

## Techniques for Vibration Suppression and End-Point Trajectory Tracking of Flexible Robot Manipulator

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**Abstract:** This study presents investigations into the development of control schemes for vibration suppression and end-point trajectory tracking of a flexible robot manipulator. A constrained planar single-link flexible robot manipulator is considered and the dynamic model of the system is derived using the assumed mode method. To study the effectiveness of the controllers, initially a collocated PD controller is developed for control of rigid body motion. This is then extended to incorporate a non-collocated PID controller and a feedforward controller based on input shaping techniques for control of vibration (flexible motion) of the system. For feedforward controller, the positive and modified Specified Negative Amplitude (SNA) input shapers are proposed and designed based on the properties of the system. Simulation results of the response of the manipulator with the controllers are presented in time and frequency domains. The performances of the control schemes are assessed in terms of level of vibration reduction, input tracking capability and time response specifications.

**Key words:** Vibration control, input shaping, non-collocated PID, flexible robot manipulator

### INTRODUCTION

An important aspect of the flexible robot manipulator control that has received little attention is the interaction between the rigid and flexible dynamics of the links. An acceptable system performance with reduced vibration that accounts for system changes can be achieved by developing a hybrid control scheme that caters for rigid body motion and vibration of the system independently. This can be realised by utilising control strategies consisting of either non-collocated with collocated feedback controllers and feedforward with feedback controllers. A hybrid collocated and non-collocated controller has previously been proposed for control of a flexible robot manipulator (Tokhi and Azad, 1996). The controller design utilises end-point acceleration feedback through a Proportional Integral Derivative (PID) control scheme and a Proportional Derivative (PD) configuration for control of rigid body motion. A PD feedback control with a feedforward control to regulate the position of a flexible manipulator has been proposed (Shchuka and Goldenberg, 1989). Simulation results have shown that although the pole-zero cancellation property of the feedforward control speeds up the system

response, it increases overshoot and oscillation. Moreover, the investigation of hybrid control schemes of flexible manipulator with different polarities of input shaping also has been discussed (Mohamed and Ahmad, 2008). Simulation results compare the performance of positive zero-vibration derivative-derivative and negative zero-vibration-derivative-derivative with collocated PD control, respectively. A control law partitioning scheme which uses end-point sensing device has been reported (Rattan *et al.*, 1990). The scheme uses end-point position signal in an outer loop controller to control the flexible modes, whereas, the inner loop controls the rigid body motion independent of the flexible dynamics of the manipulator. Performance of the scheme has been demonstrated in both simulation and experimental trials incorporating the first 2 flexible modes. A combined feedforward and feedback method in which the end-point position is sensed by an accelerometer and fed back to the motor controller, operating as a velocity servo, has been proposed in the control a flexible manipulator system (Wells and Schueller, 1990). This method uses a single mass-spring-damper system to represent the manipulator and thus the technique is not suitable for high speed operation.

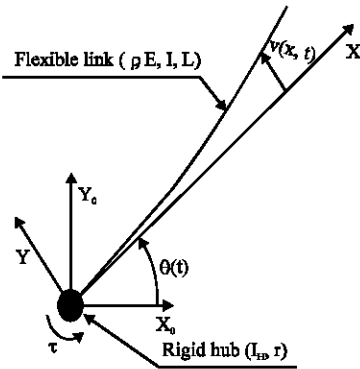


Fig. 1: Description of the flexible manipulator system

This study presents investigations into the development of techniques for vibration suppression and end-point trajectory tracking of a flexible robot manipulator. Control strategies based on feedforward with PD controllers and with combined non-collocated and PD controllers are investigated. A simulation environment is developed within Simulink and Matlab for evaluation of performance of the control schemes.

Simulation results of the response of the manipulator with the controllers are presented in time and frequency domains. The performances of the control schemes are assessed in terms of level of vibration reduction, input tracking capability and time response specifications.

Figure 1 shows the single-link flexible robot manipulator system considered in this research, where,  $X_oOY_o$  and  $XOY$  represent the stationary and moving coordinates frames, respectively,  $\tau$  represents the applied torque at the hub.  $E$ ,  $I$ ,  $\rho$ ,  $L$ ,  $A$  and  $I_h$  represent the Young modulus, area moment of inertia, mass density per unit volume, length, cross-sectional area and hub inertia of the manipulator, respectively.

