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Finite Element Formulation of a Beam with Piezoelectric Patch

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Abstract: A finite element model is used to analyze beams with piezoelectric sensors and actuators. The formulation is based on first order Kirchoff theory and accounts for lateral strains. Various parametric studies are conducted to demonstrate the effectiveness of piezoelectrics in actively controlling the vibration of beams.

Key words: Piezoelectric materials, beams, vibration control, lateral strain, sensors, actuators

INTRODUCTION

To satisfy the increasing demands for obtaining a high structural performances, the conception and the control of the light composite structures attracted a large number of researches these last years. The active control can eliminate the unwanted forces by mechanisms as sensors, actuators and feedback controllers. The piezoelectric materials which deform when an electric field is applied and conversely, generate a load in answer to a mechanical deformation can be used as actuators and sensors respectively. In the present study, the control of the vibrations of beams with integrated piezoelectrics sensors and actuators is studied.

The aspect of vibrations control of plates by piezoelectric materials was studied by Yang and Huang (1997) and Piéfort *et al.* (1998). These models are based on the classic theory of laminated plates which neglects the effects of the transverse shear. Finite element model to predict the vibrations of the piezoelectric actuators are presented by Taleghani and Campbell (1999).

The aim of the present study, is to develop a model of finite elements for beams analysis with piezoelectric sensors and actuators based on first order Kirchoff theory. The present model takes into account the lateral strains which are often neglected in the conventional models of beams. Numerical results are presented to study the efficiency of piezoelectric sensors and actuators in the active control of beam's vibrations.

MATERIALS AND METHODS

Constitutive equations are developed for beams with integrated sensors and actuators. The formulation accounts for lateral strains and shear deformations. A beam with length (L), width (b) and thickness (h) is considered. The electric field is applied through the thickness of the piezoelectric material.

Piezoelectric constitutive equations, with neglected thermal effects, can be expressed as (Crawley and Anderson, 1990; Law *et al.*, 1996; Rao and Sunar, 1994; Wang *et al.*, 1994):

$$\begin{aligned} \{D\}_m &= [e]_{mi}\{\varepsilon\}_i + [\bar{\varepsilon}]_{mk}\{E\}_k \\ \{\sigma\}_i &= [Q]^E_{ij}\{\varepsilon\}_j - [e]^t_{mi}\{E\}_m \end{aligned} \quad (1)$$

where:

$\{\varepsilon\}$	=	The strain
$\{\sigma\}$	=	The stress
$\{D\}$	=	Electric displacement
$\{E\}$	=	Electric field
$[Q]$	=	Elastic stiffness matrix
$[e]$	=	Piezoelectric stress coefficient matrix
$[\bar{\varepsilon}]$	=	Permittivity constant matrices

Piezoelectric materials possess anisotropic properties. Piézocéramics as Titanate of Zirconate (PZT) are excellent candidates for piezoelectric sensors and actuators. Piézocéramics are polarized in the thickness direction and exhibit transversely isotropic properties in the xy-plane.

It should be noted that the piezoelectric stress coefficient matrix [e] is expressed in terms of the commonly available strain coefficient matrix [d] using the relation:

$$[e] = [D].[d] \quad (2)$$

For a beam problem, one can use $\sigma_y = \tau_{yz} = \tau_{xy} = 0$ to obtain the following reduced constitutive equations from Eq. 1:

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$$\begin{Bmatrix}\sigma_x \\ \tau_{xz}\end{Bmatrix} = \begin{bmatrix}\bar{Q}_{11} & 0 \\ 0 & \bar{Q}_{55}\end{bmatrix}\cdot\left(\begin{Bmatrix}\varepsilon_x \\ \gamma_{xz}\end{Bmatrix} - \begin{Bmatrix}d_{31} \\ 0\end{Bmatrix}\cdot E_3\right) \quad (3)$$

where, $\bar{Q}_{11}$ and $\bar{Q}_{55}$ in terms of Q_{ij} (i, j = 1 ÷ 6) are given by

$$\begin{cases}\bar{Q}_{11} = Q_{11} + Q_{12}\cdot\left(\dfrac{Q_{16}Q_{26} - Q_{12}Q_{66}}{Q_{22}Q_{66} - Q_{26}^2}\right) + Q_{16}\cdot\left(\dfrac{Q_{12}Q_{26} - Q_{16}Q_{22}}{Q_{22}Q_{66} - Q_{26}^2}\right) \\ \bar{Q}_{55} = Q_{55} - \dfrac{Q_{45}^2}{Q_{44}}\end{cases} \quad (4)$$

