we conducted a sensitivity analysis considering the variation of the most relevant parameters and compared to the results from a simulator. More than 500 simulation runs were performed to estimate the results obtained by the improved correlation introduced in this work and the existing ones. Several plots of productivity index versus each one of the model variables were constructed for comparison purposes leading to obtain an application range for the existing correlations; among these, we observed that Joshi's correlation matches well with the simulated results. However, the proposed correlation provides much better results than those provided by Joshi's within a very wide range of variation of the parameters involved in the different correlations. We did not provide any value of the deviation error since this normally increases as the the studied parameter increases. However, comparative plots speak by themselves.

**Key words:** Darcy's law, numerical simulation, productivity index, steady state, radial flow

### INTRODUCTION

Throughout the years, several investigations on the determination of the productivity index for horizontal wells have been developed. Most of the researches have been focused on finding a steady-state flow solution which has led to the development of different correlations. These correlations were introduced by such well-known researchers as Giger (1983), Merkulov (Borisov, 1954), Renard and Dupuy (1990), Joshi (1988, 1991), Penmatcha *et al.* (1997). However, there are remarkable differences among their results which do not

allow us to clearly establish which one matches closely the actual value since not accurate comparison point has been given.

In this study, based on numerical simulation, we define which existing correlation is the most appropriate for the estimation of the productivity index in horizontal wells. For this purpose, numerical experiments were conducted using a popular commercial software which uses the power of PEBI grids which provide a very convenient way to numerically represent horizontal wells, faults and pinchouts (Escobar and Tiab, 2002). After evaluating the impact of several parameters, a range of

application was properly defined for each correlation. Afterwards, a new correlation was developed with the goal of overcoming most of the limitations of the previously ones.

Productivity index,  $J$ , for a well is defined as the ratio  $q/\Delta p$ , which means, the volume of produced oil per unit of pressure drawdown. To simply estimate  $J$ , it must be assumed that steady-state conditions are met, this means, there exists a constant-pressure value at the external reservoir boundary. By definition, the productivity index of a vertical well under steady-state flow is estimated by Eq. 1:

$$J = \frac{q}{\Delta p} = \frac{0.007078 kh/(\mu_o B_o)}{\ln(r_e/r_w)} \quad (1)$$

Several steady-state productivity-index analytical solutions have been presented for horizontal wells. These solutions can be extended for both transient and pseudosteady-state conditions using concepts of drainage boundary expansion with respect to time, shape factors and effective wellbore radius. Because of the mathematical relation of Fourier's law and Ohm's law with Darcy's law, this type of equations have been tested with physical models. The reported solutions are as follows: Borisov's equation (Borisov, 1954):

$$q_h = \frac{2\pi k_h h \Delta p / (\mu_o B_o)}{\ln[(4r_{eh}/L)] + (h/L)\ln[h/(2\pi r_w)]} \quad (2)$$

Giger's equation (Giger, 1983; Penmatcha *et al.*, 1997):

$$q_h = \frac{2\pi k_h L \Delta p / (\mu_o B_o)}{(L/h)\ln\left[\frac{1 + \sqrt{1 - [L/(2r_{eh})]^2}}{L/(2r_{eh})}\right] + \ln[h/(2\pi r_w)]} \quad (3)$$

Renard and Dupuy's equation (Joshi, 1988, 1991; Penmatcha *et al.*, 1997):

$$q_h = \frac{2\pi k_h h \Delta p}{\mu_o B_o} \left[ \frac{1}{\cosh^{-1}(X) + (h/L)\ln[h/(2\pi r_w)]} \right] \quad (4)$$

Where:

- $X = 2a/L$  for an elliptic drainage area
- $a =$  Half of the ellipse's mayor axis

Joshi's equation (1988, 1991; Penmatcha *et al.*, 1997)

$$q_h = \frac{2\pi k_h h \Delta p / (\mu_o B_o)}{\ln\left[\frac{a + \sqrt{a^2 - (L/2)^2}}{L/2}\right] + (h/L)\ln[h/(2r_w)]} \quad (5)$$

Where:

$$a = (L/2) \left[ 0.5 + \sqrt{0.25 + (2r_{eh}/L)^4} \right]^{0.5}$$

The productivity index,  $J_h$ , for the horizontal well can be estimated by dividing  $q_h$  by  $\Delta p$ . All the above relationships were developed for isotropic reservoirs ( $k_h = k_v$ ). To convert these equations into oil-field units,  $2\pi$  in the numerator has to be changed by constant 0,007078.

