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Volterra-Fredholm Integral Equation with Carleman Kernel in Position and Time

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Abstract: In this study, we introduce an efficient method to find and discuss an approximate solution of the integral equation of type Volterra-Fredholm in the space $L_2[a, b] \times [0, T]$. The kernel of Fredholm is considered in position and represented in a logarithmic form, while the kernel of Volterra is taken in time as a continuous function. Using a numerical method we obtain a linear system of Fredholm integral equations is position which will be solved.

Key words: Volterra-Fredholm Integral Equation (V-FIE), contact problem, potential theory method, legendre polynomials

INTRODUCTION

The mathematical physics and contact problems in the theory of elasticity are modeled by an integral equation of the first or second kind (Abdou, 2002a, b, 2001). Mkhtarian and Abdou (1990a, b) discussed some different methods to solve Fredholm Integral Equation (FIE) of the first kind with logarithmic kernel and Carleman function respectively. Abdou (2002c) obtained the spectral relationships for the FIE of the first kind in one, two and three dimensional. Abdou and Salama (2004) obtained many spectral relationships for an integral equation of V-FIE of the first kind. Devles and Mohamed (1985) and Atkinson (1997) many different methods are used to solve the Fredholm integral equation of the second kind numerically. Arutiunian (1959) and Abdou (2002c), used orthogonal polynomial method of type Legendre polynomials to obtain, numerically, the solution of Fredholm-Volterra integral equation of the second kind with singular kernel with respect to position.

In this research, we consider the V-FIE of the first kind:

$$\int_0^t\int_{-1}^1 F(t,\tau)|x-y|^{-\upsilon}\phi(y,t)dyd\tau + \int_0^t G(t,\tau)\phi(x,\tau)d\tau = f(x,t) \quad (0<\upsilon<1) \tag{1}$$

Under the condition:

$$\int_{-1}^1 \phi(x,t)dx = P(t) \tag{2}$$

The integral Eq. 1 under Eq. 2 is investigated from the c ontact problem of a rigid surface (G, υ) h aving an elastic material, where G is the displacement magnitude and υ is poisson's coefficient. If a stamp of length 2 unit, where its surface is describing by $f_*(x)$, is impressed into an elastic layer surface of a variable force P(t), whose eccentricity of application e(t), that cases rigid displacement γ(t). Therefore, we define the free term of (1) as:

$$f(x,t) = \pi\theta[\delta(t) - f_*(x)], \quad (\theta = \frac{G}{2(1-\upsilon)}, \quad 0 \le t \le T \langle\infty) \tag{3}$$

Here, Eq. 1, the given function of time F(t, τ) represents the resistance forces of the lower material, while G(t, τ) is called the supplied external force in the contact domain of the upper and lower surfaces.

In this research, a numerical method is used to obtain a system of FIEs of the first kind or second kind depending of the relation between the derivatives of the two functions F(t, τ) and G(t, τ) for all values of t, τ∈[0, T].

Then using potential theory method, the spectral relationships for the Gegenbauer operator are obtained for the system of FIEs of the first kind. Finally, we use Nystrom product method and Toeplitz matrix method to obtain the numerical solution for the linear system of FIE of the second kind with Carleman kernel. The error estimate, in each case is computed.

MATERIALS AND METHODS

In order to guarantee the existence of a unique solution of Eq. 1, under the condition Eq. 2, we assume the following:

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- The kernel

$$k\left(\left|\frac{x-y}{\lambda}\right|\right)$$

satisfies the discontinuity condition

$$\left\{\int_{-1}^{1}\int_{-1}^{1}k^2\left(\left|\frac{x-y}{\lambda}\right|\right)dxdy\right\}=A$$

(A is a constant)

- For all values of t, $\tau\in[0, T]$ the two continuous function of time $F(t, \tau)$ and $G(t, \tau)$ satisfy, $|F(t, \tau)|< B$, $|G(t, \tau)| < C$, B and C are constants
- The known function: $f(x, t)\in L_2[-1, 1] \times C[0, T]$ and its norm defined as:

$$\|f(x,t)\|_{L_2\times C}=\max_{0\le t\le T}\int_0^t\left\{f^2(x,\tau)dx\right\}^{\frac{1}{2}}d\tau$$

