Journal of Engineering and Applied Sciences 4 (3): 189-196, 2009
ISSN: 1816-949X

Characteristic Behavior of High-Speed Craft at Transition from Bow-Wetting to Full Planing

[1]Daniel T. Tamunodukobipi, [2]Ezenwa A. Ogbonnaya and [2]Kokouvi E.E. Koumako
[1]Department of Energy Mechanics, Korean Institute of Science and Technology, 31-1, Hanolgok-don, South Korea
[2]Department of Marine Engineering, Rivers State University of Science and Technology, Nkpolu-Port Harcourt, Rivers State, Nigeria

Abstract: Predicting the forces acting on a high-speed craft and the characteristic behavior of the vessel is critical, especially during transition. Researchers, over the years, have employed various means to solve this problem but with low precision on the behavior of such vessel at transition. This study, however, employed appropriate theory to simulate the character of high-speed craft as she transits from bow-wetting to full planing. The variations of performance parameters and their characteristic contours were computed using computer-based analytical model. The obtained results and graphs showed clearly the dramatic changes in trend of resistance variables, trim and Reynolds' number, particularly during transition. These generated results compared with those obtained by model tests and numerical simulations were found to be in good agreement. Finally, it established that during the short period of transitions all hydrodynamic performance variables altered characteristically to depict the transition phase before regaining their normal contours.

Key words: High speed craft, behavior at transition, hydrodynamic design, performance characteristics of planing craft, bow-wetting and full-planing of high speed craft

INTRODUCTION

Researchers and design engineers over the years have been challenged by the issue of predicting the exact motion of a high-speed planing craft in the three phases of her operation. A planing craft as shown in Fig. 1 is a hard-chine, structurally reinforced fiberglass or aluminum, ultra-light autonomous, high-speed, surface marine vessel, which is commonly powered by a diesel engine. Planing boats, unlike the displacement vessels, are capable of traversing expanse of water, shallow shores and swamps with no serious operational difficulties. In fact, these autonomous-surface vehicles are highly adaptive crafts employed mostly for swift operation in coastal/inland waters. They play safely in swampy and shallow types of waters, which are filled with all forms of nasty obstructions and obstacles.

Several theoretical methods employed either led to intricate computational complexity or generated impractical results particularly at transition from bow-wetting to full planing motion. Bow-wetting occurs at very low velocity, when the mean wetted-length L_m is greater than the keel-wetted length L_k. The bow wades through water with increased wetting of the hull length. Full

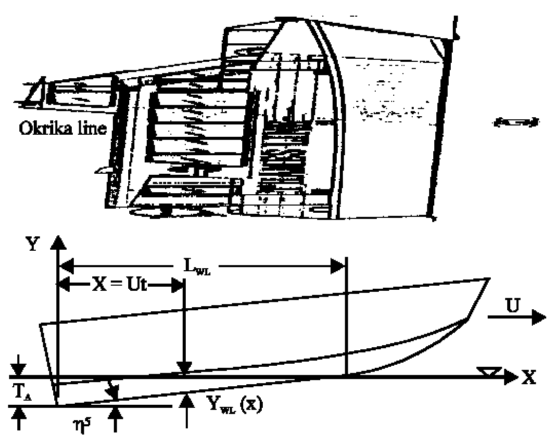


Fig. 1: Description of the high-speed craft

planing is when the vessel lifts to skim on the water surface by the joint efforts to buoyancy and hydrodynamic forces. The vessel at planing maintains high speed, low draft and trim with a resultant reduced wetted length, since the bow is completely clear-off water. The use of semi-empirical means produced results valid only within the domain of the data obtained from the

Corresponding Author: Ezenwa A. Ogbonnaya, Department of Marine Engineering, Rivers State University of Science and Technology, Nkpolu-Port Harcourt, Rivers State, Nigeria

specific model. Nonetheless experimental techniques seemed most appropriate but are limited in scope and versatility as a result of cost, variation of forms and speed range. In spite of the experimental contributions being extensively used in design, prediction of performance and validation of simulated results, their graphical presentations have not shown the expected variations of hull performance parameters during transition. The shortcoming came as consequence of the very brief period of this all important transient phase.

This study therefore, applied appropriate theory to simulate the character of high-speed craft as she transits from bow-wetting to full planing. The changes in performance parameters and their characteristic contours were calculated using computer-based analytical model. The analytical results were presented graphical alongside with Society of Naval Architects and Marine Engineers (SNAME) model tests results to show the trend and variation of parameters particularly at transition. The approach produced appropriate contours that depict the exact behavior of planing craft at transition.

