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Orbital Mechanism and Satellite Orbital Location

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Abstract: The main focus of the study is to present the mechanism of the orbit in terms of time and position in a more simplified fashion using Kepler's laws. It also shows the various planes transformation between the rotating earth and the orbiting satellite. An illustrative location of a satellite is also presented. The researchers use the geocentric coordinate system that has the centre of mass of the earth as the origin of the coordinate which is the equatorial polar coordinate system where the z-axis extends through the north geographic pole.

Key words: Satellite location, orbital elements, orbital mechanism, GEO, MEO, LEO, julian day, Kepler's law

INTRODUCTION

In modern times, Kepler's laws (1571-1630) are used to calculate approximate orbits for artificial satellite and bodies orbiting the Sun of which Kepler was unaware (Timothy and Charlse, 1986) (such as the outer planets and smaller asteroids).

They apply where any relatively small body is orbiting a larger, relatively massive body, though the effects of atmospheric drag (e.g., in a low orbit), relativity (for example, Perihelion precession of Mercury) and other nearby bodies can make the results insufficiently accurate for a specific purpose.

Information about the location and movement helps to receive direct broadcast data from a satellite as it passes overhead. Information on the time at which the satellite will rise over the horizon helps to direct a receiving antenna (a satellite dish) at that location. Once the path the satellite takes through the sky is known, it enables the ground receiving antenna to track the satellite and therefore receive data continuously until the satellite sets at the completion of the overpass.

Different orbits are used for different satellites for different purposes. A satellite to observe a particular region of the earth every day at approximately the same local time each day throughout the annual cycle require a special orbit with a particular orbital plane precession different from the one to observe events that evolve relatively rapidly in time like natural disasters. The various artificial satellite orbits that are available (Meriam, 1966) are:

Geosynchronous Orbit (GEO), 35,786 km above the earth: Orbiting at the height of 22,282 miles above the equator (35,786 km), the satellite travels in the same direction and at the same speed as the earth's rotation on its axis, taking either 24 h (for solar day) or 23 h 56 min 4.091 sec (for sidereal day) to complete a full trip around the globe. Thus, as long as a satellite is positioned over the equator in an assigned orbital location, it will appear to be stationary with respect to a specific location on the earth. A single geostationary satellite can view approximately one third of the earth's surface. If three satellites are placed at the proper longitude, the height of this orbit allows almost the earth's entire surface to be covered by the satellites.

Medium Earth Orbit (MEO), 8,000-20,000 km above the earth: These orbits are primarily reserved for communications satellites that cover the North and South pole. Unlike the circular orbit of the geostationary satellites, MEO's are placed in an elliptical (oval-shaped) orbit.

Low Earth Orbit (LEO), 500-2,000 km above the earth: These orbits are much closer to the earth, requiring

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satellites to travel at a very high speed in order to avoid being pulled out of orbit by earth's gravity. At LEO, a satellite can circle the earth in approximately one and a half hours.

Orbital mechanism: Orbital mechanics is a subfield which focuses on spacecraft trajectories, motion including orbital maneuvers, orbit plane changes and interplanetary transfers and is used by mission planners to predict the results of propulsion (Bate *et al*., 1971). The motion of these objects is usually calculated from Newton's law of motion and Newton's law of universal gravitation that results in a Kepler orbit.

The orbital equations derived in this study modelled the earth and the satellite as point masses acted upon only by mutual gravitational attraction (www.wikipedIa. com/kepler's laws of plaetary motion). The researchers shall therefore take the shape of the satellite orbit as an ellipse (osculating orbit) and it follows from the mathematics of an ellipse that:

$$r = \frac{p}{1+\varepsilon\cos v} \quad (1)$$

Where r and v form the polar co-ordinates system and p is the semilatus rectum and is given as:

$$p = \frac{h^2}{\mu} \quad (2)$$

Where:

h = A constant called the orbital angular momentum of the satellite

μ = The Kepler's constant (GM_E = 3.9861352E5 km^3 sec^{-2})