### EVALUATION OF CORRELATIONS

Figure 1-7 show the respective influence of each variable involved in the correlations on the productivity index for a horizontal well. These plots were built using the correlations for productivity index by Borisov (1954), Giger (1983), Renard and Dupuy (1990) and Joshi (1988, 1991). The simulated results obtained using a well-known reservoir simulator are also presented. We clearly observe in these plots that the Joshi's correlation matches more closely the results obtained from the simulator. It is also observed that such variables as horizontal wellbore length,  $L$  and reservoir external radius,  $r_e$ , cause instability in the results from the correlations, especially, Giger's and Borisov's. Notice in Fig. 5, for  $L > 1000$  ft (304.8 m), Giger's correlation presents too much deviation from the simulated curve. Same situation takes place in the correlations of Borisov (1954), Renard and Dupuy (1990) for  $L > 1200$  (396.4 m) and Joshi's for  $L > 1600$  (487.7 m). It must take into account that this behavior depends upon the drainage radius,  $r_e$ . As observed in Fig. 7, Borisov's and Giger's correlations become unstable for  $r_e < 1000$  ft (304.8 m). Only the correlations of Joshi (1988, 1991), Renard and Dupuy (1990) behave steadily, even though, the error increases progressively as  $r_e$  decreases.

As seen in Fig. 2, reservoir thickness is another parameter affecting directly the results. The thicker the formation, the higher the deviation error. Results from Joshi's correlation deviates from the simulation model for  $h > 200$  ft (60.9 m). For this thickness value, the results from the remaining correlations deviate about 10 % from the expected behavior.

The influence of horizontal wellbore length on the productivity index can be observed in Fig. 5  $r_e = 1000$  ft (304.8 m). This plot permits to establish a relationship between  $L$  and  $r_e$ , in order to define the recommended application range for each correlation based upon the maximum value of the ratio  $L/r_e$ , as follows:

Correlation of	Range of application
Borisov	$L/r_e \leq 1,2$
Giger	$L/r_e \leq 0,8$
Renard and Dupuy	$L/r_e \leq 1,2$
Joshi	$L/r_e \leq 1,6$
This study	$L/r_e \leq 2,0$