A brief historical background: Planing craft was designed primarily to overcome the inherent hydrodynamic limitations associated with high speed operations for most displacement vessels especially in shallow coastal regions (SNAME, 1957). The hull form is designed to have positive hydrodynamic pressures. The hull lifts out of water with increasing speed to avoid the large drag force associated with displacement vessels with both longitudinal and transverse convex-curvatures. The wetted surface at full planing speed is often <40% of the original wetted area at rest. This value is dependent largely on the position of the longitudinal centre of gravity.

Performance prediction methods: A number of methods exist to predict the performance of planing craft. These range from model testing, analytical approach, experimental data from geometric model series and regression analysis of random model data (Blount and Bartee, 1997). Oldest and most reliable technique is the towing tank tests and results. Towing tanks results have not only predicted resistance by simulation but provided an understanding of the actual hydrodynamics of the hull when operating in seaway.

In the past, analytical models were inadequate to deal with the several peculiar problems of planing hull hydrodynamics. Emphasis on high speed planing boat was left in the field of aerodynamics of water-based aircraft. Aircraft researchers made remarkable progress in analytical studies of such fundamental parameters as planing lift, stability and wave impact. Most remarkable is the advance knowledge gathered from model tests in towing tanks. Nonetheless, the towing tanks up till date are still used to define the basic static and dynamic phenomena associated with calm and rough water operation of planing hulls.

More so, earliest experimental studies of prismatic planing boats with constant beam and deadrise were undertaken by Baker (1912) in England. However, this was followed by several more extensive researches by Scottorf (1934) in Germany; Korvin-Kroukovsky *et al.* (1949); in USA; Savitsky and Neidinger (1954) in USA and Shuford (1958) in USA.

Their research efforts yielded an enormous collection of relevant data on planing boat's loads, wetted area, trim angle, speed, deadrise angle, etc. The compendium of their results led to the generation of vital empirical and semi-empirical formulas for performance prediction. Those technical data, also, formed the basis of most design and serve as a yardstick for validation of any analysis. In spite of these advantages, this method is limited by several factors, such as cost, inflexibility and inability to correctly capture the transient transition-phase.

Semi-empirical means combined principles of physics and empirical measurements to predict performance and motion of a planing craft. Most successful of these was developed by Savitsky (1964) and modified in Savitsky and Brown (1976). Their iterative procedure and formulas predict the running attitude of a prismatic planing hull, the trim, draft and the thrust required. Their method gained wide acceptance because of the ease and versatility of its application. However, it inherited lots of limitations because of its gross empirical origin. Akers (2002) pointed out the following shortcomings: It is not adaptive to hull that have variable deadrise along her length. Its approach is quasi-static and can mot predict directly the transient behavior of the vessel. It aggregates several hydrodynamic forces into a series of empirical relationships. This makes it difficult to analyze point or panel loading of the planing craft at transition.

However, researchers on planing hulls, from 1930s-1960s, engaged in linearized two-dimensional analysis of the hydrodynamic parameters of planing hulls (Squire, 1957; Cumberbatch, 1958). Most investigators in 1960s related the pressure distribution on a planing surface to its hull form by various integral equations. Nonetheless, this three-dimensional approach of 1960s had setbacks on either the speed or aspect ratio of the planing surface (Wang, 1971; Tuck, 1975). However, Doctors (1975) finite pressure element approach on three-dimensional analysis overcame these delimitations. His technique adopted an iterative procedure to adjust the wetted surface area to satisfy trailing-edge Kutta condition until a constant value was reached. The limitation of this method was the undesirable pressure oscillation at the side edges and downstream. Such oscillation could be attributed to the pressure discontinuities at those regions.

Tong *et al*. (1989) proposed that the wetted surface should be predetermined before proceeding to compute the pressure distribution and the shape of the transom. His proposal solved the pressure oscillation issue that saddled Doctors' procedure except for number of buttocks beyond six. Cheng and Wellicome (1994) developed a model for evaluating the hydrodynamic forces on a planing hull without aspect ratio and speed restrictions. In accordance with Kutta's condition on the transom edge, Cheng formulated his transverse strips of variable pressure technique to tackle the problems of pressure discontinuities and oscillation at the side edges. The shortcoming of this method is its inability to prescribe the shape of the transom from the computed wetted surface and the transient hydrodynamic patterns at transition.

Furthermore, Vorus (1993), using slender body theory, developed an analytical model Hydrodynamic Impact and Penetration of Flat Cylinders. He combined the hydrodynamic and hydrostatic pressure components to determine the running attitude and powering requirement of a vessel. However, Vorus' (1993) model did not gain wide acceptance as Savisky's (1964) because it appeared more complex to apply.