G = The gravitational constant and M_E is the mass of the earth

ε = The eccentricity of the ellipse

ε ranges between 0 and 1 with the limiting condition value of 0 in which the ellipse would be a circle with the earth at centre of it. v ranges between 0° and 180°. When v is 0°, the satellite is at the perigee (the point where the satellite is closest to the earth is called perigee) and we have:

$$r_{min} = \frac{p}{1+\varepsilon} \quad (3)$$

And when v is 180°, the satellite is at the apogee (the point where the satellite is farthest from earth is called apogee) and then we have:

$$r_{max} = \frac{p}{1-\varepsilon} \quad (4)$$

The arithmetic mean of Eq. 1 and 2 gives the semi major axis-a;

$$a = \frac{1}{2}\left\{\frac{p}{1+\varepsilon} + \frac{p}{1-\varepsilon}\right\} \qquad \Rightarrow a = \frac{p}{1-\varepsilon^2} \quad (5)$$

Also, the geometric mean of Eq. 3 and 4 gives the semi minor axis-b;

$$b = \sqrt{\left\{\frac{p}{1+\varepsilon} \times \frac{p}{1-\varepsilon}\right\}} \qquad \Rightarrow b = \frac{p}{\sqrt{1-\varepsilon^2}} \text{ or } a\left(1-\varepsilon^2\right)^{\frac{1}{2}} \quad (6)$$

By simple arithmetic,

$$\frac{a}{b} = \frac{b}{p} \quad (7)$$

Combining Eq. 1 and 5, we have:

$$r = \frac{a\left(1-\varepsilon^2\right)}{1+\varepsilon\cos v} \quad (8)$$

Equation 8 gives the equation of the orbit. The rectangular co-ordinates of the satellite from Fig. 1 are:

$$x_0 = r\cos v, \quad y_0 = r\sin v \quad (9)$$

The general equation relating the orbital period with the semi major axis, obtain from Newton's law of gravity is:

$$T^2 = \frac{4\pi^2 a^3}{\mu} \quad (10)$$

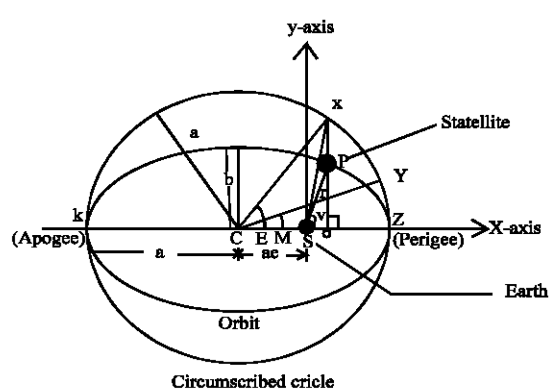

Fig. 1: The orbit as it appears in the orbital plane showing the circumscribed circle

The orbital period is the time required for the satellite to complete one revolution and travelled 2π rad. Therefore, the average angular velocity is given as:

$$\eta = \frac{2\pi}{T} \tag{11}$$

The following expressions are deducted from the geometry of Fig. 1: x is the projection of the planet to the auxiliary circle then the area:

$$|zsx| = \frac{a}{b}.|zsp| \tag{12}$$

y is a point on the auxiliary circle such that the area;

$$|zcy| = |zsx| \tag{13}$$

and M = ∠zcy, where is the mean anomaly. The area of the circular sector is:

$$|zcy| = \frac{a^2M}{2} \quad \text{from} \left(\frac{M}{2\pi} \times \pi a^2\right) \tag{14}$$

and the area swept since perigee is:

$$|zsp| = \frac{b}{a} \times |zsx| = \frac{b}{a} \times \frac{a^2M}{2} = \frac{abM}{2} \tag{15}$$

Also, we have E∠zcx, where E is the eccentric anomaly which is related to the radius r by:

$$r = a(1 - \varepsilon \cos E) \tag{16}$$

$$\Rightarrow a - r = a\varepsilon \cos E \tag{17}$$

The researchers can now establish the relationship between M and E according to Fig. 1, where:

$$|zcy| = |zsx| = |zcx| - |scx| \tag{18}$$

By substitution, we have:

$$\frac{a^2M}{2} = \frac{a^2E}{2} - \frac{a\varepsilon \times a\sin E}{2} \tag{19}$$