- The unknown function $\phi(x, t)$ behaves like $f(x, t)$ and satisfies Lipshitz condition with respect to the first argument and Holder condition for the second argument

To obtain the solution of Eq. 3, under Eq. 2, we divide the interval [0,T], as follows:

$$0=t_0<t_1<\ldots<t_N=T$$

where,

$$t=t_j,\ j=0,1,2,\ldots,N$$

to get

$$\int_0^{t_j}G(t_j,\tau)\phi(x,\tau)d\tau+\int_0^{t_j}\int_{-1}^{1}F(t,\tau)|x-y|^{-\upsilon}dyd\tau=f(x,t_j)\tag{4}$$

Under the condition:

$$\int_{-1}^{1}\phi(x,t_j)dx=P(t_j)\tag{5}$$

Hence we have,

$$\sum_{i=0}^{j}\upsilon_iG(t_k,t_i)\phi(x,t_i)+\sum_{l=0}^{j}u_lF(t_j,t_l)\int_{-1}^{1}|x-y|^{-\upsilon}\phi(y,t_l)dy+O(\hbar_j^p)+O(\hbar_j^{\tilde{p}})=f(x,t_j),\tag{6}$$

$$(\hbar_j=\max_{0\le i;l}h_j;\ h_i=t_{i+1}-t_i)$$

where, $O(\hbar_j^p)$ the estimate error deduced from the approximate integral of the function $G(t, \tau)$ and

$$O(\hbar_j^{\tilde{p}})$$

depends of $F(t, \tau)$. The values of weight functions u_i, u_l and p, $\tilde{p}$, depending on the number of derivatives of $G(t, \tau)$ and $F(t, \tau)$, for all $\tau\in[0, T]$, with respect to t. For example, if $G(t, \tau)\in C^4[0, T]$, then, we have P = 4, $j\approx 4$ and

$$\upsilon_0=\frac{h_0}{2},\ \upsilon_4=\frac{h_4}{2},\ \upsilon_n=h_n,\ n=1,2,3,\ \upsilon_n=0$$

for n>4. While, if $F(t, \tau)\in C^3[0, T]$, we have p = 3, k ≈ 3,

$$u_0=\frac{h_0}{2},u_3=\frac{h_3}{2},u_m=h_m,\ m=1,2$$

and u_m = 0 for m>3. More information for the characteristic points and quadrature coefficient are found (Atkinson, 1997; Abdou, 2003). Using the following notations:

$$G(t_j,t_l)=G_{j,l},\ F(t_j,t_l)=F_{j,l},\ \phi(x,t_i)=\phi_i(x)$$
$$f(x,t_i)=f_i(x),\ (i,j,l=0,1,2,\ldots,N)\tag{7}$$

the Eq. 6, after neglecting the error, becomes

$$\sum_{i=0}^{j}\upsilon_iG_{j,i}\phi_i(x)+\sum_{l=0}^{j}u_lF_{j,l}\int_{-1}^{1}|x-y|^{-\upsilon}\phi_l(y)dy=f_j(x)\tag{8}$$

Under the condition:

$$\int_{-1}^{1}\phi_j(x)dx=P_j,\ (P_j\ \text{are constants}\ j=0,1,\ldots,N)\tag{9}$$

- The Eq. 8 represents a linear system of FIEs of the second kind, for all cases when the two functions $G(t, \tau)$ and $F(t, \tau)$ have the same derivatives with respect to time $t\in[0, T]$. Hence we have,

$$\mu_j\phi_j(x)+\mu_j'\int_{-1}^{1}|x-y|^{-\upsilon}\phi_j(y)dy=g_j(x)\tag{10}$$

where,

$$g_j(x) = f_j(x) - \sum_{i=0}^{j-1} u_i G_{j,i} \phi_i(x) - \sum_{i=0}^{j-1} u_i F_{j,i} \int_{-1}^{1} |x-y|^{-\upsilon} \phi_j(y)dy$$
$$(\mu_j = \frac{h_i}{2} G_{j,j},\ \mu'_j = \frac{h_j}{2} F_{j,j},\ G_{j,j} \neq 0,\ u_i = \upsilon_i)$$