Royce (1994) formulated a graphic model to predict horizontal skin friction, pressure drag, hydrostatic lift and hydrodynamic lift of a planing vessel. He used the principal dimensions, speed and position of centre of gravity to predict the operating attitude of the craft. His rational two-dimensional graphic model although logical in sequence has not gained wide acceptance as Savitsky's because of being recent and its iterative procedure of formulas and graphs is susceptible to producing results that are less exact to published experimental data. His method did inherit some of the limitations of Savitsky's semi-empirical approach and does not reflect the transition behavior of the vessel.

All the referenced contributions lacked in expounding the exact behavior of high speed craft as she transits from bow-wetting to full planing motion. This study however, developed a computer-aided mathematical model based on similarity principle to predict the exact behavior of the planing crew-boat, particularly as she transits from bow-wetting to full planing. The results of this study clearly show the pre-eminent role played by fluid flow patterns in determining performance characteristics at the various

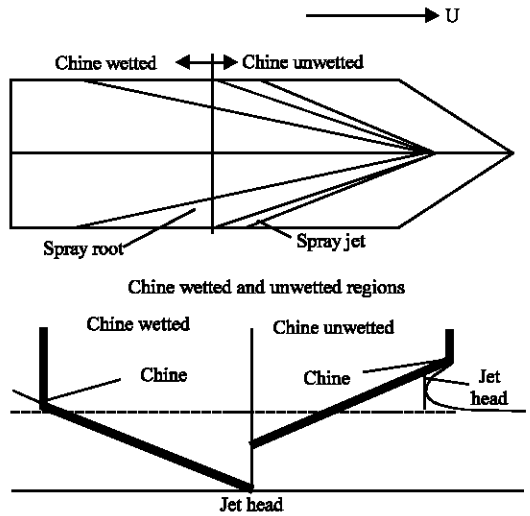


Fig. 2: Hydrodynamic flow patterns around the hull, from Roycc (1994)

phases of her motion. The graphical results compared favorably with other established methods commonly invoked by designers except during the transition phase.

Description of planing boat bottom fluid flow characteristics: The planing boat has two distinct regions: chine unwetted and chine wetted as shown in Fig. 2. During motion, there occur the spray jet and the spray root. The jet head develops at the point where, the spray root feeds into the spray jet. The region aft of the merging point of the jet heads is known as chine wetted. However, the chine unwetted region experiences the largest pressure impact force, while the chine wetted region is linked with subsequent penetration of cylinder. Fig. 2: Hydrodynamic Flow Patterns around the hull, from Royce (1994)

During motion, especially when accelerating, there is a significant build-up of water or spray root, to the sides of the hull. This invariably adds to the hydrostatic lift. However, the spray root increased the complexity of numerical solution, since the exact water-hull interface is undefined. In spite of this, the added buoyancy due to spray root is too small to be considered for hydrostatic lift- but forms a significant part of the hydrodynamic lift assessment.

The vessel's performance in relation to her hydrodynamic character is largely influenced by factors such as speed (v), rake, deadrise (β), trim angle (τ), the draught (d), mean wetted length (L_m) and wetting factor (W_f). W_f can be defined as the ratio of the height of the intersection of the spray root with the hull to the height of the free water surface above the base-line as common datum.

$$W_f = \left[\frac{Y`}{Y_{WL}}\right] \approx \frac{\pi}{2} \quad (1)$$

The wetting factor can be expressed as Wf = $\pi/2$ and was adopted as correction factor by Wagner (1967). However, experience has shown that Wf varies inversely as the deadrise angle.

Mean wetted length (L_m) is the average wetted lengths along the keel and the chine. It was developed to facilitate calculation of friction drag for better prediction of power, forces and pitching moments during operation. It can be expressed also as the mean wetted length-beam ratio (λ_m) as given:

$$L_m = 0.5\times(L_k + L_c)$$

let,

$$\lambda_m = L_m/B$$

therefore,

$$\lambda_m = 0.5\times(L_k + L_c)/B$$

MATERIALS AND METHODS

Below are set of mathematical relationships required for the hydrodynamic design and simulation of the performance of a crew boat as shown in Fig. 1. These equations are derived based on International Towing Tank Conference (ITTC) extrapolation mechanism and the established theories of naval architecture.