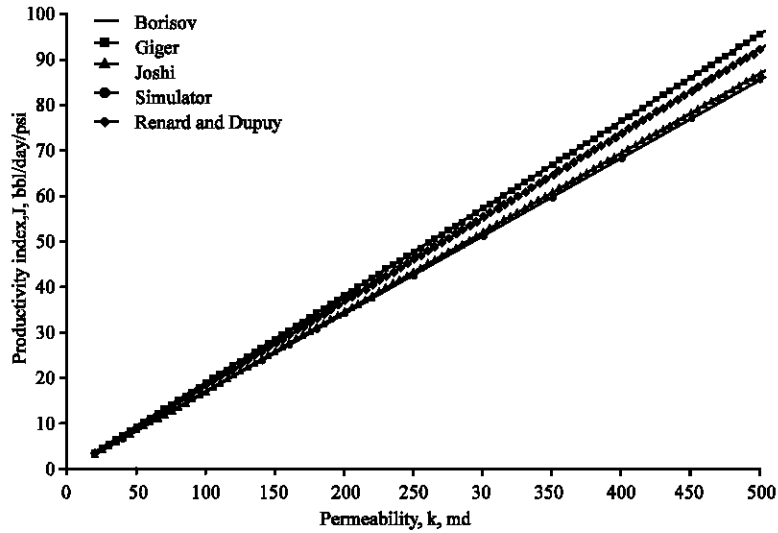


Fig. 1: Permeability vs. productivity index

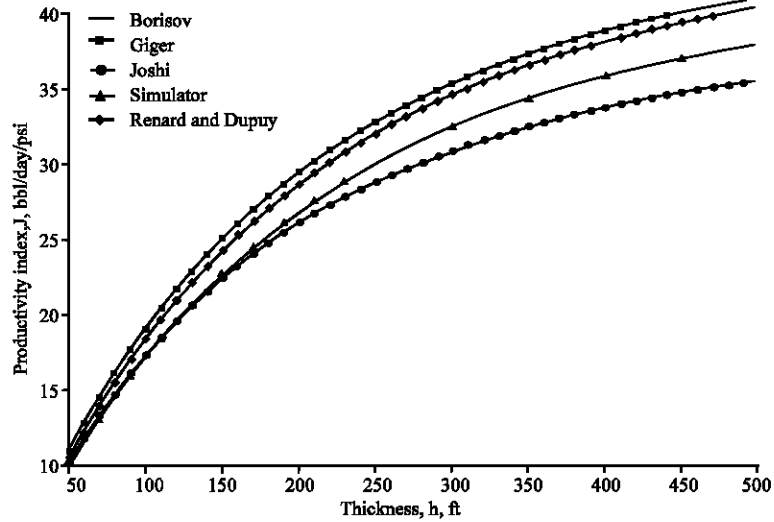


Fig. 2: Reservoir thickness vs. productivity index

As previously mentioned, the model proposed by Joshi more closely follows the results from the numerical simulation. However, a new model with a much lower deviation error than Joshi's is presented here:

$$J = \frac{0.007078k_h h / (\mu_o \beta_o)}{\cosh^{-1} \left[ 1.075 \left( 0.5 + \sqrt{0.25 + (2r_e/L)^4} \right)^{0.5} \right] + 0.874(h/L) \ln [h/(2r_w)]} \quad (6)$$

Differently to other correlations, our model includes two constants to allow an optimum match with respect to the simulated results. These constants were found by statistical regression. Equation 6 can also be written as a steady-state flow equation as follows:

$$q_h = \frac{0.007078k_h h \Delta p / (\mu_o \beta_o)}{\cosh^{-1} \left[ 1.075 \left( 0.5 + \sqrt{0.25 + (2r_e/L)^4} \right)^{0.5} \right] + 0.874(h/L) \ln [h/(2r_w)]} \quad (7)$$

A comparison of results of our equation against Joshi's and the simulator are given in Fig. 8-12. We only compare to the results from Joshi since it has been demonstrated that this one provides better results. However, the correlation presented in this study provides the best results. As observed before, the volumetric factor, viscosity and permeability do not affect significantly the productivity index result. Therefore, they are not plotted. As for the case of formation

