Journal of Engineering and Applied Sciences 5 (6): 403-412, 2010
ISSN: 1816-949X

Optimal Production Mix for Bread Industries: A Case Study of Selected Bakery Industries in Eastern Nigeria

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Abstract: This research focuses on developing a production mix which will give maximum profit at the lowest input cost, it analyze various process time and profit margin for two bread industries in Eastern Nigeria namely, Premier bread industry and Stephens bread industry. Linear programming model was employed in determining production mix and associated total profit for the two bread industries. A simulator (referred here as BREADPROD) was programmed in the MATLAB™ Graphic Users Integrated Development Environment (GUIDE) windows and used to validate the production mix and accrued total profit for the 2 bread industries. Both the simulator and LP model was subjected to various initial input conditions in bread production processes which include mixing, matching, molding and baking processes with 3 different sizes of bread loaves: the giant, the long and small loaves with reference to non-basic variables x_1, x_2, x_3, respectively. The study gave a production mix of 51% giant loaf, 26% long loaf and 23% small loaf with a profit margin of over 100%. The analysis of LP model and the simulation which operates on GUIDE window of MATLAB™ and runs on run on either cost minimization or profit maximization will help any bread industry to estimate the quantity of bread to be produced in order to maximize profit and minimize production cost. The model adopted gave 202 giant loaf size, 102 long loaf and 92 small loaf for the optimum process time production batch which corresponds with the results of BREADPROD simulator.

Key words: Linear programming, simulation, optimality, duality, feasibility range, optimization

INTRODUCTION

Every industry is concerned with the rate of productivity which can be related to the efficiency of the production system. Productivity is a measure of output from production process, per unit of input. It could equally be seen as a ratio to measure how well an organization (or individual, industry, country) converts input resources (labour, materials, machines, etc.) into goods and services.

In this modern day, manufacturing industries at all levels are faced with the challenges of producing goods (cars, machines, breads, etc.) of right quality and quantity and at right time and more especially at minimum cost (minimized cost) and maximum profit for their survival and growth. Thus, this demands an increase in productive efficiency of the industry. According to Mustapha (1998), the primary responsibility of any production manager is to make decisions that will answer the question of what size of bread should I produce more? Should I invest in new machinery or technology? What price should I try to negotiate for this item? What production mix will yield the highest profit? etc., and of course, maximizing profits is generally one of the most important objectives of any operation. But not only are managers faced with making decisions that potentially impact revenues and costs of the firm, they must implement solutions in a manner that is both efficient and effective. One of the most important decisions facing the production manager is determining the optimum production mix. But what determines whether or not a product mix is optimum or not? This question is complicated by the fact that an optimal production mix for one product may not be the optimal production mix for another (Bender, 2000).

In reality, optimum may encompass many things including: utilizing resources in their most efficient and productive manner; providing favorable cash flows; satisfying attractiveness constraints of buyers and or production; maximizing profits in the short and long run and satisfying current demand trends and preventing oversupply situation. The real challenge is to find a production mix that accomplishes all or most of these things. It is certainly not easy. But there are procedural steps that managers can follow to determine an optimum production mix for their particular operations; one of them is developing a linear programming analysis to aid in product mix decision making. Dibua explains that linear programming model is best applied where a manufacturer

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wants to develop a production schedule/target and an inventory policy that will satisfy sales demand in future period. Ideally, the schedule and policy will enable the production company to satisfy demand and at the same time, minimize the total production and inventory costs. Sonder reported that linear programming works by searching for the basic feasible solution and ends with the search of optimum solution.

This goes to explain simplex method in a more understandable manner. Martand opined that linear programming could be used to solve the task of production planning and control in production processes which can be seen as highly complex in manufacturing environments. Everette said that linear programming could be used to provide uninterrupted production by optimizing production processes for efficiency. However, LP is both a science and an art.

It is a science by virtue of the embodying mathematical techniques present and it is an art because the success of all the phases that precede and succeed the solution of the mathematical model depends largely on the creativity and experience of the production manager and his team (Taha, 2004).

The objectives of this study is to apply Linear Programming (LP) as a mathematical model to optimize the production processes of selected bread bakery industries with a view to proffering applicable production mix and hence, design software using a Graphical Users Integrated Development Environment (GUIDE) which will generate a quicker output and effectively validate the established mathematical model.