- When the function G(t, τ) has n derivatives with respect to t, n<j, therefore the Eq. 8 takes the following forms:

$$\sum_{i=0}^{n} u_i \left\{ G_{n,i}\phi_i(x) + F_{n,i} \int_{-1}^{1} |x-y|^{-\upsilon} \phi_i(y)dy \right\} = f_n(x) \quad (11)$$

$$\sum_{i=n+1}^{j} u_i F_{J,i} \int_{-1}^{1} |x-y|^{-\upsilon} \phi_i(y)dy = f_3 - \sum_{i=0}^{n} u_i \left\{ G_{n,i}\phi_i + F_{n,j} \int_{-1}^{1} |x-y|^{-\upsilon} \phi_i(y)dy \right\} \quad (12a)$$

The Eq. 11 represents a linear system of FIEs of the second kind, while Eq. 12a of the first kind, $\phi_i(x)$, i = 0, 1, ..., n in the R.H.S of Eq. 12a represent, the recurrence solution of integral Eq. 11.

- When the function F(t, τ) has n derivatives such that n⟨k, hence we have;

$$\sum_{i=n+1}^{j} u_i G_{j,i}\phi_i(x) = f_j(x) - \sum_{i=0}^{n} \gamma_i(u_i, G_{n,i}, F_{n,i})\phi_i(x) \quad (12b)$$

where, φ, (x) in the R.H.S is the solution of Eq. 11 and γ_i in Eq. 12b are distinct points.

RESULTS AND DISCUSSION

Spectral relationships for carleman integral equation: In this study, using potential theory method by Abdou (2001, 2002b), we obtain the Spectral relationships for the FIE of the first kind with Carleman kernel. The importance of Carleman kernel came from the research of Arutinuian (1959), who has showed that the plane contact problem of the nonlinear theory of plasticity, in its first approximation can be reduced to FIE of the first kind with Carleman kernel. Consider the integral equation:

$$\int_{-1}^{1} |x-y|^{-\upsilon} \phi(y)dy = f(x) (0 < \upsilon < 1) \quad (13)$$

Under the static condition:

$$\int_{-1}^{1} \phi(y)dy = P, (P \text{ is constant}) \quad (14)$$

To solve Eq. 13, under the condition Eq. 14, we introduce the general Carleman function;

$$U(x,t) = \int_{-1}^{1} \frac{\phi(y)dy}{\left[(x-y)^2 + t^2\right]^{\frac{\upsilon}{2}}} \quad (15)$$

The solution of Eq. 15, under 14, is equivalent to the boundary value problem:

$$\begin{aligned} &\Delta U + \frac{\mu}{t}\frac{\partial U}{\partial t} = 0, \\ &((x,t) \notin (-1,1),\ \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial t^2}) \\ &U(x,0) = f(x), \\ &(x,t) \in (-1,1) \\ &U(x,t) \cong P_r^{-\upsilon}, \\ &(P \to 0 \text{ as } r = \sqrt{a^2+t^2} \to \infty) \end{aligned} \quad (16)$$

The complete solution of Eq. 13 is given by Mkhtarian and Abdou (1990a, b).

$$\phi(x) = \frac{\Gamma(\frac{\upsilon}{2})}{\sqrt{\pi}\Gamma(\frac{1+\upsilon}{2})} \lim_{t\to 0} t|y|^{\upsilon} \frac{\partial U}{\partial t},\ x \in (-1,1) \quad (17)$$

where, Γ(n) is the gamma function. Using the substitution:

$$U(x,t) = |t|^{-\frac{\upsilon}{2}} V(x,t) \quad (18)$$

and the transformation mapping

$$z = \frac{1}{2}w(\zeta) = \frac{1}{2}(\zeta + \frac{1}{\zeta}), \quad (\zeta = \rho e^{i\theta},\ z = x+iy,\ i = \sqrt{-1}) \quad (19)$$

the boundary value problem Eq. 16, yields

$$\begin{aligned} &\Delta V(\rho,\theta) + \upsilon(2-\upsilon)\left[\frac{1}{(\rho^2-1)^2} + \frac{1}{4\rho^2 \sin^2\theta}\right] V(\rho,\theta) = 0, (\rho < 1) \\ &\left[\frac{1}{2}(\rho - \frac{1}{\rho})\sin\theta\right]^{-\frac{\upsilon}{2}} V(\rho,\theta) = 0\Big|_{\rho=1} = f(\cos\theta) \\ &(-\pi \langle \theta \langle \pi),\ V(0,\theta) = 0,\ (\Delta = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial \theta^2}) \end{aligned} \quad (20)$$