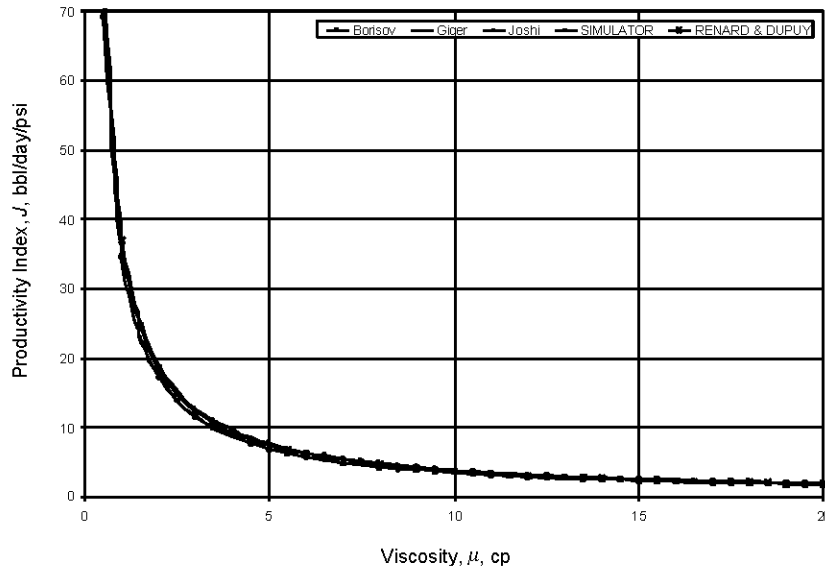


Fig. 3: Oil viscosity vs. productivity index

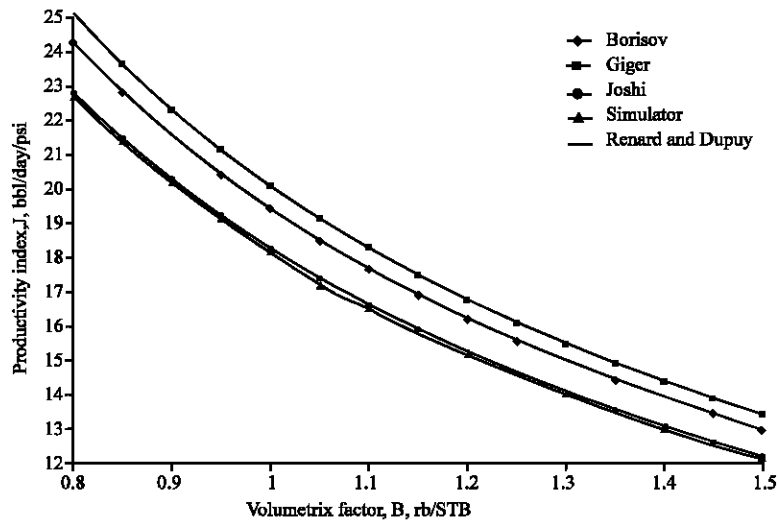


Fig. 4: Oil volume factor vs. productivity index

thickness, see Eq. 8, our model provides the best match with the simulated solution with an error less than 1 % as far as reservoir thickness is concerned. Besides that, our model allows us to extend the range of application to a ratio  $L/r_e \leq 2$  based on the behavior observed in Fig. 9 and 10.

Figure 11 and 12 show the effect of wellbore radius and drainage radius, respectively. Our model not only behaves more steadily but also produces the best match with the simulated curve.

**The proposed correlation:** Estimation of productivity index in horizontal wells is directly affected by two key parameters which determine flow direction toward the well: one in the vertical direction and another in the horizontal direction. We can see these two factors in any of the productivity-index correlations, Eq. 3-5. For example, looking at the Joshi's correlation, Eq. 5, there are two terms in the denominator: The first one (left-hand side) responds for flow in the horizontal direction and the second one is responsible for flow in the vertical direction. Notice that this last term is a function of

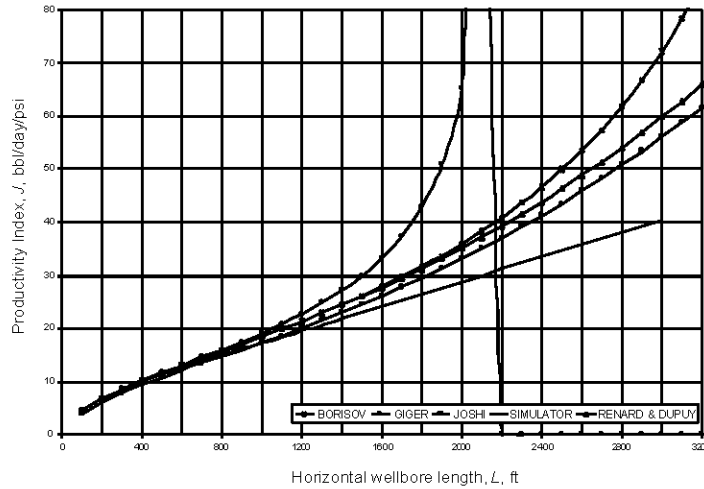


Fig. 5: Horizontal wellbore length vs. productivity index ( $r_e = 1000$  ft)