The strain displacement relations based on a first order shear deformation theory associated with the displacement field are given by

$$\begin{cases}\varepsilon_x = \dfrac{\partial u}{\partial x} - z\dfrac{\partial \psi}{\partial x} \\ \gamma_{xz} = -\psi + \dfrac{\partial w}{\partial x} = \beta\end{cases} \quad (5)$$

with $\psi + \beta = \partial w/\partial x$

where:

u and w = Displacements in the x and z directions, respectively

ψ = The rotation around the y axis

Finite element formulation: The variation kinetic energy can be expressed as:

$$T = \frac{1}{2}\int \rho\{\dot{u} \quad \dot{w} \quad \dot{\psi}\}\cdot[\bar{T}]\cdot\begin{Bmatrix}\dot{u} \\ \dot{w} \\ \dot{\psi}\end{Bmatrix}\cdot b\cdot dx \quad (6)$$

with

$$[\bar{T}] = \begin{bmatrix}h & 0 & 0 \\ 0 & h & 0 \\ 0 & 0 & \dfrac{h^3}{12}\end{bmatrix} \quad (7)$$

Integrating with respect to x, the kinetic energy can be expressed by:

$$T = \frac{1}{2}\{\dot{q}\}^T\cdot[M]\cdot\{\dot{q}\} \quad (8)$$

where (M) is element mass matrix.

The variation in the strain energy is expressed by

$$U = \frac{1}{2}\int\left(\begin{array}{c}\bar{Q}_{11}\left\{\dfrac{\partial u}{\partial x} \quad \dfrac{\partial \psi}{\partial x}\right\}^T\begin{bmatrix}1 & -z \\ -z & z^2\end{bmatrix}\begin{Bmatrix}\dfrac{\partial u}{\partial x} \\ \dfrac{\partial \psi}{\partial x}\end{Bmatrix} \\ + K\bar{Q}_{55}\{\gamma_{xz}\}^T\{\gamma_{xz}\}\end{array}\right)dv \quad (9)$$

Integrating with respect to y and z first and then the result should be integrated with respect to x, strain energy can be expressed by

$$U = \frac{1}{2}\{q\}^T\cdot[K]\cdot\{q\} \quad (10)$$

where (K) is the element stiffness matrix

The differential of the total energy expressed in terms of T, U, q, $\dot{q}$ and t is given by

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial U}{\partial q_i} = F_i \quad (11)$$

Where:

$$\{q_i\} = \{u_1, w_1, \psi_1, \ldots, u_N, w_N, \psi_N\}$$
$$\{\dot{q}_i\} = \{\dot{u}_1, \dot{w}_1, \dot{\psi}_1, \ldots, \dot{u}_N, \dot{w}_N, \dot{\psi}_N\}$$

Replacing U and T by their expressions in the Eq. 11 and 12 of motion are obtained as:

$$[M]\{\ddot{q}\} + [K]\{q\} = \{F\} \quad (12)$$

The approximation of the solution is made by polynomials functions called interpolation functions as:

$$u = [N_x].\{q\} \quad (13)$$

$$w = [P_x].[B]^{-1}.\{q\} \quad (14)$$

$$\Psi = [R_x].[B]^{-1}.\{q\} \quad (15)$$

Substituting these equations, the element mass and stiffness matrices can be express as:

$$[M] = \left(\rho h\int_0^1 \begin{array}{c}([N_x]^T[N_x]) + \left(([B]^{-1})^T[P_x]^T[P_x][B]^{-1}\right) \\ + \left(\dfrac{h^2}{12}([B]^{-1})^T[R_x]^T[R_x][B]^{-1}\right)\end{array} dx\right) \quad (16)$$

$$[K] = bh\left(\begin{array}{c}\int_0^1\left(\bar{Q}_{11}[N_x']^T[N_x']\right) + \left(\begin{array}{c}\dfrac{h^2}{12}\bar{Q}_{11}([A_x]^{-1})^T \\ [R_x']^T[R_x'][A_x]^{-1}\end{array}\right) \\ + \left(K\bar{Q}_{55}\dfrac{\phi l^2}{12}([A_x]^{-1})^T[P_x''']^T[P_x'''][A_x]^{-1}dx\right)\end{array}\right) \quad (17)$$

Where:

$$\phi = \frac{12EI}{K\bar{Q}_{55}Al^2}$$

In this study, we consider 2 types of loads. Concentrated and uniformly distributed loads.