Frictional resistance C_F according to ITTC (1957) is given by:

$$C_F = \frac{3}{40}\left|LOG_{10} R_e - 2\right|^{-2} = f(R_e) \quad (2)$$

Total resistance coefficient of model by ITTC extrapolation technique is:

$$C_{Tm} = C_{Rm} + C_{Fm} = C_{Rm} + f(R_{em}) \quad (3)$$

It follows that the residuary resistance coefficient of model C_{Rm}

$$C_{Rm} = C_{Tm} - C_{Fm} = C_{Tm} - f(R_{em})$$

Assuming the residuary resistance coefficients of model and crew-boat are equal, then:

$$C_{RS} = C_{Rm} = C_{Tm} - f(R_{em}) \quad (4)$$

The total resistance coefficient of crew-boat becomes;

$$C_{Ts} = C_{Tm} + C_{Fs} - C_{Fm}$$

which is transformed into a total resistance coefficient formula as:

$$C_{Ts} = C_{Tm} + \frac{3}{40}\left[\begin{array}{l}(LOG_{10}R_{es} - 2)^{-2}\\ -(LOG_{10}R_{em} - 2)^{-2}\end{array}\right] \quad (5)$$

Total resistance of crew boat in tonnes by ITTC formula:

$$R_{Ts} = \left\{\frac{1}{2}C_T\left(\frac{\rho}{g}\right)SU^2\right\}_s \quad (6)$$

Considering 5 and 6 gives the total resistance as:

$$R_{Ts} = 0.5\times\left[C_{Tm} + \frac{3}{40}\left[\begin{array}{l}(LOG_{10}R_{es} - 2)^{-2}\\ -(LOG_{10}R_{em} - 2)^{-2}\end{array}\right]\right]SU^2 \quad (7)$$

Also, by scale factor method of extrapolation R_{TS} can be given as:

$$R_{Ts}1 = \lambda^3 R_{Tm} \quad (8)$$

At this point, it becomes pertinent to consider Reynolds in two different media, i.e. fresh and salt water. By similarity principles the craft Reynolds number becomes:

$$R_{es} = \lambda^{1.5}\left|\frac{\upsilon_m}{\upsilon_s}\right|\cdot R_{em} \quad (9)$$

where, υ_m and υ_s are kinematic viscosities of model and crew boat, respectively.

If we assume that $\mu_s = \mu_m$, then:

$$R_{es} = \lambda^{1.5}\left|\frac{\rho_s}{\rho_m}\right|\cdot R_{em} \quad (10)$$

where, μ_s and μ_f are the dynamic viscosities of salt and fresh water, respectively

RESULTS AND DISCUSSION

The wetted surface curve slopes downward from left to right as speed increases. This is accounted for by the rise in dynamic pressure lift with increasing speed, which gradually forces down the mean wetted length-beam ratio. The total resistance coefficient curve, consequently,

Table 1: The performance data for power and resistance of the planing craft

Speed (m sec^{-1})	L/B	Re. No. (x10^{-9})	Wetted surface (m^2)	Cts (x10^3)	Rts (tons)	Rts[1] (tons)	EHP	Power (kW)
4.490	3.54	5.71	33.72	14.03	0.48	0.48	12.97	0.73
5.610	3.33	6.64	33.35	14.11	0.75	2.00	25.27	18.95
6.730	3.19	7.62	31.17	11.92	0.86	2.62	34.37	25.78
7.870	3.10	8.65	30.08	9.81	0.93	2.77	43.62	32.71
9.010	2.96	9.46	28.89	8.46	1.01	3.07	54.21	40.66
10.13	2.80	10.06	26.59	7.84	1.09	3.57	65.84	49.38
11.24	2.59	10.22	23.06	7.78	1.15	3.78	77.29	57.97
12.36	2.45	10.76	21.55	7.19	1.21	3.80	88.98	66.74
13.50	2.36	11.30	20.55	6.59	1.26	3.67	101.24	75.93
14.61	2.27	11.78	19.58	6.15	1.31	3.53	113.94	85.45
15.69	2.22	12.37	19.11	5.70	1.36	3.32	127.77	95.83
16.86	2.15	12.87	18.42	5.39	1.44	3.12	144.44	108.33
18.01	2.11	13.46	17.95	5.05	1.5	3.97	160.79	120.59
15.64	2.08	14.10	17.7	4.79	1.06	2.73	98.61	73.96
20.24	2.06	14.79	17.49	4.62	1.69	2.65	203.62	152.71
21.34	2.04	15.41	17.23	4.48	1.79	2.53	227.62	170.72