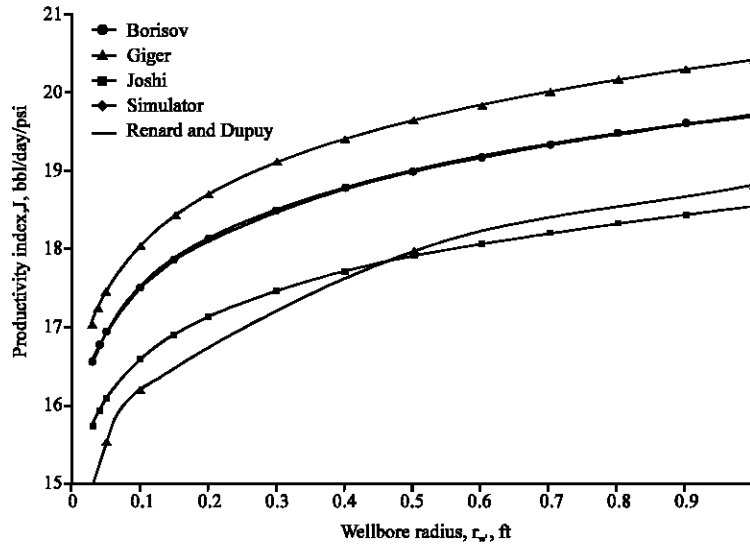


Fig. 6: Wellbore radius vs. productivity index

reservoir thickness. This term also has a direct relationship with the term of the denominator in Eq. 1 which denominator refers to horizontal flow. By analogy, flow in a horizontal well is the same as flow in a vertical well rotated 90°. We can state that  $r_e$  for a vertical well is equivalent to  $h/2$  for a horizontal well. After replacing  $r_e$  by  $h/2$  into Eq. 1, it yields:

$$q = \frac{2\pi kh\Delta p / (\mu_o B_o)}{\ln((h/2)/r_w)} \quad (8)$$

The influence of vertical flow in horizontal wells is closely linked to the relation between reservoir thickness and wellbore length,  $h/L$ , which means, the lower  $h/L$ , the lower the influence of this type of flow.

Applying this concept to Eq. 8, we obtain:

$$q = \frac{2\pi kh\Delta p / (\mu_o B_o)}{(h/L)\ln[h/(2r_w)]} \quad (9)$$

This analysis coincides with the one applied on the second term in the denominator of Joshi's correlation. To perform the analysis for horizontal flow in horizontal wells, all the factors of the studied correlations were tested. We found that the horizontal flow factor proposed by Renard and Dupuy (1990) behaves in the same manner as the one in Joshi's correlation. Therefore, we took the factor of the correlation proposed by Renard and Dupuy (1990) to represent this type of flow. It was necessary to introduce a weighting