In this research, the motion of the manipulator is confined to  $X_oOY_o$  plane. Transverse shear and rotary inertia effects are neglected, since the manipulator is long and slender. Thus, the Bernoulli-Euler beam theory is allowed to be used to model the elastic behavior of the manipulator. The manipulator is assumed to be stiff in vertical bending and torsion, allowing it to vibrate dominantly in the horizontal direction and thus, the gravity effects are neglected.

Moreover, the manipulator is considered to have a constant cross-section and uniform material properties throughout. In this study, an aluminium type flexible robot manipulator of dimensions  $900 \times 19.008 \times 3.2004 \text{ mm}^3$ ,  $E = 71 \times 10^9 \text{ N m}^{-2}$ ,  $I = 5.1924 \times 10^{11} \text{ m}^4$ ,  $\rho = 2710 \text{ kg m}^{-3}$  and  $I_h = 5.8598 \times 10^{-4} \text{ kg m}^{-2}$  is considered. These

parameters constitute a single link flexible robot manipulator experimental-rig developed for test and verification of control algorithms (Tokhi *et al.*, 2001).

## MATERIALS AND METHODS

A brief description on the modelling of the flexible manipulator system is provided, as a basis of a simulation environment for development and assessment of the hybrid control techniques. The assume mode method with 2 modal displacement is considered in characterizing the dynamic behaviour of the manipulator incorporating structural damping. The dynamic model has been validated with experimental exercises where a close agreement between both theoretical and experimental results has been achieved (Martins *et al.*, 2003).

Considering revolute joints and motion of the manipulator on a 2-dimensional plane, the kinetic energy of the system can thus be formulated as:

$$T = \frac{1}{2}(I_h + I_b)\dot{\theta}^2 + \frac{1}{2}\rho \int_0^L (\dot{v}^2 + 2\dot{v}x\dot{\theta})dx \quad (1)$$

where,  $I_b$  is the beam rotation inertia about the origin  $O_o$  as if it were rigid. The potential energy of the beam can be formulated as:

$$U = \frac{1}{2} \int_0^L EI \left( \frac{\partial^2 v}{\partial x^2} \right)^2 dx \quad (2)$$

This expression states the internal energy due to the elastic deformation of the link as it bends. The potential energy due to gravity is not accounted for since only motion in the plane perpendicular to the gravitational field is considered.

To obtain a closed-form dynamic model of the manipulator, the energy expressions in Eq. 1 and 2 are used to formulate the Lagrangian  $L = T - U$ . Assembling the mass and stiffness matrices and utilizing the Euler Lagrange equation of motion, the dynamic equation of motion of the flexible manipulator system can be obtained as:

$$M\ddot{Q}(t) + D\dot{Q}(t) + KQ(t) = F(t) \quad (3)$$

where,  $M$ ,  $D$  and  $K$  are global mass, damping and stiffness matrices of the manipulator, respectively. The damping matrix is obtained by assuming the manipulator exhibit the characteristic of Rayleigh damping.  $F(t)$  is a vector of external forces and  $Q(t)$  is a modal displacement vector given as:

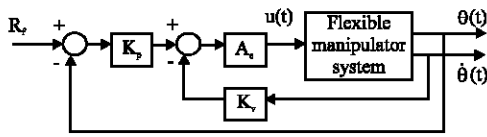


Fig. 2: Block diagram of collocated PD control structure

$$Q(t) = [\theta \ q_1 \ q_2 \ \dots \ q_n]^T = [\theta \ q^T]^T \quad (4)$$

$$F(t) = [\tau \ 0 \ 0 \ \dots \ 0]^T \quad (5)$$

Here,  $q_n$  is the modal amplitude of the  $i$ th clamped-free mode considered in the assumed modes method procedure and  $n$  represents the total number of assumed modes. The model of the uncontrolled system can be represented in a state-space form as:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \quad (6)$$

with the vector

$$x = [\theta \ \dot{\theta} \ q_1 \ q_2 \ \dot{q}_1 \ \dot{q}_2]^T$$

and the matrices  $A$  and  $B$  are given by

$$A = \begin{bmatrix} 0_{3 \times 3} & I_{3 \times 3} \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0_{3 \times 1} \\ M^{-1} \end{bmatrix} \quad (7)$$

$$C = [I_{1 \times 3} \ | \ 0_{1 \times 3}], \quad D = [0]$$

The control schemes for rigid body motion control and vibration suppression of a flexible robot manipulator are proposed in this study. Initially, a collocated PD controller is designed for rigid body motion control. Then a non-collocated PID control and feedforward control based on input shaping are incorporated in the closed-loop system for control of vibration of the system.