where,

$$V(x,y) = V((\rho + \frac{1}{\rho})\cos\theta,$$

$$(\rho - \frac{1}{\rho})\sin\theta) = V(\rho,\theta)$$

The transformation mapping (Eq. 19) maps the region in x-y plane into the region outside the unit circle γ, such that $w'(\zeta)$ does not vanish or become infinite outside γ. The mapping function (Eq. 19) maps the upper and the lower half-plane $(x, y)\in(-1, 1)$ into the lower and upper of semi-circle $\rho = 1$, respectively. Moreover, the point $z = \infty$ will be mapped onto the point $\zeta = 0$. Using the separation of variable:

$$V(\rho,\theta) = R(\rho)z(\theta) \qquad (21)$$

the first integral Eq. 20 becomes:

$$\rho^2 \frac{d^2R}{d\rho^2} + \rho\frac{dR}{d\rho} + \left[\upsilon(2-\upsilon)\frac{\rho^2}{(1-\rho^2)} - \alpha^2\right] \qquad (22)$$

$$R(\rho) = 0 \ (\ 0 \le \rho \langle \ 1 \ )$$

and

$$\frac{d^2z}{d\theta^2} + \left[\alpha^2 + \frac{\upsilon(2-\upsilon)}{4\sin^2\theta}\right]z(\theta) = 0 - \pi \langle \ \theta \le \pi \qquad (23)$$

where, α^2 is the constant of separation.

As Abdou (2001), the general solution of Eq. 22 and 23, respectively takes the form:

$$R(\rho) = \rho^{n+\upsilon L2}(1-\rho^2)^{\upsilon L2}F(\frac{\upsilon}{2}, n+\upsilon; n+1; \rho^2) \qquad (24)$$

$$(R(0) = 0, 0 \le \rho < 1, n = 0,1,2,...)$$

and

$$z(\theta) = |\sin\theta|^{\frac{\upsilon}{2}} C_n^{\frac{\upsilon}{2}}(\cos\theta) \qquad (25)$$

$$(-\pi \langle \theta \le \pi, \ n = 0,1,2,...)$$

Here, F (a, b, c, z) is the Hypergeometric function and

$$C_n^{\frac{\upsilon}{2}}(x)$$

is the Gegenbauer polynomial. Using Eq. 24 and 25 in Eq. 21, then using the result in Eq. 18, we have

$$U(\rho,\theta) = \rho^{n+\upsilon}F\left(\frac{\upsilon}{2}, n+\upsilon; n+1+\frac{\upsilon}{2}; \rho^2\right)C_n^{\frac{\upsilon}{2}}(\cos\theta) \qquad (26)$$

$$U(\rho,\theta) = U\left(\frac{1}{2}(\rho+\frac{1}{\rho}), \frac{1}{2}(\rho-\frac{1}{\rho})\sin\theta\right) = U(x,y)$$

The complete solution of the problem, can be obtained, by writing Eq. 17 in polar coordinates:

$$\phi(\cos\theta) = \frac{\Gamma(\frac{\upsilon}{2})(\sin\theta)^{\upsilon-1}}{\sqrt{\pi}2^{\upsilon+1}\Gamma\left(\frac{1+\upsilon}{2}\right)}\lim_{\rho\to 1}(1-\rho^2)^{\upsilon}\frac{\partial U}{\partial U}, \ (0 < \theta < \pi) \qquad (27)$$

then using Eq. 26, in Eq. 27, to obtain

$$\phi(\cos\theta) = \frac{\Gamma(\upsilon)\Gamma(n+1+\frac{\upsilon}{2})}{\sqrt{\pi}2^{\upsilon}\Gamma\left(\frac{1+\frac{\upsilon}{2}}{2}\right)\Gamma(n+\upsilon)}(\sin\theta)C_n^{\frac{\upsilon}{2}}(\cos\theta) \qquad (28)$$