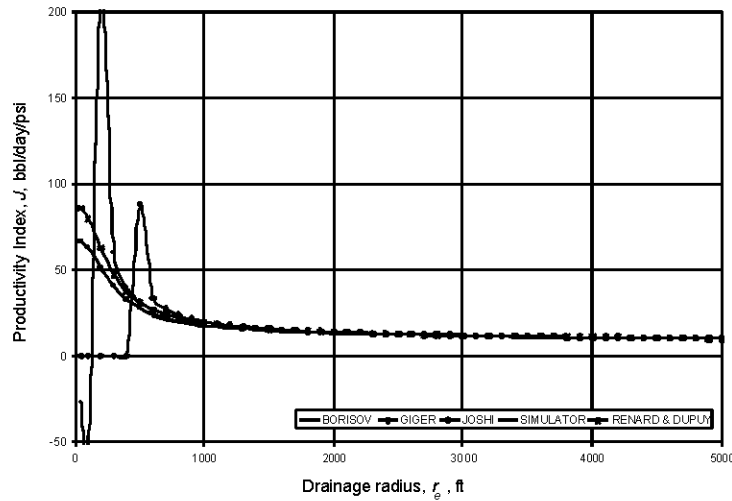


Fig. 7: Drainage radius vs. productivity index

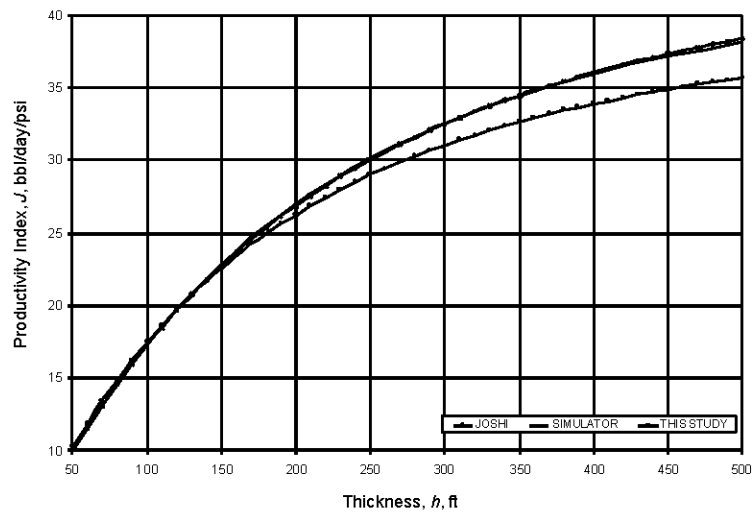


Fig. 8: Reservoir thickness vs. productivity index

coefficient to both terms in the denominator in order to adjust the correlation to the simulated results, thus:

$$q = \frac{2\pi kh\Delta p / (\mu_o B_o)}{\cosh^{-1}(\psi X) + \omega(h/L)\ln[h/(2r_w)]} \quad (10)$$

being X same variable as in Joshi's correlation.

Using a trial-and-error procedure, the values of constants  $\psi$  and  $\omega$  were determined when the lowest deviation error was attained. These values were found to be  $\psi = 1.075$  and  $\omega = 0.874$ . Plugging these values into Eq. 10 and replacing a and X by their respective values, the final form of the equation is the one given in Eq. 6.

**Field example:** Joshi (1991) presented in his book an example of the determination of the productivity index for an horizontal well if its drainage area is 80 acres. Other relevant information regarding this example is listed as follows:

$$k_v = k_h = 75 \text{ md } \mu_o = 0.62 \text{ cph} = 160 \text{ ft}$$

$$B_o = 1.34 \text{ rb/STB } \phi = 3.8 \% r_w = 0.365 \text{ ft}$$

It is required to estimate the productivity index with the here proposed correlation and compare to the results obtained from Joshi (1991).

**Solution:** The drainage radius,  $r_{eh}$ , of a horizontal well results to be 1053 ft if circular geometry is assumed. Using the correlation of this study:

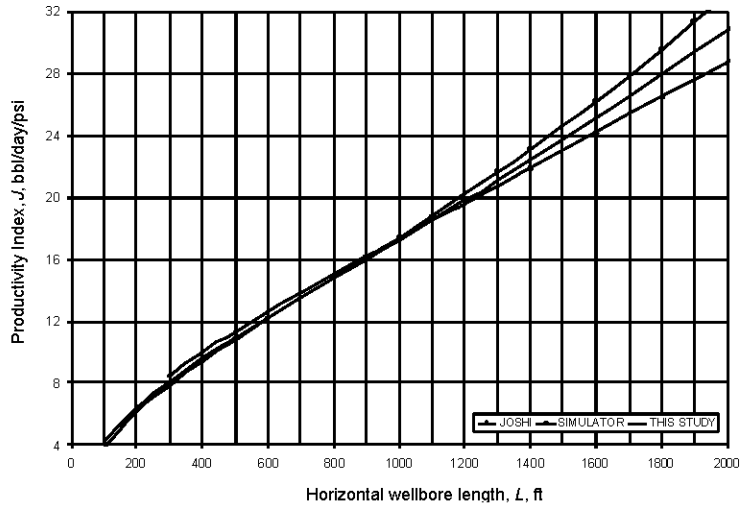


Fig. 9: Horizontal wellbore length vs. productivity index ( $r_e = 1000$  ft)