Dividing through by, $a^2/2$ we have:

$$M = E\text{-}\varepsilon \times \sin E \tag{20}$$

Also, we have:

$$M = \eta(t - t_p) \Rightarrow M = \eta t \tag{21}$$

When t_p is zero, the time of perigee. E is obtained from Eq. 19 by an infinite series, given as:

$$E \approx M + \left(\varepsilon - \frac{1}{8}\varepsilon^3\right)\sin M + \frac{1}{2}\varepsilon^2 \sin 2M + \frac{3}{8}\varepsilon^3 \sin 3M + ... \tag{22}$$

For the small ε typical of an orbit, such series are quite accurate with only a few terms. We can now relate v and E. From the geometry of Fig. 1, we have:

$$a \times \cos E = a \times \varepsilon + r \times \cos v \tag{23}$$

dividing by a and combining it with Eq. 8 to get:

$$\cos E = \varepsilon + \frac{1 - \varepsilon^2}{1 + \varepsilon \times \cos v} \times \cos v \tag{24}$$

$$\Rightarrow \cos E = \frac{\varepsilon + \cos v}{1 + \varepsilon \times \cos v} \tag{25}$$

Equation 24 can be simplified further using the trigonometric identity:

$$\tan^2 \frac{A}{2} = \frac{1 - \cos A}{1 + \cos A} \tag{26}$$

to get;

$$\tan \frac{E}{2} = \sqrt{\frac{1 - \varepsilon}{1 + \varepsilon}} \times \tan \frac{v}{2} \tag{27}$$

Equation 1-27 have provided us the neccesary expressions for the orbital mechanism in terms of position and time.

Satellite orbital location: At any point in the space, all the artificial satellites whether in GEO, MEO or LEO maintain a specific relationship with the earth as shown in Fig 1.

Point on the trajectory close to the earth is term Perigee and that farther away is term Apogee. The orbit could be termed Geo, Meo or Leo-stationary orbit as the case may be. But sometimes the orbital plane tilts away from the equatorial plane. Such orbit is known as synchronous orbit. The relationships between the equatorial plane and the synchronous orbital plane are shown in Fig. 2. The points where the planes intersect are

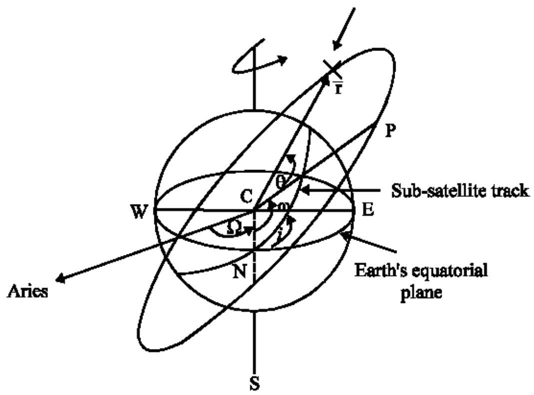

Fig. 2: Parameters describing the relationship between the satellite and the earth

called nodes and the line joining these points of intersection is called the line of nodes, as it connects the center of mass with the ascending and descending nodes. For an inclination between 0 and 90° a satellite motion is towards the east and it is called a direct motion. For an inclination between 90 and 180° the motion is oriented westward and it is called a retrograde motion. Also from Fig. 2, the coordinates of the earth has its x-axis points toward a fixed location termed Aries. This is the direction of a line connecting the centre of the sun and the centre of the earth at the vernal equinox (about March 21). This is the instance when the sub-satellite point crosses the equator South to North (Timothy and Charlse, 1986).

The sub-satellite track defines an imaginary line on the earth's surface such as would be traced out by a straight line drawn from the earth's centre C to the instantaneous position of the satellite. When the sub-satellite track intersects the earth's equatorial plane, as the satellite moves from South to North, it defines the location of the ascending node N. The orbit's inclination, i is the angle between the plane of the satellite orbit and the earth's equatorial plane measured anticlockwise from the latter. The right ascension of the ascending node Ω is the angle measured eastward from the direction of Aries to the ascending node N.

The angle ω, the argument of Perigee, measures the angle in the satellite's orbital plane between the ascending node and the point of perigee P. The satellite's instantaneous position in the orbital plane, the true anomaly is measured by the angle 0 from the point of perigee in the sense of the direction of satellite motion to the vector r. The term Mean Anomaly is substituted mostly for the true anomaly.