Hence, inserting Eq. 28 in Eq. 13, we arrive to the following spectral relationships:

$$\int_{-1}^{1}\frac{C_n^{\frac{\upsilon}{2}}(u)du}{|x-u|^{\upsilon}(1-u^2)^{\frac{1-\upsilon}{2}}} = \lambda_n C_n^{\frac{\upsilon}{2}}(x) \qquad (29)$$

$$\lambda_n = \pi\Gamma(n+\upsilon)[n!\Gamma(\upsilon)\cos(\pi\frac{\upsilon}{2})]^{-1}$$

where, λ_n are called the eigenvalues of the integral operator. For a Volterra-Fredholm integral operator, we have,

$$\sum_{j=0}^{k}u_j F_{j,k}\int_{-1}^{1}\frac{C_{n_j}^{\frac{\upsilon}{2}}(u)du}{|x-u|^{\upsilon}(1-u^2)^{\frac{1-\upsilon}{2}}} = \sum_{j=0}^{k}u_j F_{j,k}\lambda_{n,j}C_{n_j}^{\frac{\upsilon}{2}}(x)(n_j \ge 0) \qquad (30)$$

Many spectral relationships can be established from Eq. 29.

- Let x = -1 in Eq. 29 and use the following relation, (Abdou and Salama, 2004):

$$C_n^{\frac{\upsilon}{2}}(-x) = (-1)^n C_n^{\frac{\upsilon}{2}}(x) \qquad (31)$$

We have,

$$\int_{-1}^{1}\frac{C_n^{\frac{\upsilon}{2}}(u)du}{|x-u|^{\upsilon}(1-u^2)^{\frac{1-\upsilon}{2}}} = (-1)^n\lambda_n C_n^{\frac{\upsilon}{2}}(x) \qquad (32)$$

Differentiating Eq. 29 with respect to x and using the relation:

$$\frac{d}{dx}C_n^{(\upsilon)}(x) = nC_{n-1}^{(\upsilon+1)}(x) \tag{33}$$

we get;

$$\int_{-1}^{1} \frac{C_n^{\frac{\upsilon}{2}}(u)du}{|x-u|^{\upsilon+1}(1-u^2)^{\frac{1-\upsilon}{2}}} = \frac{-\pi\Gamma(n+\upsilon)C_{n-1}^{(\upsilon+1)}(x)}{(n-1)!\Gamma(1+\upsilon)\cos(\frac{\pi\upsilon}{2})} \tag{34}$$

- Using the Gegenbauer

$$C_n^{\frac{\upsilon}{2}}(x)$$

and Jacobi $P^{\alpha,\beta}{}_n(x)$ relation

$$C_n^{\frac{\upsilon}{2}}(x) = \frac{\Gamma(\frac{\upsilon+1}{2})\Gamma(n+\upsilon)}{\Gamma(\upsilon)\Gamma(n+\frac{\upsilon+1}{2})} P_n^{(\frac{\upsilon-1}{2},\frac{\upsilon-1}{2})}(x) \tag{35}$$

we have the following spectral relationships

$$\int_{-1}^{1} \frac{P_n^{(\frac{\upsilon-1}{2},\frac{\upsilon-1}{2})}(u)du}{|x-y|^{\upsilon}(1-u^2)^{\frac{1-\upsilon}{2}}} = \lambda_n P_n^{(\frac{\upsilon-1}{2},\frac{\upsilon-1}{2})}(x) \tag{36}$$

Using the famous formulas (Gradchtein and Rezuk, 1971),

$$\lim_{\upsilon\to 0}\Gamma(\frac{\upsilon}{2})C_{2n}^{\frac{\upsilon}{2}}(x) = \frac{1}{n}T_{2n}(x)$$

and

$$\ln\frac{1}{|x-y|} = \lim_{\upsilon\to 0}\left(|x-y|^{-\upsilon}-1\right)^{-\upsilon} \tag{37}$$

we arrive to the following spectral relationships

$$\int_{-1}^{1} \ln\frac{1}{|x-y|}\frac{T_{2m}(y)}{\sqrt{1-y^2}}dy = \begin{cases} \pi\ln 2 & m=0 \\ \dfrac{\pi T_{2m}(x)}{2m} & m\ge 0 \end{cases} \tag{38}$$

where T_{2m} (x) is the Chebyshev polynomial of the first kind.