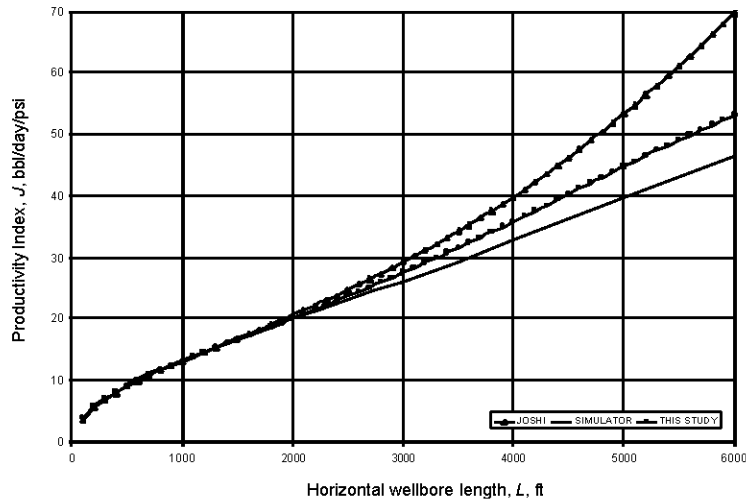


Fig. 10: Horizontal wellbore length vs. productivity index ( $r_e = 2000$  ft)

$$J_h = \frac{0.007078(75)(160)/(0.62 \times 1.34)}{\cosh^{-1} \left[ 1.075 \left( 0.5 + \sqrt{0.25 + \left( \frac{2 \times 1053}{1000} \right)^4} \right)^{0.5} \right]} + 0.874 \left( \frac{160}{1000} \right) \ln \left[ \frac{160}{2 \times 0.365} \right]$$

$J_h = 44.96$  STB/(day-psi)

The following are the results obtained by Joshi (1991):

Correlación de	$J_h$ , STB/(day-psi)	% Deviation
Borisov	48.3	7.43
Giger	49.8	10.77
Joshi	44.4	1.25

We estimated the absolute deviation with respect to the correlation of this research. As exposed before, the second best option is the correlation of Joshi. Since  $L/r_e = 0,95$ , as expected, Giger’s correlation is outside the range, then, its result is considered inaccurate.

**RESULTS**

The more significant impact on the productivity index is caused by the horizontal wellbore length, drainage radius, wellbore radius and reservoir thickness as observed in Fig. 2, 5-12. Besides, instability in the productivity index is introduced at low values of drainage radius and high values of wellbore length. As