Having established elements mass and stiffness matrices and element loads vector, it's necessary to assemble them to obtain the global matrices. The Eq. 18 of motion becomes:

$$[M_G]\{\ddot{q}\} + [C_G]\{\dot{q}\} + [K_G]\{q\} = \{F_G\} + \{F_p\} \qquad (18)$$

where:

$[M_G]$	=	Global mass matrix
$[K_G]$	=	Global stiffness matrix
$\{F_G\}$	=	Global loads vector
$[C_G]$	=	Global damping matrix

RESULTS AND DISCUSSION

To validate the present model, the results obtained for a clamped-free beam are compared with those of (Sunar and Rao, 1997). The observed error, according to the number of elements, varies from 0.009-3%.

We considered several parameters to analyze their influence on the behaviour of the beam: The structural damping coefficient, the thickness and the length of the piezoelectric patch with various boundary conditions. Results obtained by the present model are shown in Fig. 1 at 12.

The beam's length, width and thickness are: L = 40 cm, b = 1 cm, e = 0.5 cm. A uniformly distributed load of 10^3 N/m^2 is applied to the beam.

As shown in Fig. 2-5, the deflection at the free end of the beam decrease upto 20%. On Fig. 6-8, the influence of the length of the piezoelectric patch is studied. We notice an increase of the deflection. These same remarks were made by Chandrashekhara and Donthireddy (1997).

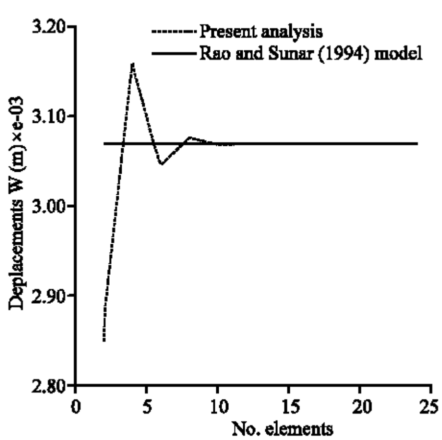

Fig. 1: Displacement w (m)