Locating the orbital plane with respect to the equatorial plane involves variables Ω and i while the location of the orbital coordinates system with respect to the equatorial coordinates system involves variable ω. The satellite coordinates x_0 y_0 z_0 in the orbital plane are related to the satellite coordinates (x_i, y_i, z_i in the equatorial plane by a linear transformation Timothy and Charlse (1986) and Marcel *et al.* (1997) given as:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} \cos\Omega \operatorname{coe}\omega - \sin\Omega\cos i\sin\omega & -\cos\Omega\sin\omega - \sin\Omega\cos i\cos\omega & \sin\Omega\sin i \\ \sin\Omega\cos\omega + \cos\Omega\cos i\sin\omega & -\sin\Omega\sin\omega + \sin\Omega\cos i\cos\omega & -\cos\Omega\sin i \\ \sin i\sin\omega & \sin i\cos\omega & \cos i \end{bmatrix} \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} \tag{28}$$

But to locate the satellite with respect to a point on the rotating earth, we need a rotating rectangular coordinates system (x_r, y_r, z_r) similar to the geocentric equatorial system. The coordinates of the satellite in the rotating system are related to the coordinates in the geocentric equatorial system by:

$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} \cos(\Omega_e T_e) & \sin(\Omega_e T_e) & 0 \\ -\sin(\Omega_e T_e) & \cos(\Omega_e T_e) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \tag{29}$$

Where Ω is the angular velocity the rotating system turns and T_e is the elapsed time when $x_r$axis coincided with the x_i axis. The coincidence occurs once with every rotation of the earth but it does not occur at the same time every day because of the earth's motion in its orbit around the sun. The value of the product $\Omega_e T_e$ at any time t expressed in minutes after midnight universal time is given as:

$$\Omega_e T_e = \alpha_{g,o} + 0.25068447t \text{ degrees} \tag{30}$$

Where $\alpha_{g,o}$ is the right ascension of the Greenwich meridian at zero hour Universal Time (UT) defined as:

$$\alpha_{g,o} = 99.6909833 + 36000.7689T_c + 0.00038708T_c^2 \text{ degrees} \tag{31}$$

And T_c is the elapsed time in Julian centuries between zero hour universal time on Julian Day and noon universal time on 1st of January, 2000 (Julian day or Julian date is a second dating system used by the Astronomers. The counting starts at the noon of universal time UT) (Lynch, 2000):

$$T_c = (JD - 2451545)/36525 \text{ Julian centuries} \tag{32}$$

The sub satellite Latitude L_s in degrees north and Longitude l_s on any quadrant on the rotating earth surface are:

$$L_s = 90^\circ - \cos^{-1}\left(\frac{z_r}{\left(x_r^2 + y_r^2 + z_r^2\right)^{\frac{1}{2}}}\right) \quad (33)$$

$$l_s = \begin{cases} -\tan^{-1}\left(\frac{y_r}{x_r}\right), & y_r \geq 0 \quad \text{and} \quad x_r \geq 0 (\text{first quadrant}) \\ 180^\circ + \tan^{-1}\left(\frac{y_r}{|x_r|}\right), & y_r \geq 0 \text{ and } x_r \leq 0 (\text{second quadrant}) \\ 90^\circ + \tan^{-1}\left|\frac{x_r}{y_r}\right|, & y_r \leq 0 \quad \text{and} \quad x_r \leq 0 (\text{third quadrant}) \\ \tan^{-1}\left(\frac{|y_r|}{x_r}\right), & y_r \leq 0 \quad \text{and} \quad x_r \geq 0 (\text{fourth quadrant}) \end{cases} \quad (34)$$

These parameters: mean anomaly (M), right ascension of ascending node (Ω), argument of perigee, (ω) inclination (i), eccentricity (ε) and semi major axis (α) define an ellipse; orient it about the earth and the place where the satellite on the ellipse is at a particular time using the relevant coordinates transformation.

Location analysis using sample data: The following data is assumed within the actual range of values to describe analytically the processes involved in locating a satellite on the rotating earth surface. It is assumed that this information is received at noon UT on the 31st August, 2008.