The two industries studied; Premier bread industry and Stephens bread industry is faced with a problem of optimization on the quantity and size of the bread to be produced in each production batch for efficient productivity as their present production is not enough to meet demand. The industry also have low profit margin due to inadequate production mix. Premier bread industry for example do not have problem with the sell of produced bread because there is high demand for their products. Hence, the need for a production mix that will ensure profit maximization and cost minimization with a view of meeting customers demand.

In any production industry like bread bakery industries, optimization is a crucial event for a high performance rate. There should be a balanced cost of production that will result to maximization of profit in most of these production industries. The actual model to adopt in order to achieve a proper optimization has also been seen as a great problem facing most bread industries in Eastern Nigeria and a cause for concern to every production engineer. This research is aimed at forecasting the number of each loaf of bread to be produced per batch in order to maximize profit and minimize cost associated with production processes.

Simulation and optimization index: The objective of simulation and optimization is minimizing the resources spent while maximizing the information obtained in a simulation experiment. The general simulation model comprises n input variables (x_1, x_2 …x_n.) and m output variables (f_1 (x), f_2 (x), ... f_m (x)) or (P_1, P_2, ...P_m) (Fig. 1). Simulation optimization entails finding optimal settings of the input variables i.e., values of x_1, x_2,… .x_n which optimize the output variable (s) (Carson and Maria, 1997). Simulation and optimization methods have been applied to applications with a single objective, applications that require the optimization of multiple criterions and applications with non-parametric objective functions. Azadivar and Lee (1988) applied a simulation optimization algorithm based on Box's complex search method to optimize the locations and inventory levels of semi-finished products in a pull-type production system. Hall *et al.* (1996) used Evolutionary Strategies (ES) with a simulation model for optimizing a Kanban sizing problem. Lutz (1995) developed a procedure that combined simulation with Tabu Search (TS) to deal with problems of Work-in-Process (WIP) inventory management.

Fu and Healy (1992) applied the Perturbation Analysis (PA) technique to inventory models where the demand has an associated renewal arrival process. In this research, simulation will be employed as an optimization tool developed in MATLAB™ Graphic Users Integrated Development Environment (GUIDE) to verify the output of classical linear programming model.

Model building: For the model construction, the general purpose MATLAB™ was selected and used because of its flexibility and low cost to simulate this type of problems. The resultant package is branded BREADPROD; the simulator has an interactive, graphical and user-friendly window designed for use in bread industries. It is believed to be useful in determining the production mix and associated total profit per batch for bread industries thus, a combination of mathematical and simulation models was therefore used to solve the decision problem. BREADPROD produces several output

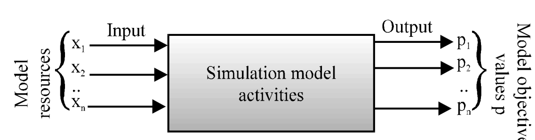

Fig. 1: Representation of the simulation model

files that contain simulation results in pie chart format representing the percentage of giant loaf (x_1), long loaf (x_2), small loaf (x_3) sizes of bread and quantity to be produced at the end of each production batch and accrued net profit. It equally has the ability of detecting non-feasible production mix. BREADPROD is a powerful tool that can be used to view analysis output in bread industries and other multi-product industries. To obtain the required result, simulation is designed to run on either cost minimization or profit maximization.

Data source and validation

Premier bread Industry: In Ekwulobia, Anambra state Nigeria bakes three sizes of bread loaves: the giant loaf, the long loaf and small loaf. These three sizes of loaves required different amounts of four kinds of labour: mixing, matching, molding and baking. Information obtained from the factory shows that the factory has 630 min of mixing labour, 560 min of matching labour, 546 min of molding labour and 560 min of baking labour each production batch. Each giant loaf requires 2 min of mixing labour, 3 min of matching labour, 3 min of molding labour and 2 min of baking labour; each long loaf requires 1 min of mixing labour, 1 min of matching labour, 2 min of molding labour and 3 min of baking labour, each small size loaf requires 2 min of mixing labour, 4 min of matching labour, 2 min of molding labour and 2 min of baking labour per bag of floor. The costing of products carried out indicated that Premier bread industry sells giant loaf ₦120 (12,000 kobo), long loaf ₦80 (8,000 kobo) and small loaf ₦60 (6,000 kobo). Thus, makes a profit of 1400 kobo per each loaf of giant size sold; 1100 kobo per each loaf of long size sold and 400 kobo per each loaf of small size sold. The data is shown in Table 1.