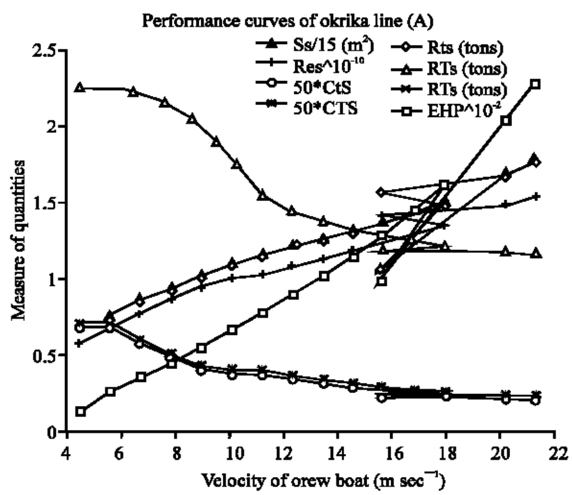


Fig. 3: The behavior of resistance and power curves at transition

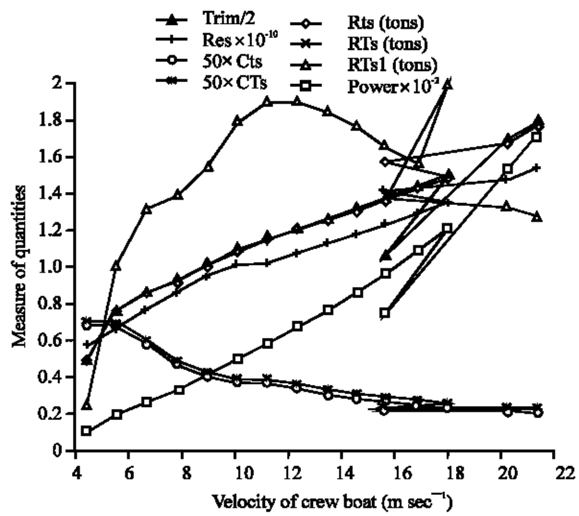


Fig. 4: The impact of transition on trim

configure itself along the same trend. Their common profile is due largely to diminishing frictional drag as a result of the decreasing wetted surface. The graphs of wetted surface and resistance coefficient reveal the impact of wetted surface on vessel's total resistance.

Tracking down the slopes, there occurs a recumbent fold on each, like the letter Z located between 15 and 18 m sec^{-1}. This region marks the transition from bow wetting to full planing. Here dynamic pressure lift coefficient dominates buoyancy lift and the vessel is being lifted out to skim freely on the surface like a water-plane. The sudden drop in friction drag typified the actual reduction of wetted surface as the craft rises to plane. This translates in a corresponding drop in total resistance curve in that region. Beyond the transition, the slopes of resistance and wetted surface assume approximately, constant values. Refer to Table 1 and Fig. 3 and 4 for results.

However, the Z in Reynolds No. curve is due to a sharp fall of wetted length during transition. This backward recumbent fold influenced the shape of the coefficient of total resistance, since the latter is a function of the former. The significance of the transition region is amplified in the total resistance and effective power curves giving a remarkable drop in each.

The character of trim during transition is dramatic and very vital for stability consideration. At transition, the trim rises and falls sharply to a plateau. This characteristic behavior is due to an abrupt increase in dynamic pressure lift at the unwetted region around the bow. This dynamic lift drives towards the transom until the transom is finally lifted to a planing level, causing a sharp fall in trim angle. With further increase in speed beyond the transition phase, the trim approximately maintains a constant value. Furthermore, the three resistance curves obtained from different extrapolation methods coincided all through except during and beyond.