A common strategy in the control of manipulator systems involves the utilization of PD feedback of collocated sensor signals. In this research, such a strategy is adopted at this stage. A sub-block diagram of the PD controller is shown in Fig. 2, where,  $K_p$  and  $K_v$  are proportional and derivative gains, respectively,  $K_p$  and  $K_v$  represent hub angle and hub velocity, respectively,  $R_r$  is the reference hub angle and  $A_c$  is the gain of the motor amplifier. Here, the motor/amplifier gain set is considered as a linear gain. To design the PD controller, a linear state space model of the flexible manipulator was obtained by linearising the equations of motion of the system.

The control signal  $u(s)$  in Fig. 2 can be written as:

$$u(s) = A_c [K_p \{R_r(s) - \theta(s)\} - K_v s \theta(s)] \quad (8)$$

where,  $s$  is the Laplace variable. The closed-loop transfer function is, therefore, obtained as:

$$\frac{\theta(s)}{R_r(s)} = \frac{K_p H(s) A_c}{1 + A_c K_v (s + K_p / K_v) H(s)} \quad (9)$$

where,  $H(s)$  is the open-loop transfer function from the input torque to hub angle, given by:

$$H(s) = C(sI - A)^{-1}B \quad (10)$$

where,  $A$ ,  $B$  and  $C$  are the characteristic matrix, input matrix and output matrix of the system, respectively and  $I$  is the identity matrix. The closed-loop poles of the system are, thus, given by the closed-loop characteristics Eq. as:

$$1 + A_c K_v (s + K_p / K_v) H(s) \quad (11)$$

where,  $Z = K_p / K_v$  represents the compensator 0 which determines the control performance and characterises the shape of root locus of the closed-loop system. In this study, the root locus approach is utilized to design the PD controller.

A combination of collocated and non-collocated control scheme for control of rigid body motion and vibration suppression of the system is presented in this study. The use of a non-collocated control system, where, the end-point of the manipulator is controlled by measuring its position, can be applied to improve the overall performance, as more reliable output measurement is obtained. The control structure comprises two feedback loops: The collocated PD control for rigid body motion control. The end-point residual (elastic deformation) as input to a separate non-collocated control law for vibration control. These 2 loops are then summed together to give a torque input to the system. A block diagram of the control scheme is shown in Fig. 3 where,  $\alpha$  represents the end-point residual. The  $r_\alpha$  represents end-point residual reference input, which is set to zero as the control objective is to have 0 vibration during movement of the manipulator.

For rigid body motion control, the collocated PD control strategy developed in the previous study is adopted whereas, for the vibration control loop, the end point residual feedback through a PID control scheme is

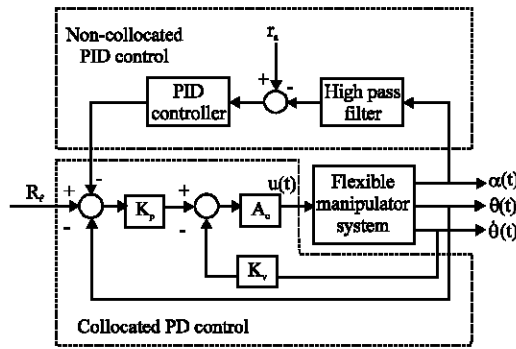


Fig. 3: Block diagram of collocated PD with non-collocated PID control structure

utilized. The PID controller parameters were tuned using the Ziegler-Nichols method using a closed-loop technique, where the proportional gain  $K_p$  was initially tuned and the integral gain  $K_i$  and derivative gain  $K_d$  were then calculated (Warwick, 1989). Accordingly, the PID parameters  $K_p$ ,  $K_i$  and  $K_d$  were deduced as 0.8, 5 and 0.03, respectively. To decouple the end-point measurement from the rigid body motion of the manipulator, a 3rd order Infinite Impulse Response (IIR) Butterworth High-pass filter was utilised. In this investigation, a High-pass filter with cut-off frequency of 5 Hz was designed.