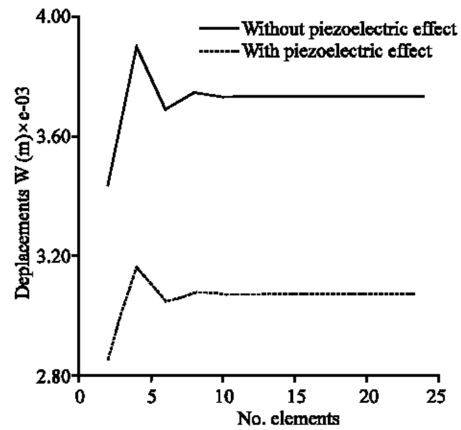

Fig. 2: Displacement w (m). Clamped-free beam

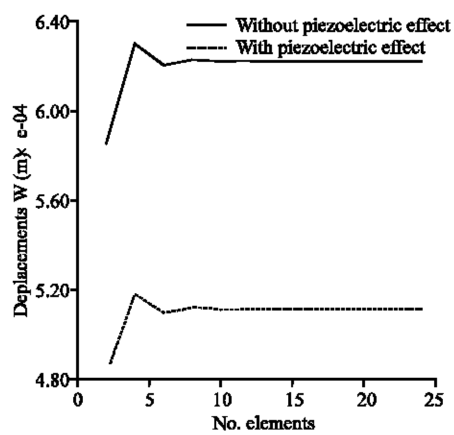

Fig. 3: Displacement w (m). Clamped-clamped beam

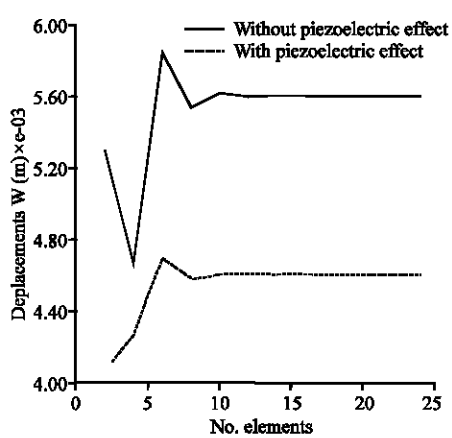

Fig. 4: Displacement w (m). Simple supported-simple supported beam

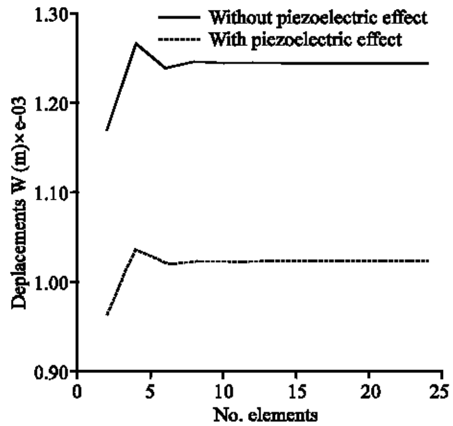

Fig. 5: Displacement w (m). Clamped-simple supported beam

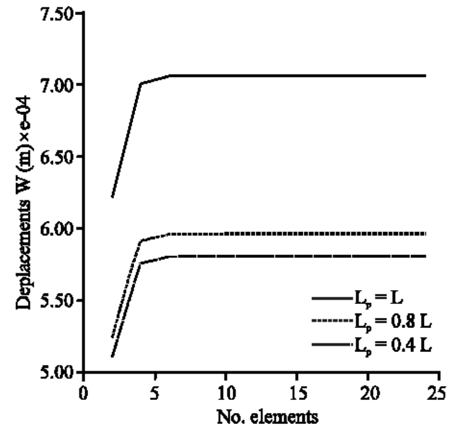

Fig. 8: Displacement w (m). Clamped-simple supported beam

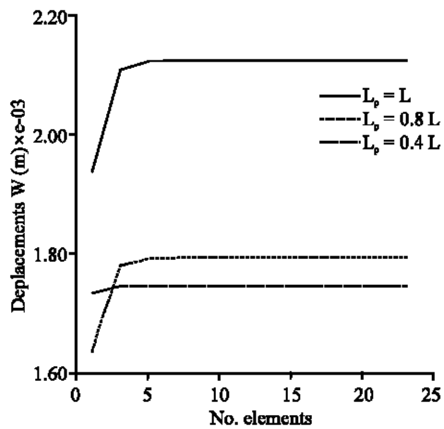

Fig. 6: Displacement w (m). Clamped-free beam

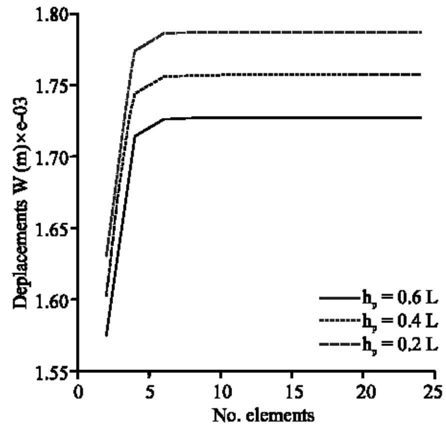

Fig. 9: Displacement w (m). Clamped-free beam

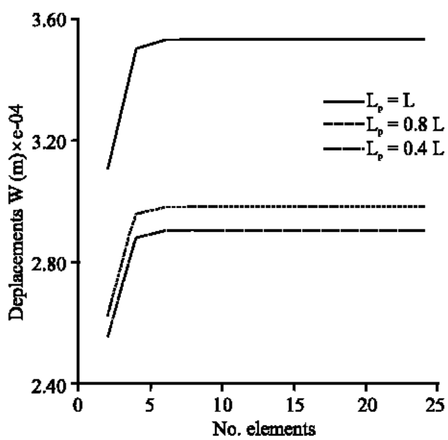

Fig. 7: Displacement w (m). Clamped-clamped beam

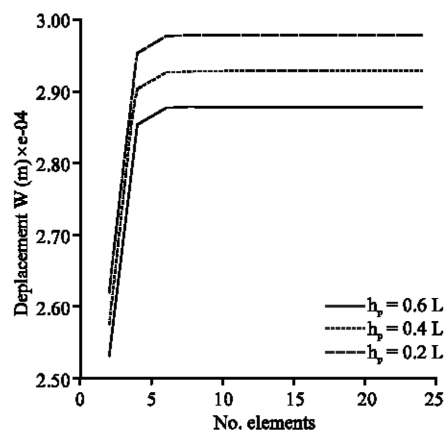

Fig. 10: Displacement w (m). Clamped-clamped beam

The results shown in Fig. 9-12 indicate a light decrease of deflection and rotation in the free end of the beam for thicknesses of the piezoelectric patch varying for $h_p = (0.2 \div 0.6)$ h. This is due to the increase of the load

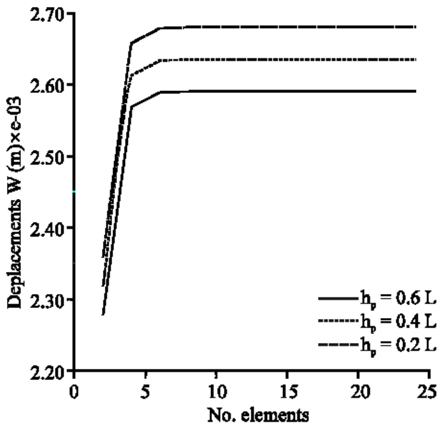

Fig. 11: Displacement w (m). Simple supported-simple supported beam

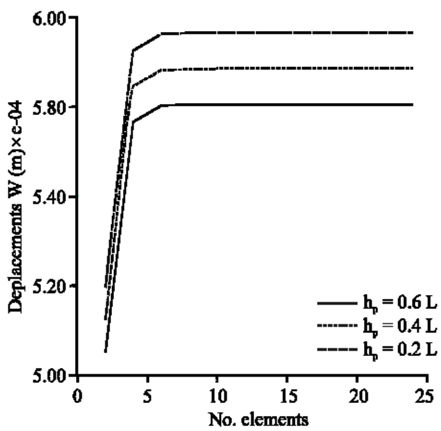

Fig. 12: Displacement w (m). Clamped-simple supported beam

vector, due to the piezoelectric element, by increasing the thickness of the piezoelectric patch what decreases the global load vector.

CONCLUSION