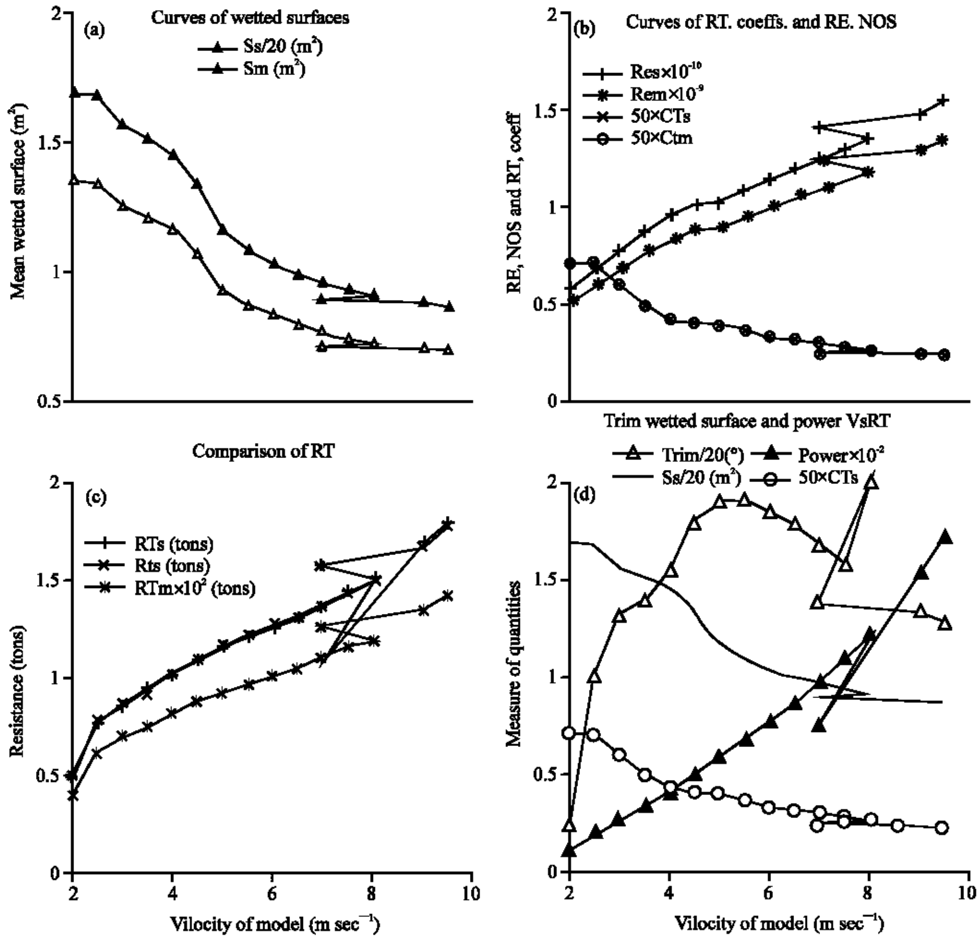


Fig. 5: Comparison of simulated performance with model test data

Comparison of simulated performance of vessel with SNAME model tests sheet: The graphs below compared the simulated performance of the vessel with SNAME Model Tests Data. The wetted surfaces of model S_m and vessel S_s were compared in Fig. 5a. The graphs are similar in gradients and contours, expect in magnitude. The same applies to Reynolds numbers and coefficients of total resistance as shown in Fig. 5b and c, respectively. Its impact on trim and other hydrodynamic variables are shown in Fig. 5d.

The above graphs clearly demonstrated dramatic but transient changes in performance parameters during the shift from displacement to full planing speed. The specific SNAME Data Sheet with, which the comparison was made is given in Appendix 1 and 2.

CONCLUSION

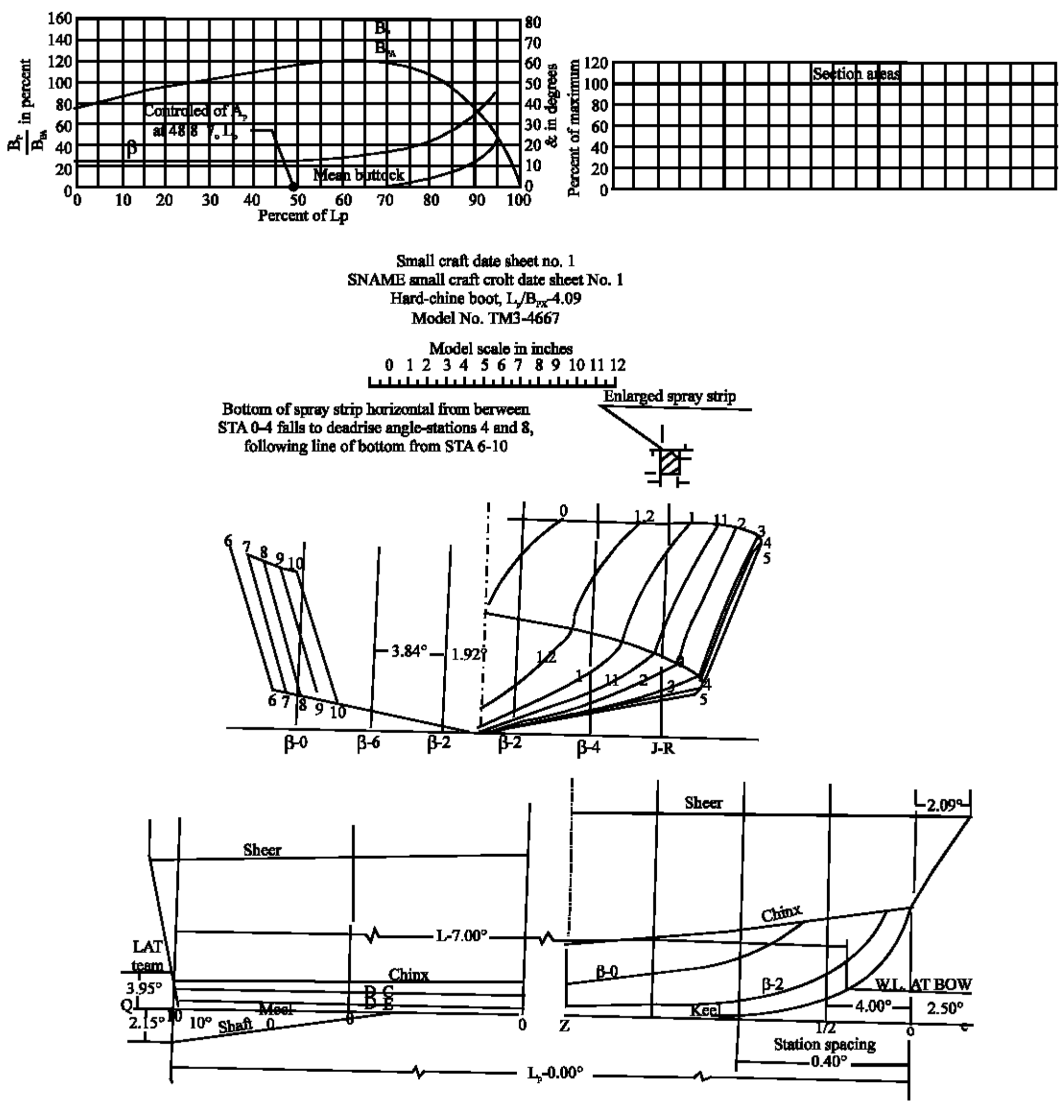
Based on the results and the discussion, this study has drawn some conclusions. The total resistance coefficient as consequence of the wetted surface drops gently with rising velocity throughout the displacement phase. During transition from bow wetting to full planing, all the performance parameters experienced a characteristic change in contours.