A control structure for control of rigid body motion and vibration suppression of the flexible robot manipulator based on PD and input shaping control is also proposed in this investigation. The positive input shapers and modified Specified Negative Amplitude (SNA) input shapers are proposed and designed based on the properties of the system. In this study, the input shaping control scheme is developed using a Zero-Vibration-Derivative-Derivative (ZVDD) input shaping technique. Previous experimental study with a flexible robot manipulator has shown that significant vibration reduction and robustness is achieved using a ZVDD technique (Mohamed and Tokhi, 2002). A block diagram of the PD with input shaping control technique is shown in Fig. 4.

The input shaping method involves convolving a desired command with a sequence of impulses known as input shaper. The design objectives are to determine the amplitude and time location of the impulses based on the natural frequencies and damping ratios of the system. The positive input shapers have been used in most input shaping schemes. The requirement of positive amplitude for the impulses is to avoid the problem of large amplitude impulses. In this study, each individual impulse must be  $<1$  to satisfy the unity magnitude constraint. In addition, the robustness of the input shaper to errors in natural

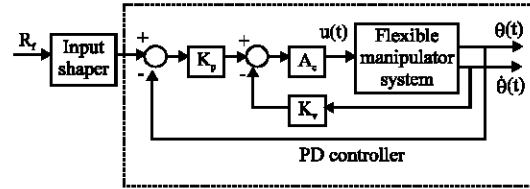


Fig. 4: Block diagram of collocated PD with input shaping control

frequencies of the system can be increased by solving the derivatives of the system vibration equation. This yields a positive ZVDD shaper with parameter as:

$$t_1 = 0, t_2 = \frac{\pi}{\omega_d}, t_3 = \frac{2\pi}{\omega_d}, t_4 = \frac{3\pi}{\omega_d}$$

$$A_1 = \frac{1}{1 + 3K + 3K^2 + K^3}, A_2 = \frac{3K}{1 + 3K + 3K^2 + K^3} \quad (12)$$

$$A_3 = \frac{3K^2}{1 + 3K + 3K^2 + K^3}, A_4 = \frac{K^3}{1 + 3K + 3K^2 + K^3}$$

Where:

$$K = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}, \omega_d = \omega_n \sqrt{1-\zeta^2}$$

$\omega_n$  and  $\zeta$  representing the natural frequency and damping ratio, respectively. For the impulses,  $t_j$  and  $A_j$  are the time location and amplitude of impulse  $j$ , respectively.

Input shaping techniques based on positive input shaper has been proved to be able to reduce vibration of a system. In order to achieve higher robustness, the duration of the shaper is increased and thus, increases the delay in the system response. By allowing the shaper to contain negative impulses, the shaper duration can be shortened, while satisfying the same robustness constraint. To include negative impulses in a shaper requires the impulse amplitudes to switch between 1 and -1 as:

$$A_i = (-1)^{i+1}, i = 1, \dots, n \quad (13)$$

The constraint in Eq. 13 yields useful shapers as they can be used with a wide variety of inputs. In this research, the previous SNA input shaper (Mohamed *et al.*, 2006) is modified by locating the negative amplitudes at the centre between each positive impulse sequences with even number of total impulses. This will result the shaper duration as one-fourth of the vibration period of an undamped system. The modified SNA ZVDD shaper is proposed and applied in this research to enhance the robustness capability of the controller while, increasing the speed of the system response (Mohamed and Ahmad, 2008). By considering

the form of modified SNA-ZVDD shaper, the amplitude summation constraints equation can be obtained as:

$$2a + 2c - 2b - 2d = 1 \quad (14)$$

The values of a, b, c and d can be set to any value that satisfy the constraint in Eq. 14. However, the suggested values of a, b, c and d are  $<|1|$  to avoid the increase of the actuator effort.

### RESULTS AND DISCUSSION

In this study, the proposed control schemes are implemented and tested within the simulation environment of the flexible robot manipulator and the corresponding results are presented. The manipulator is required to follow a trajectory within the range of  $\pm 8$  radian. System responses namely the end-point trajectory and its acceleration are observed. To investigate the vibration of the system in the frequency domain, Power Spectral Density (PSD) of the end-point acceleration response is obtained. The performances of the control schemes are assessed in terms of vibration suppression, input tracking and time response specifications.