A finite element formulation based on the first order theory of Kirchoff is used to analyze beams with integrated piezoelectric actuators and sensors. The lateral strains are taken into account in the equations of beams. Several parameters are taken into account to study their influence and to demonstrate the efficiency of the piezoelectric materials in controlling the vibrations of beams.

REFERENCES

Chandrashekhara, K. and P. Donthireddy, 1997. Vibration suppression of composite beams with devices using a higher order theory. Eu. J. Mech., A/Solids, 16 (4): 709-721.

Crawley, E.F. and E.H. Anderson, 1990. Detailed models of piezoceramic actuation of beams. J. Intell. Mater. Syst. Struct., 1 (1): 4-25.

Law, H.H., P.L. Rossiter, G.P. Simon and L.L. Koss, 1996. Characterization of mechanical vibration damping by piezoelectric materials. J. Sound Vib., 197 (4): 489-513.

Piéfort, V., N. Loix and A. Preumont, 1998. Modeling of piezolaminated composite shells for vibration contrôle. NASA/TM 1998-352487.

Rao, S.S. and M. Sunar, 1994. Piezoelectricity and its use in disturbance sensing and contrôle of flexible structures: A survey. Applied Mech. Rev., 47 (4): 113-123.

Sunar, M. and S.S. Rao, 1997. Thermopiezoelectric control design and actuator placement. AIAA J., 35 (3): 534-539.

Taleghani, B.K. and J.F. Campbell, 1999. Non linear finite element modeling of THUNDER piezoelectric actuators. NASA/TM 1999-245712.

Wang, B.T., R.A. Burdisson and C.R. Fuller, 1994. Optimal placement of piezoelectric actuators for active structural acoustic control. Intell. Mater. Sys. Struct., 5 (1): 67-77.

Yang, S. and W. Huang, 1997. Piezoelectric constitutive equations for a plate shape Sensor/Actuator. AIAA J., 35 (12): 1894-1895.