Stephens bread industry: In Orlu, Imo state Nigeria bakes three sizes of bread loaves: Table 2 shows that the factory allocates 490 min for mixing labour, 300 min for matching labour, 725 min for molding labour and 800 min for baking labour each production batch.

It was also gathered that each giant loaf requires 2 min of mixing labour, 1 min of matching labour, 3 min of molding labour and 4 min of baking labour; each long loaf requires 1 min of mixing labour, 1 min of matching labour, 4 min of molding labour and 3 min of baking labour, each small size loaf requires 2 min of mixing labour, 1 min of matching labour, 2 min of molding labour and 2 min of baking labour per bag of floor. The costing of products carried out indicated that Stephens bread industry sales giant loaf ₦120 (12,000 kobo), long loaf ₦80 (8,000 kobo) and small loaf ₦60 (6,000 kobo). Thus, makes a profit of 1500 kobo per each loaf of giant size sold; 1200 kobo per each loaf of long size sold and 500 kobo per each loaf of small size sold. The data is shown in Table 2. It was observed that the production processes of these industries and most other bread bakeries followed the same operational procedure from raw material to finished product as shown in Fig. 2.

Table 1: Process data table for Premier bread industry

	Process time (min) per loaf size				
Size of loaves	(T_1) mixing	(T_2) matching	(T_3) molding	(T_4) baking	Profit (P) per loaf (kobo)
Giant loaf (x_1)	2	3	3	2	1400
Long loaf (x_2)	1	1	2	3	1100
Small loaf (x_3)	2	4	2	2	400
Total available time (min batch^{-1})	630	560	546	560	-

Table 2: Process data table for Stephens bread industry

	Process time (min) per loaf size				
Size of loaves	(T_1) mixing	(T_2) matching	(T_3) molding	(T_4) baking	Profit (P) per loaf (kobo)
Giant loaf (x_1)	2	1	3	4	1500
Long loaf (x_2)	1	1	4	3	1200
Small loaf (x_3)	2	1	2	2	500
Total available time (min batch^{-1})	490	300	725	800	-

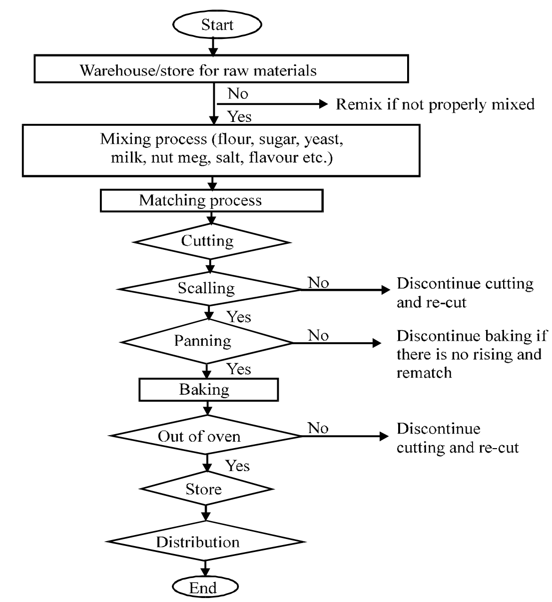

Fig. 2: Operational procedure from raw material to finished product

MATERIALS AND METHODS

The situation in the selected bread bakery industries seeks to determine the optimal production mix, leading to the following definition of terms, x_1-x_3 as the non-basic variables and T_1-T_4 as the total available operation time.

For the construction of the objective function, it is required to increase profit as much as possible so let P represent the total profit (k), the objective is thus expressed as:

$$\text{Max. } P = ax_1 + bx_2 + cx_3 \quad (1)$$

$$\text{Subject to } dx_1 + ex_2 + fx_3 \leq T_1$$

$$gx_1 + hx_2 + ix_3 \leq T_2 \quad (2)$$
$$jx_1 + kx_2 + lx_3 \leq T_3$$
$$mx_1 + nx_2 + qx_3 \leq T_4$$

$$(x_1, x_2, x_3 \geq 0)$$

To solve the mathematical set up model shown above, slack variables are introduced to eliminate the inequalities. Thus:

$$\text{Max. } P = ax_1 + bx_2 + cx_3 + OS_1 + OS_2 + OS_3 + OS_4 \quad (3)$$

$$\text{Subject to } dx_1 + ex_2 + fx_3 + S_1 + OS_2 + OS_3 + OS_4 = T_1$$
$$gx_1 + hx_2 + ix_3 + OS_1 + S_2 + OS_3 + OS_4 = T_2 \quad (4)$$
$$jx_1 + kx_2 + lx_3 + OS_1 + OS_2 + S_3 + OS_4 = T_3$$
$$mx_1 + nx_2 + qx_3 + OS_1 + OS_2 + OS_3 + S_4 = T_4$$

$$x_1, x_2, x_3, S_1, S_2, S_3, S_4 \geq 0 \text{ [Non negative]}$$

Where, x_1-x_3 is quantities of the giant loaf, long loaf and small loaf, respectively called the non-basic variables. S_1-S_4 is the slack variables used to eliminate the inequalities generated in the objective function of the LP model set up. P_{max} is expected profit to be made after optimization called the total gross amount for outgoing profit. C_j-P_{max} is net evaluation row for the objective function of the LP model called decision variable. C_j is objective function coefficients. T_1-T_4 is total available time available for mixing, matching, molding and baking, respectively a, b, c, d, e, f, g, h, i, j, k, I, m, n, p is process available time constants.

Iterative formulation format for the linear programming model:

- Design the sample problem
- Setup the inequalities describing the problem
- Convert the inequalities to equations adding slack variables
- Enter the inequalities in a table for initial basic feasible solutions with all slack variables as basic variables. The table is called simplex table
- Compute C_j and P_{max} values for this solution where C_j is objective function coefficient for variable j and P_{max} represents the decrease in the value of the objective function that will result if one unit of variable corresponding to the column is brought into the solution
- Determine the entering variable (key column) by choosing the one with the highest C_j-P_{max} value
- Determine the key row (outgoing variable) by dividing the solution quantity values by their corresponding solution quantity values by their corresponding key column values and choosing the smallest positive quotient. This means that we compute the ratios for rows where elements in the key column are greater than zero
- Identify the pivot element and compute the values of the key row by dividing all the numbers in the key row by the pivot element. Then change the product mix to the heading of the key column
- Compute the values of the other non-key rows
- Compute the P_{max} and C_j-P_{max} values for this solution
- If the column value in the C_j-P_{max} row is positive, return to step (vi)

If there is no positive C_j-P_{max}, then the final solution has been reached. Mathematical equation as suggested by Dibua was adopted for selection of non-basic variables; it entails subtraction of number of variables from number of constrain equations. Linear programming model was applied as an optimization tool to ascertain the acceptable optimum, the problem thus was solved algebraically as shown in Table 3 and the successive approximations at the 4th iteration gave the values for giant loaf (x_1), long loaf (x_2) and small loaf (x_3), respectively which denotes 1400 x_1 for the giant loaf having 82 loaves, 1100 x_2 for long loaf having 96 loaves and 400 x_3 for small loaf having 55 loaves. The total profit associated with this production mix per batch is 241500 kobo (₦2415.00) which is the number of bread produced per size multiplied by the corresponding profit. The results obtained via this model which represents the number of bread produced per size of loaf was validated using the simulator as shown in Fig. 3.

Simulation and validation: The various process time and profit for the 3 sizes of the bread associated with this model were inputted into the simulator and the simulator returned production mix of 82 giant loaves, 96 long loaves and 55 small loaves. The giant loaf, long loaf and small loaf are 35, 41 and 24% of the total production, respectively. This production mix yielded a maximum profit of ₦2415.00 per production batch. This simulated result agrees with Simplex method result obtained for Premier bread industry. It was, however observed from their demand data that there is high demand for all sizes of