The change in trim has an initial sharp rise at the inception of transition before dropping steeply to a near constant value at full planing. This behavior depicts a pounding motion. The results also proved that the various extrapolation techniques employed could suitably predict resistance, except during and beyond transition when significant deviance occurs. The paper has brought to fore the obscured but enormous hydrodynamic influence on performance parameters a transient phase of transition.

Notation: As far as possible, the notation used is consistent with the society's Explanatory Notes for Resistance and Propulsion Data Sheets (Technical and Research Bulletin No. 13). Exceptions and additions. The subscript designates the planing bottom which is the portion bounded by the chine and transom.

Appendix 1: SNAME data sheet for the planing Crew-boat model

V knots	Rt (kg)	Δ (kg)	Rt/Δ	Lk (m)	Lc (m)	Lm (m)	Re×E-8	S (m^2)	$S/\Delta^{2/3}$	Ct×E3	Δτ (É)	CG rise (m)	Fv
-	0	69.81	0	2.38	1.65	2.01	-	0	0	-	0	0	0
3.9	3.887	69.81	0.055679702	2.38	1.68	2.03	4.965	1.349	0.0796	14.056	0.48	-0.0097	1
4.88	6.046	69.81	0.086606503	2.38	2.01	2.19	5.774	1.334	0.0787	14.122	2	-0.014	1.25
5.85	6.858	69.81	0.098238075	2.35	1.86	2.1	6.632	1.247	0.0735	11.926	2.62	-0.0058	1.5
6.84	7.321	69.81	0.104870362	2.32	1.77	2.04	7.531	1.203	0.071	9.803	2.77	0.00102	1.75
7.83	8.069	69.81	0.11558516	2.26	1.65	1.95	8.234	1.156	0.0682	8.448	3.07	0.00737	2.09
8.81	8.704	69.81	0.124681278	2.16	1.52	1.84	8.758	1.064	0.0627	7.821	3.57	0.01473	2.26
9.77	9.212	69.81	0.131958172	2.01	1.4	1.71	8.893	0.923	0.0544	7.761	3.78	0.02565	2.5
10.75	9.643	69.81	0.138132073	1.89	1.34	1.62	9.363	0.862	0.0509	7.173	3.8	0.03327	2.76
11.74	10.03	69.81	0.143675691	1.83	1.28	1.55	9.837	0.822	0.0485	6.565	3.67	0.03937	3.01
12.7	10.41	69.81	0.149119037	1.77	1.22	1.49	10.25	0.783	0.0462	6.123	3.53	0.04267	3.25
13.64	10.86	69.81	0.155565105	1.77	1.16	1.46	10.76	0.765	0.0451	5.672	3.32	0.05029	3.5
14.66	11.42	69.81	0.163586879	1.74	1.1	1.42	11.2	0.737	0.0435	5.356	3.12	0.05309	3.76
15.66	11.87	69.81	0.170032947	1.74	1.04	1.39	11.71	0.718	0.0424	5.013	3.97	0.05512	4.01
16.6	12.49	69.81	0.178914196	1.74	1.01	1.37	12.27	0.708	0.0418	4.757	2.73	0.05791	4.25
17.6	13.35	69.81	0.191233348	1.74	0.98	1.36	12.87	0.7	0.0413	4.582	2.65	0.05842	4.51
18.55	14.17	69.81	0.202979516	1.74	0.94	1.34	13.41	0.689	0.0407	4.436	2.53	0.05969	4.75