Figure 5-7 show the responses of the flexible robot manipulator to the reference input trajectory using collocated PD controller in time-domain and frequency domain (PSD). These results were considered as the system response under rigid body motion control and will be used to evaluate the performance of the non-collocated PID and input shaping control. The steady-state end point trajectory of  $+0.8$  radian for the flexible manipulator was achieved within the rise and settling times and overshoot of 0.506 sec, 0.800 sec and 0.5%, respectively. It is noted that the manipulator reaches the required

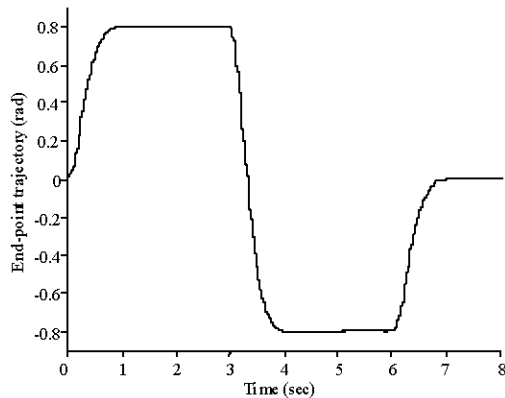


Fig. 5: End-point trajectory response with collocated PD

position from  $+0.8$  rad to  $-0.8$  rad within 1 sec, with little overshoot. However, a noticeable amount of vibration occurs during movement of the manipulator. It is noted from end-point acceleration response, the vibration of the system settles within 2 sec with a maximum acceleration of  $\pm 600$  m sec<sup>-2</sup>. Moreover, from the PSD of the end-point acceleration response the vibrations at the end-point are dominated by the first 2 vibration modes, which are obtained as 13.18 and 55.91 Hz.

The end-point trajectory, end-point acceleration and power spectral density responses of the flexible robot manipulator using PD with non-collocated PID (PD-PID), positive (PD-PZVDD) and negative (PD-NZVDD) input shaping control are shown in Fig. 8-10, respectively. Similar, end-point trajectory, displacement and end-point acceleration responses were observed as compared to the PD controller. Table 1 summarises the levels of vibration reduction of the system responses at the first 2 modes in comparison to the PD control.

In overall, the highest levels of vibration reduction for the first 2 modes were obtained using PD-PZVDD

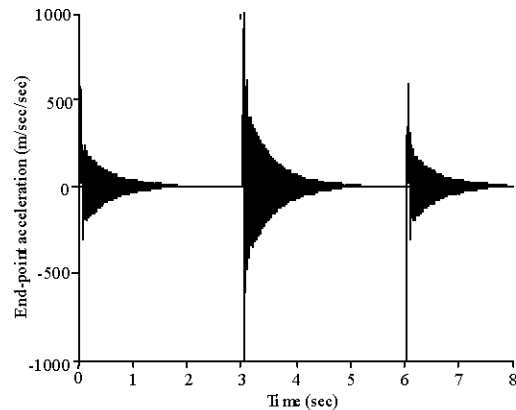


Fig. 6: End-point acceleration response with collocated PD

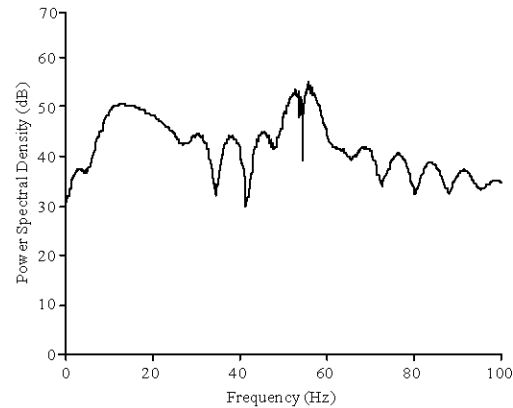


Fig. 7: Power spectral density response with collocated PD

Table 1: Level of vibration reduction of the end-point acceleration and specifications of end-point trajectory response for PD-PZVDD, PD-NZVDD and PD-PID control schemes

Controller	Attenuation (dB) of vibration end-point acceleration		Specifications of end-point trajectory response		
	Mode 1	Mode 2	Rise time (s)	Settling time (s)	Overshoot (%)
PD-PZVDD	43.68	77.02	0.522	0.879	0.49
PD-NZVDD	32.87	42.49	0.523	0.867	0.48
PD-PID	2.96	11.31	0.511	0.858	0.49