Table 3: Primal result of iteration techniques for Premier bread industry

Cj	1400 X_1	1100 X_2	400 X_3	$0S_1$	$0S_2$	$0S_3$	$0S_4$	Values	Interation no.
OS_1	2	1	2	1	0	0	0	630	1st Interation
OS_2	3	1	4	0	1	0	0	560	
OS_3	3	2	2	0	0	1	0	546	
OS_4	2	3	2	0	0	0	1	0	
Pj	0	0	0	0	0	0	0	-	
Cj-Pj	1400*	1100	400	0	0	0	0	-	
↑Highest C_j-Pj									
$0S_1$	2	1	2	1	0	0	0	630	
OS_2	3	1	4	0	1	0	0	560	
$1400x_1$	1	2/3	2/3	0	0	½	0	182	
OS_4	2	2	2	0	0	0	1	560	
Key columns									
$0S_1$	0	-1/3	2/3	1	0	-1	0	266	2nd Interation
OS_2	0	-1	2	0	1	-3/2	0	14	
$1400x_1$	1	2/3	2/3	0	0	½	0	182	
OS_4	0	5/3	2/3	0	0	-1	1	196	
P_j	1400	280/3	2800/3	0	0	700	0	254800	
C_j-P	0	500/3	-1600/3	0	0	-700	0	-	
OS_1	0	-1/3	2/3	1	0	-1	0	266	
OS_2	0	-1	2	0	1	-3/2	0	14	
$1400X_1$	1	2/3	2/3	0	0	½	0	182	
$1100x_2$	0	1	2/5	0	0	½ -3/5	3/5	588/5	
Key columns									
OS_1	0	0	4/5	1	0	-6/5	-1/5	1526/5	3rd Interation
OS_2	0	0	12/5	0	1	-21/10	3/5	658/5	
$1400X_1$	1	0	2/5	0	0	1/5	-2/5	518/5	
$1100x_2$	0	1	2/5	0	0	-3/5	3/5	588/5	
P_j	0	1100	440	0	0	-660	660	129360	
C_j-Pmax	1400	0	-40	0	0	660	-660	-	
OS_1	0	0	4/5	1	0	-6/7	1/5	1526/5	
$400X_3$	0	0	½	0	5/12	-7/8	1/4	55	
$1400X_1$	1	0	2/5	0	0	1/5	-1/10	518/5	
$1100x_2$	0	1	2/5	0	0	-3/5	3/5	588/5	
OS_1	0	0	0	1	-1/3	½	0	261	4th Interation
$400X_3$	0	0	1	0	5/12	-7/8	1/4	55	
$1400X_1$	1	0	0	0	-1/6	11/20	0	82	
$1100x_2$	0	1	0	0	-1/6	-1/4	2/5	96	
Pmax	-	-	-	-	-	-	-	2441500	