Solution for the system of Fredholm integral equation of the second kind: In this study, we discuss the Toeplitz matrix method (Abdou *et al*., 2003) and product Nystrom method of Atkinson (1997) and Devles and Mohamed (1985) to obtain the numerical solution of the system of FIEs of the second kind (10).

Toeplitz matrix method: The idea of this method is to obtain 2N+1 linear algebraic equations, the coefficients matrix is expressed as sum of two matrices one of them is the Toeplitz matrix and the other is a matrix with zero elements except the first and last columns (rows). Consider the integral equation:

$$\mu\phi(x) - \lambda\int_{-a}^{a} k(x,y)\phi(y)dy = f(x) \tag{39}$$

Then, assume that

$$\int_{-a}^{a} k(x,y)\phi(y)dy = \sum_{m=-N}^{N-1}[A_n(x)\phi(a) + B_n(x)\phi(a+h)]R_N \tag{40}$$

$$(a = nh,\ h = \frac{2a}{N},\ x = mk)$$

Here, A_n (x) and B_n (x) are arbitrary functions, will be determined and R_N is the error of order $O(h^3)$. Using the principal idea of the Toeplitz matrix, we can arrive to the following linear algebraic system:

$$\begin{gathered}\Phi(mh) = U^{-1}f(mh), \\ U = \mu I - \lambda(G_{mn} - E_{mn}), |U| \ne 0\end{gathered} \tag{41}$$

where,

$$\begin{gathered}G_{mn} = A_n(mh) + B_{n-1}(mh), \\ -N \le n, m \le N\end{gathered} \tag{42}$$

is a Toeplitz matrix of order 2N+1,

$$E_{mn} = \begin{cases} B_{-N-1}(mn) & n=-N \\ 0 & -N<n<N \\ A_N(mh) & n=N \end{cases} \tag{43}$$

represents a matrix of order (2N+1) whose elements are zeros except the first and the last columns and I is the identity matrix.

Definition (39): The Toeplitz matrix method is said to be convergent of order r in [-α, α], if and only if for N sufficiently large, there exist a constant D>0 independent on N such that:

$$\|\phi(x) - \phi_N(x)\| < DN^{-1} \tag{44}$$

The product Nystrom method: This method is described by Atkinson (1997) and Devles and Mohamed (1985). Consider the integral equation:

$$\mu\phi(x) - \lambda\int_{a}^{b} p(x,y)k^2(x,y)\phi(y)dy = f(x) \tag{45}$$

where, p and $\tilde{k}$ are respectively badly behaved and well behaved functions of their arguments. According to the product Nystrom method, the Eq. 45 takes the form:

$$\mu\phi(x_i)-\lambda\sum_{J=0}^{N}w_{ij}\tilde{k}(x_i,x_j)\phi(x_j)=f(x_i) \quad (46)$$

where, $x_i = y_i = \alpha + ih$, i = 1, 2,..., N with

$$h=(\frac{b-a}{N})$$

and N even. The weight functions w_{ij} are defined as:

$$\begin{aligned} &w_{i0}=\beta_1(y_i),\ w_{i,2j+1}=2\gamma_{j+1}(y_i),\\ &w_{i,2j}=\alpha_j(y_i)+\beta_{j+1}(y_i),\ w_{i,N}=\alpha_{\frac{N}{2}}(y) \end{aligned} \quad (47)$$

$$\begin{aligned} &\alpha_j(y_i)=\frac{1}{2h^2}\int_{y_{2j-2}}^{y_{2j}}p(y_i,y)(y-y_{2j-2})(y-y_{2j-1})dy\\ &\beta_j(y_i)=\frac{1}{2h^2}\int_{y_{2j-2}}^{y_{2j}}p(y_i,y)(y_{2j-1}-y)(y_{2j}-y)dy \end{aligned} \quad (48)$$

$$\gamma_j(y_i)=\frac{1}{2h^2}\int_{y_{2j-2}}^{y_{2j}}p(y_i,y)(y-y_{2j-2})(y_{2j}-y)dy$$

The solution of the linear system (Eq. 46) may take the form:

$$\Phi=[\mu I-\lambda w]^{-1}F,\ \ |\mu I-\lambda w|\neq 0 \quad (49)$$

As Atkinson (1997) and Devlas and Mohamed (1985), the integral Eq. 10 can be solved using Nystrom method. The following numerical results are obtained, when the exact solution $\phi(x, t) = x+t$ and the Volterra kernel $F(t, \tau) = t^2$ $G(t, \tau) = t$, $\upsilon = 0.8$.