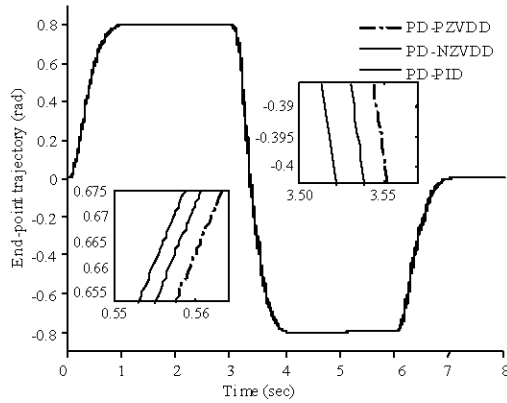


Fig. 8: End-point trajectory response with PD-PZVDD, PD-NZVDD and PD-PID

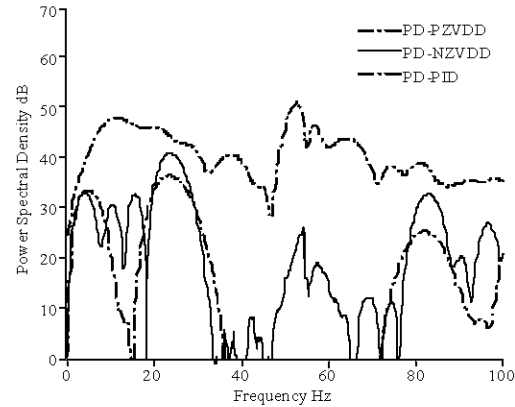


Fig. 10: Power spectral density response with PD-PZVDD, PD-NZVDD and PD-PID

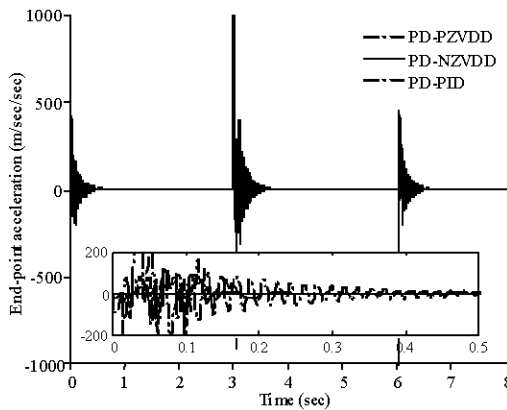


Fig. 9: End-point acceleration response with PD-PZVDD, PD-NZVDD and PD-PID

followed by PD-NZVDD and PD-PID. However, the fastest system response was obtained using PD-PID followed by PD-NZVDD and PD-PZVDD. It is noted with the feedforward controller, the impulses sequence in input shaper increase the delay in the system response. The corresponding rise time, settling time and overshoot of the end-point trajectory response using PD-PZVDD, PD-NZVDD and PD-PID is depicted in Table 1. Moreover, as demonstrated in the end-point trajectory response with PD-PID control, the minimum phase behaviour of the manipulator is unaffected. A significant amount of

vibration reduction was demonstrated at the end-point of the manipulator with both control schemes. With the PD PID control, the maximum end-point acceleration is  $\pm 500 \text{ m sec}^{-2}$  while with the PD-PZVDD and PD-NZVDD control is  $\pm 400$  and  $\pm 200 \text{ m sec}^{-2}$ , respectively. Hence, it is noted that the magnitude of oscillation was significantly reduced by using PD with feedforward control as compared to the case of PD with non-collocated PID control. In overall, the performance of the control schemes at input tracking capability is maintained as the PD control.

### CONCLUSION

The development of techniques for vibration suppression and end-point trajectory tracking of a flexible robot manipulator has been presented. Acceptable performance in end-point vibration suppression and input tracking control has been achieved with proposed control strategies. A comparative assessment of the control schemes has shown that the PD control with input shaping (feedforward) performs better than the PD with non-collocated PID control in respect of vibration reduction at the end-point of the manipulator. However, the speed of the response is slightly improved at the expenses of decrease in the level of vibration reduction by

using the PD with non-collocated PID control. It is concluded that the proposed controllers are capable of reducing the system vibration while maintaining the input tracking performance of the manipulator.