*Highest C_i-P_j

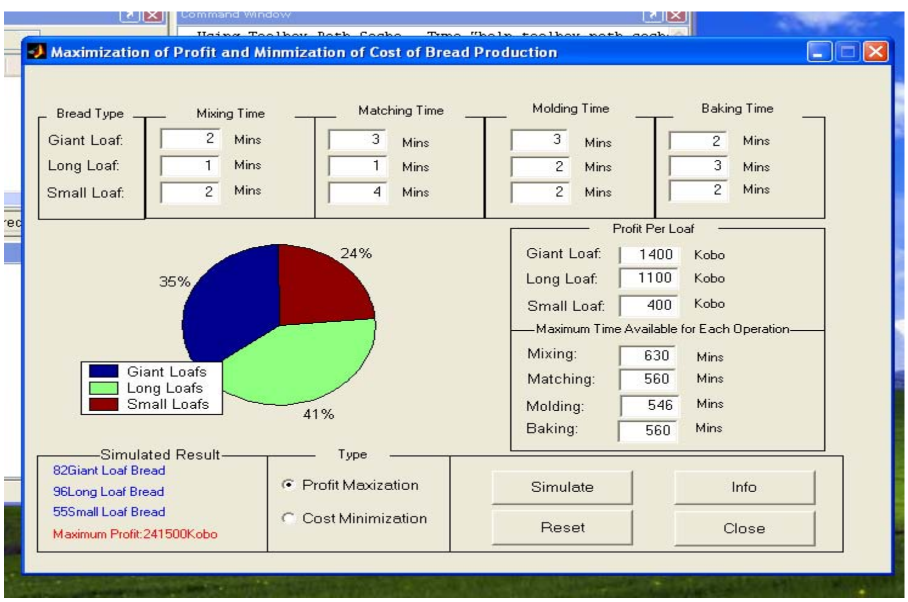

Fig. 3: Results of the simulator for Premier bread industry

Table 4: Primal result of iteration techniques for Stephens bread industry

C_j	1500 X_1	1200 X_2	500 X_3	$0S_1$	$0S_2$	$0S_3$	$0S_4$	Values	Iteration no.
OS_1	2	1	2	1	0	0	0	490	1st Iteration
OS_2	1	1	1	0	1	0	0	300	
OS_3	3	4	2	0	0	1	0	725	
OS_4	4	3	2	0	0	0	1	800	
P_j	4	0	0	0	0	0	0	-	
C_j-P_j	1500*	1200	500	0	0	0	0	-	
↑Highest C_j-Pj									
OS_1	2	1	2	1	0	0	0	490	
OS_2	1	1	1	0	0	0	0	300	
OS_3	3	4	2	0	1	0	0	725	
1500x_1	1	3/4	½	0	0	1/4	1/4	200	
↑Highest C_j-Pj									
OS_1	0	-1/2	1	1	0	0	-1/2	90	2nd Iteration
OS_2	0	1/4	½	0	1	0	-1/4	100	
OS_3	0	7/4	½	0	0	1	-3/4	125	
1500x_1	1	3/4	½	0	0	0	1/4	200	
P_j	1500	1125	750	0	0	0	375	30000	
C_j-P_j	0	75*	-250	0	0	0	-375	0	
OS_1	0	-1/2	1	1	0	0	-1/2	90	
OS_2	0	1/4	½	0	1	0	-1/4	100	
1200x_2	1	1	2/7	0	0	4/7	-3/7	500/7	
1500x_1	0	3/4	½	0	0	0	-1/4	200	
OS_1	0	0	8/7	1	0	2/7	-5/7	880/7	3rd Iteration
OS_2	0	0	3/7	0	1	-1/7	-1/7	575/7	
1200x_2	0	1	2/7	0	0	4/7	-3/7	500/7	
1500x_1	1	0	2/7	0	0	-3/7	4/7	1025/7	
P_j	0	1200	2400/7	0	0	4800/7	-3600/7	6000000/7	
C_j-P_j	1500	0	1100/7	0	0	4800/7	3600/7	-	
500x_3	0	0	1	7/8	0	1/4	-5/8	110	
OS_2	0	0	3/7	0	1	-1/7	-1/7	575/7	
1200x_2	0	1	2/7	0	0	4/7	-3/7	500/7	
1500x_1	1	0	2/7	0	0	-3/7	4/7	1025/5	
500x_3	0	0	1	7/8	0	1/5	-5/8	110	4th Iteration
OS_2	0	0	0	-3/8	1	-1/4	1/8	35	
1200x_2	0	1	0	-1/4	0	½	-1/4	40	
1500x_1	1	0	0	-1/4	0	-1/2	3/4	115	
P_j	-	-	-	-	-	-	-	275500	

*Highest C_i-P_j

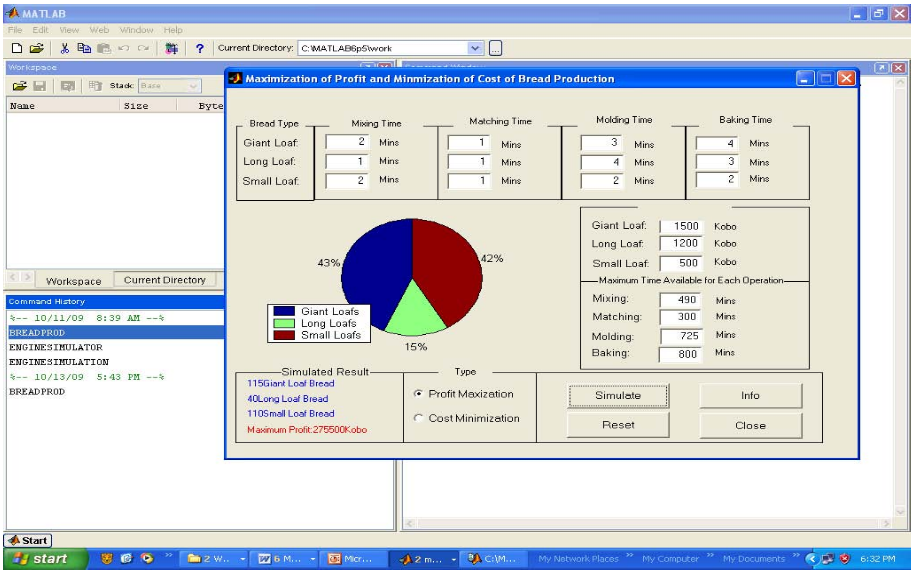

Fig. 4: Results of the simulator for Stephens bread industry

bread of which their present production capacity could not meet. Hence, there is obvious need to increase the quantity of bread produced within the production confine. Applying a linear programming as a required model for simulation to ascertain an acceptable optimization yielded an optimal at the 4th iteration as shown in Table 4 for which there is 115 giant loaves per batch, 40 long loaves per batch and 110 small loaves per batch. The total profit associated with this production mix per batch is 275500 kobo (₦2755.00). The value obtained via this model which represents the number of bread produced per size of loaf was validated using the simulator as shown in Fig. 4.