Here, ϕ^T_{mn} means numerical method using Toeplitz matrix where, R^T is the resulting error, while ϕ^N_{mn} for the Nystrom method and the resulting error R_N. The dividing interval is considered when h = 0.25, t = 0.3 and t = 0.8, N = 40 (Table 1 and 2).

Table 1: The results, when t = 0.3

X	$\phi^{(T)}_{mn}$	$R^{(T)}_N$	$\phi^{(T)}_{mn}$	$R^{(T)}_N$
-1.00	-0.70E+03	-0.70E-03	-0.70E+00	0.33E-05
-0.75	-0.45E+00	0.46E-04	-0.45E+00	0.83E-04
-0.50	-0.20E+00	0.53E-04	-0.20E+00	0.15E-03
-0.25	0.50E-01	0.54E-04	-0.50E-01	0.22E-03
0.00	0.30E+00	0.53E-04	0.30E+00	0.27E-03
0.25	0.55E+00	0.51E-04	0.55E+00	0.32E-03
0.50	0.80E+00	0.47E-04	0.80E+00	0.35E-03
0.75	0.10E+00	0.41E-04	0.10E+01	0.37E-03
1.00	0.13E+01	0.14E-04	0.13E+01	0.20E-03

Table 2: The results, when t = 0.8

X	$\phi^{(T)}_{mn}$	$R^{(T)}_N$	$\phi^{(T)}_{mn}$	$R^{(T)}_N$
-1.00	-0.20E+00	0.84E-03	-0.20E+00	0.41E-05
-0.75	0.50E-01	0.99E-04	0.50E-01	0.23E-03
-0.50	0.30E+00	0.12E+03	0.30E+00	0.41E-03
-0.25	0.55E+00	0.13E-03	0.55E+00	0.58E-03
0.00	0.80E+00	0.12E-03	0.80E+00	0.72E-03
0.25	0.10E+01	0.12E-03	0.10E+01	0.84E-03
0.50	0.13E+01	0.11E-03	0.13E+01	0.93E-03
0.75	0.15E+01	0.97E-03	0.15E+01	0.96E-03
1.00	0.18E+01	0.32E-04	0.18E+01	0.51E-03

CONCLUSION

From the results and discussions, the following may be concluded:

- The contact problems of a rigid surface having an elastic material, when a stamp of lenth 2 unit is impressed into an elastic layer surface of a strip, which has a resistance force F (t, τ), by a variable force p(t), in time, $0 \le t \le T < \infty$, whose eccentricity of application e(t) represents a Volterra-Fredholm integral equation
- The kind of the system of Fredholm integral equations depends on the relation between the number of the derivatives of F (t, τ) and the external force of resistance G (t, τ), $t \in [0, T]$
- When, there is no external force of resistance i.e., G (t, τ) = 0, we have a system of Fredholm integral equations of the first kind
- The numerical method used gives us a system of FIEs, where the solution of the system depends on the kind of the system. For this, we use potential theory method to solve the Fredholm system of the first kind. And, for the second kind we use Toeplitz matrix method and product Nystrom method
- In the numerical results we must note that: (i) when υ taken the values of 0.1, 0.22. 0.32 (υ is called Poisson's ratio in the theory of elasticity, the error R(υ, t, N), for the two numerical methods, follows the inequality $R(0.1,t,N) < R(0.32,t,N) < \ldots < R(0.8,t,N)$, (ii) Also, for the time t, when t = 0.1,0.3,…,0.8 and fixed values υ and N, we have:

  $$R(\upsilon, 0.1, N) < R(\upsilon, 0.3, N) < R(\upsilon, 0.44, N) < R(\upsilon, 0.8, N)$$

  (iii) Also, for increasing N and fixing υ and t the error decreases

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