#### REFERENCES

- Martins, J.M., Z. Mohamed, M.O. Tokhi, J. Sa da Costa and M.A. Botto, 2003. Approaches for dynamic modelling of flexible manipulator systems. *IEE. Proc. Control Theory Appl.*, 150 (4): 401-411. DOI: 10.1049/ip-cta:20030496. <http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=1232628&isnumber=27622>.
- Mohamed, Z. and M.O. Tokhi, 2002. Vibration control of a single-link flexible manipulator using command shaping techniques. *Proceedings IMechE-I. J. Syst. Control Eng.*, 216: 191-210. DOI: 10.1243/0959651021541552. <http://journals.pepublishing.com/content/pv01863j84g80127/fulltext.pdf>.
- Mohamed, Z., A.K. Chee, Mohd A.W.I. Hashim, M.O. Tokhi, S.H.M. Amin and R. Mamat, 2006. Techniques for vibration control of a flexible manipulator. *Robotica*, 24: 499-511. DOI: 10.1017/S0263574705002511. <http://journals.cambridge.org/action/displayAbstract?fromPage=online&aid=451257>.
- Mohamed, Z. and M.A. Ahmad, 2008. Hybrid Input Shaping and Feedback Control Schemes of a Flexible Robot Manipulator. *Proceedings of the 17th World Congress The International Federation of Automatic Control*, Seoul, Korea, July 6-11, pp: 11714-11719. DOI:10.3182/20080706-5-KR-1001.1008.ISBN:978-3-902661-00-5. [http://myais.fsktm.um.edu.my/4172/1/IFAC\\_2008.pdf](http://myais.fsktm.um.edu.my/4172/1/IFAC_2008.pdf).
- Rattan, K.S., V. Feliu and H.B. Brown, 1990. Tip position control of flexible arms. *Proceedings of the IEEE Conf. Robotics and Automation*, Cincinnati, Ohio, May 13-18, pp: 1803-1808. DOI: 10.1109/ROBOT.1990.126270. <http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=126270&isnumber=3534>.
- Shchuka, A. and A.A. Goldenberg, 1989. Tip control of a single-link flexible arm using feedforward technique. *Mechanism Machine Theory*, 24 (5): 439-455. DOI: 10.1016/0094-114X(89)90072-4. [http://www.science-direct.com/science?\\_ob=MIimg&\\_imagekey=B6V46-482GS7Y-H6-1&\\_cdi=5750&\\_user=4406426&\\_orig=search&\\_coverDate=12%2F31%2F1989&\\_sk=999759994&view=c&wchp=dGLzVtb-zSkzS&md5=79135df7f9a8892b92558f134981ee32&ie=/sdarticle.pdf](http://www.science-direct.com/science?_ob=MIimg&_imagekey=B6V46-482GS7Y-H6-1&_cdi=5750&_user=4406426&_orig=search&_coverDate=12%2F31%2F1989&_sk=999759994&view=c&wchp=dGLzVtb-zSkzS&md5=79135df7f9a8892b92558f134981ee32&ie=/sdarticle.pdf).
- Tokhi, M.O. and A.K.M. Azad, 1996. Control of flexible manipulator systems. *Proceedings IMechE-I. J. Syst. Control Eng.*, 210: 113-130. DOI: 10.1243/PIME\_PROC\_1996\_210\_444\_02. <http://archive.pepublishing.com/content/a726382726014865/fulltext.pdf>.
- Tokhi, M.O., Z. Mohamed and M.H. Shaheed, 2001. Dynamic characterisation of a flexible manipulator system. *Robotica*, 19 (5): 571-580. DOI: 10.1017/S0263574700003209. <http://journals.cambridge.org/action/displayAbstract?fromPage=online&aid=83379>.
- Warwick, K., 1989. *An introduction to Control systems*. 2nd Edn. Prentice Hall, London, pp: 362. ISBN: 9810225970, 9789810225971. [http://books.google.com.my/books?id=EvgjDFf84I8C&pg=PR7&source=gbs\\_selected\\_pages&cad=0\\_1](http://books.google.com.my/books?id=EvgjDFf84I8C&pg=PR7&source=gbs_selected_pages&cad=0_1).
- Wells, R.L. and J.K. Schueller, 1990. Feedforward and feedback control of a flexible manipulator. *IEEE Control Syst. Magazine*, 10: 9-15. DOI: 10.1109/37.50663. <http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=50663&isnumber=1851>.