Appendix 2: SNAME data sheet for planing the crew-boat

A_P : Projected planing area, excluding area of external strips
B_P : Beam or breadth over chines excluding external spray strips
B_{PA} : Main breadth over chines excluding external spray strips
B_{px} : Maximum breadth over chines, excluding external spray strips
L_P : Projected chine length
S : Area of wetted surface (This is the actual wetted surface underway including the area of the sides which is wetted at low speeds and the wetted bottom area of external spray strips; however, the area wetted by the spray is excluded)
α : Angle of attack of after portion of mean buttock in degrees
β : Dead rise angle of planing bottom in degrees This is obtained by approximating eaach body plan section by a straight line
Δ : Displacement of rest, weight of
T : Trim angle of hull with respect to attitude
∇ : Displacement at rest, volume of Subscript O indicates when hull is at rest in water

REFERENCES

Akers, R., 2002. Wave Piercers and Fast Cats, Professional Boatbuilders, No. 74, pp: 54-67. US 2002/0096098 Al Kingsbury (43).

Baker, G.S., 1912. Some Experiments in Connection with the Design of Floats for Hydro-Aeroplanes. ARC (British) R and M, No 70.

Blount, D.L. and R.J. Bartee, 1997. Design of propulsion system for high speed craft. Marine Technol., 34 (4): 276-292.

Cheng, X. and J.F. Wellicome, 1994. Study of planing hydrodynamics sing strips of transversely variable pressure. Journ of Ship Res., 38 (1): 30-41.

Cumberbatch, E., 1958. Two-dimensional Planing at froude number. J. Fluid Mechanics, 4 (5): 466-478.

Doctors, L., 1975. Representation of three dimensional planing surfaces by finite elements. Proc.1st Conf. Numerical Ship Hydrodynamics, 32: 517-537.

ITTC, 1957. Proceedings of the 8th ITTC, Madrid, Spain, Published by Canal de.

Korvin-Kroukovsky, B.B., D. Savitsky and W.F. Lehman, 1949. Wetted Area and Centre of pressure of Planing Surfaces. Report 360, Davidson Laboratory, Stevens Institute of Technology.

Royce, R.A., 1994. A Rational Prismatic Hull Approach for Planing Hull Analysis. Great Lakes and Great Rivers Section, SNAME, Cleveland, Ohio.

Savitsky, D., 1964. Hydrodynamic Design of Planing Hulls. Marine Technol., 32 (3): 78-88.

Savitsky, D. and P.W. Brown, 1976. Procedure for hydrodynamic evaluationations of planing hulls in smooth and rough water. Marine Technol., 13 (4): 381-400.

Savitsky, D. and J.W. Neidinger, 1954. Wetted Area and Centre of Pressure at Planing Surfaces at Very Low Speed Coefficients. Report 493, Davidson Laboratory, Stevens Institute of Technology.

Scottorf, W., 1934. Experiments with Planing Surfaces. NACA TM 739 and NACA TM 661, 1932.

Shuford, C.L. Jr., 1958. A Theoretical and Experimental Study of Planing Surfaces Including Effects of Cross-Section and Plan Form. NACA Report 1355.

SNAME, 1957. Small Craft Data Sheets: Hard-chine Boat. 74 Trinity Place, New York, pp: 1006.

Squire, H.B., 1957. The motion of a simple wedge along the water surface. Proc. Royal Soc. London, Series A, 243: 48-64. http://rspa.royalsocietypublishing.org/content/243/1232/48.abstract.

Tong, J., C.K. Lee and S.C. Wong, 1989. A finite element approach to planing problem. Int. J. Num. Methods in Eng., 43 (7): 1253-1273; John Wiley and Sons Ltd. DOI: 10.1002/(SICI)1097-0207(19981215) 43:7.

Tuck, F.O., 1975. Low aspect ratio flat ship theory. J. Hydrodynamics, 9 (1, 9): 3-12, DOI: 10.2514/3.63010.

Vorus, W., 1993. Hydrodynamic Impact and Penetration of Flat Cylinders. Unpublished, J. Eng. Maths, Vol. 48. ISBN: 0022-0833. DOI: 10.1023: engi. 0000018173. 66342.9f.

Wagner, H., 1967. Uber stoss-und gleitvorgange an der oberflanche von flussigkeitenZeitschrift fur Angewandle Mechanik, 12 (193): 822-825. DOI: 10. 1007/BF00887648. http://www.springerlink.com/content/t085j5026681407. Publisher: Springer-New York.

Wang, D.P. and P. Rispen, 1971. Three dimensional planing at high froude number. J. Ship Res., 15 (3): 221-230.