- Eccentricity, ε = 0.001
- Mean anomaly, M = 118°
- Inclination, i = 1°
- Argument of perigee, ω =140°
- Semi major axis, a = 42164.8 km
- Right ascension of ascending node, Ω = 84°
- True anomaly, v = 117°

Computation

Mean angular velocity η: From Eq. 10 and 11:

$$\eta = \frac{1}{a}\left(\frac{\mu}{a}\right)^{\frac{1}{2}} = \frac{1}{42164.8}\left(\frac{3.9861352E5}{42164.8}\right)^{\frac{1}{2}}$$

$$\Rightarrow \eta = 7.292071954E-5 \text{ rds}^{-1}$$

Converting this to degree/s, we have 4.178049469E-3 degree s^{-1}.

Elapsed time since the passage of perigee (t-t_p): From Eq. 21:

$$\left(t - t_p\right) = \frac{M}{\eta}$$

$$\Rightarrow \left(t - t_p\right) = \frac{117^\circ}{(4.178049469E-3)^\circ / s} = 28{,}003.49801 s$$

$$\Rightarrow \left(t - t_p\right) = 7 \text{ h } 46 \text{ min } 43.50 \text{ sec}$$

That is perigee passed at 04:14:43.6 UT of 31st August, 2008 (Hours:Minutes:Seconds).

Eccentric anomaly E: From Eq. 27:

$$\tan\frac{E}{2} = \sqrt{\frac{1-\varepsilon}{1+\varepsilon}} \times \tan\frac{v}{2}$$

$$\Rightarrow E = 2\tan^{-1}\left(\left(\sqrt{\frac{1-0.001}{1+0.001}}\right) \times \tan\frac{117^\circ}{2}\right)$$

$$\Rightarrow E = 116.92^\circ$$

Coordinates systems: We can use the value for E to calculater, the orbital plane radial coordinate, using Eq. 16:

$$r = a\left(1 - \varepsilon \cos E\right)$$

$$\Rightarrow r = 42164.8\left(1 - 0.001\cos 116.92^\circ\right)$$

$$\Rightarrow r = 42183.89 \text{ km}$$

With this we can now transform the polar plane of the satellite into a rectangular plane using Eq .9 yielding:

$$x_o = -19151.09 \text{ km}$$
$$y_o = 37586.12 \text{ km}$$

The satellite coordinates in the equatorial plane can now be obtained using Eq. 28 to get:

$$x_i = 39879.43 \text{ km}$$
$$y_i = -51677.93 \text{ km}$$
$$z_i = -717.34 \text{ km}$$

The coordinates of the satellite in the rotating earth are related to its coordinates in the equatorial plane using Eq. 29. First, we need to calculate $\Omega_e T_e$ from Eq. 30.

The Julian Day (JD) of noon UT on the 31st August, 2008 is 2454710 (from Julian Day Table). Therefore, the elapsed time in Julian centuries since noon UT of 1st January, 2000 T_c is obtained from Eq. 32:

$$\Rightarrow T_c = (2454710 - 2451545)/36525$$
$$\Rightarrow T_c = 0.086652977 \text{ Julian centuries}$$

Then from Eq. 31, the right ascension of the Greenwich meridian at zero hour universal time, $\alpha_{g,o}$ is gotten as:

$$\alpha_{g,o} = 99.6909833 + 36000.7689T_c + 0.00038708T_c^2$$
$$\Rightarrow \alpha_{g,o} = 3{,}219.26 \text{ degree}$$

Since the midnight UT of 31st August, 2008 is not yet reached, then from Eq. 30:

$$\Omega_e T_e = \alpha_{g,o} = 3{,}219.26 \text{ degrees}$$

Converting this to numbers of revolution, we have 56.18668648 revolutions. The decimal fraction in degrees gives the required value of $\Omega_e T_e$. That is:

$$\Omega_e T_e = 10.69634727 \text{ degrees}$$

Now using Eq. 29, the satellite coordinates in the rotating earth is:

$$x_r = -5671.52 \text{ km}$$
$$y_r = -58181.79 \text{ km}$$
$$z_r = -717.34 \text{ km}$$

These values show that the coordinates are in the third quadrant according to Eq. 34. The Latitude L_s in degrees north and the Longitude l_s in degrees of the sub satellite location on the rotating earth. From Eq. 33:

$$L_s = -0.703°N$$

$$l_s = 95.57°$$

The intersection of these lines gives the point of the sub satellite point on the rotating earth.

CONCLUSION

The orbital mechanism and the location of a satellite in a rotating earth have been shown in this study. An illustrative example on how to locate a satellite on a rotating earth was also presented.

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