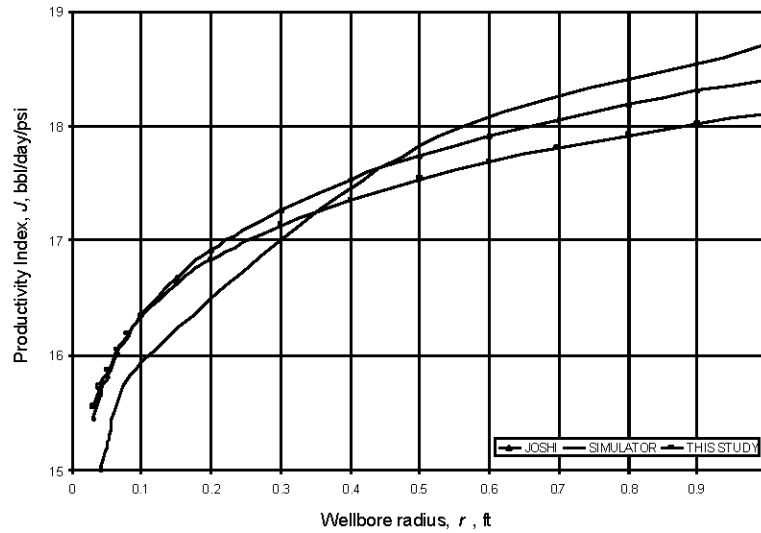


Fig. 11: Wellbore radius vs. productivity index

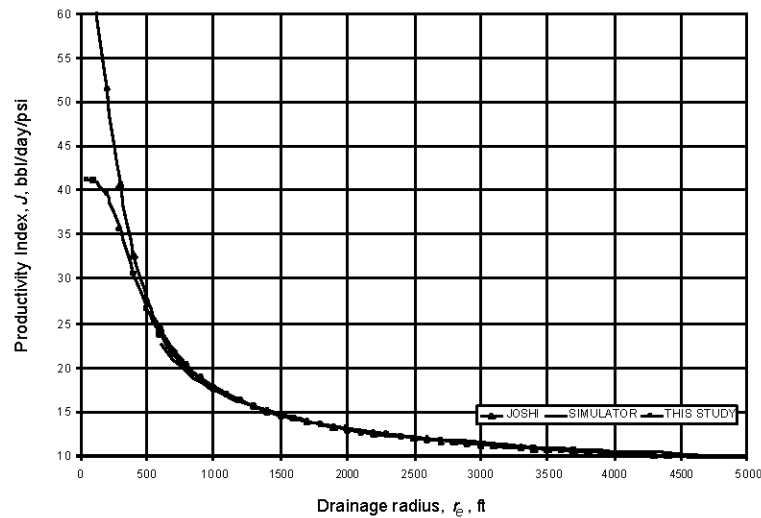


Fig. 12: Drainage radius vs. productivity index

observed in Fig. 8-12, our model most closely follows the numerical solution. This makes our model the best option to predict the productivity index of a horizontal well and Joshi's the second one.

Therefore, our model was taken as the reference point in the worked example and we found that, in fact, the result from Joshi's closely matches ours.

### CONCLUSION

The best correlation, so far, to estimate productivity index in horizontal wells is presented. This correlation behaves steadier for a wider range of

application and presents the lowest deviation error among the studied correlations. According to the proximity to the simulated results, Joshi's correlation is the second option to estimate the productivity index in horizontal wells.

The limit for all the studied correlations is presented. The maximum  $L/r_e$  ratio resulted to be 1,2 for Borisov, 0,8 for Giger, 1,2 for Renard and Dupuy, 1,6 for Joshi and 2,0 for this study.

Horizontal wellbore length,  $L$ , drainage radius,  $r_e$ , wellbore radius,  $r_w$  and reservoir thickness,  $h$ , were found to affect more the results of all the studied correlations.



**Nomenclature:**

A	:	Well drainage area, Ac.
$B_o$	:	Oil volumetric factor, rb/STB.
$c_t$	:	Total compressibility, $\text{psi}^{-1}$ .
h	:	Formation thickness, ft.
J	:	Productivity index, bbl/day/psi.
$J_h$	:	Horizontal well productivity index, bbl/day/psi.
k	:	Permeability, md.
$k_v$	:	Vertical permeability, md.
$k_h$	:	Horizontal permeability, md.
L	:	Horizontal wellbore length, ft.
$p_i$	:	Initial reservoir pressure, psi.
$p_{wf}$	:	Well flowing pressure, psi.
$p_e$	:	Pressure at the drainage radius, psi.
q	:	Oil production rate, STB/day.
$q_h$	:	Horizontal well oil production rate, STB/day.
$r_e$	:	Drainage radius or reservoir external radius, ft.
$r_{eh}$	:	Horizontal well drainage radius, ft.
$r_w$	:	Wellbore radius, ft.
$r_w$	:	Effective wellbore radius, ft.
s	:	Skin factor.
$\Delta p$	:	Pressure drop, psi.
$\mu_o$	:	Oil viscosity, cp (cp).

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