Simulation and validation: The simulator equally returned production mix having 115 giant loaves, 40 long loaves and 110 small loaves. The giant loaf, long loaf and small loaf are 43, 42 and 15% of the total production, respectively with an associated total profit of ₦2755.00 per production batch. This simulated result agrees with Simplex method result obtained for Stephens bread industry. It was also observed from available demand data that there is high demand for all their products of which this present production capacity could not meet. Hence, there is a persistent need to increase the quantity of bread produced which could also be adjusted when demand fluctuates so as to satisfy customers and maintain steady production.

Model solution and analysis: Observation shows that both bread industries has not been able to meet the demand for bread in their area, hence this study seeks the determination of optimal production mix that can meet the ever increasing demand for the products of Stephens and Premier bread in particular and all other similar industry in general. After gathering necessary information, it is therefore estimated that since these bread industries follow similar operational procedure as in Fig. 2 and bakes three sizes of bread loaves, increasing demand and stiff competition requires that 900 min be used for mixing, 800 min for matching, 780 min for molding and 800 min for baking for each production batch. It also provide that each giant loaf takes 3 min of mixing labour, 4 min of matching labour, 2 min of molding labour and 2 min of baking labour; each long loaf takes 2 min of mixing labour, 2 min of matching labour, 1 min of molding labour and 3 min of baking labour; each small loaf takes 1 min of mixing labour, 3 min of matching labour, 3 min of molding labour and 1 min of baking labour. The profit made by the two bread industries was compared and a profit of 1500 kobo for giant loaf, 1200 kobo for long loaf and 500 kobo for small loaf were used. Table 5 shows the problem information. The problem in Table 5 was further shown

Table 5: Data set for optimum process time

	Process time (min) per loaf size				
Size of loaves	(T_1) mixing	(T_2) matching	(T_3) molding	(T_4) baking	Profit (P) per loaf (kobo)
Giant loaf (x_1)	3	4	2	2	1500
Long loaf (x_2)	2	2	1	3	1200
Small loaf (x_3)	1	3	3	1	500
Total available time (min batch^{-1})	900	800	780	800	

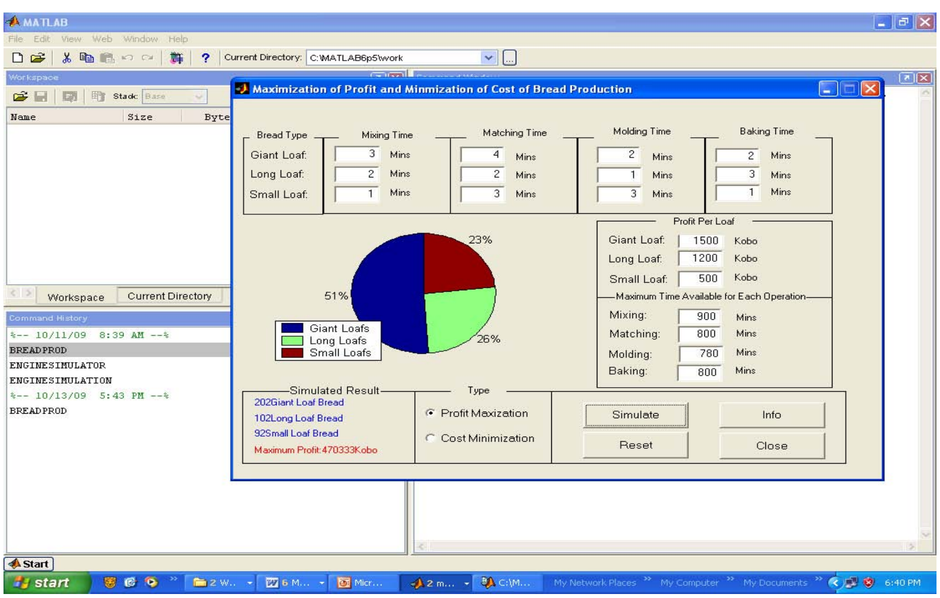

Fig. 5: Results of the simulator for recommended optimum process time

mathematically as in Eq. 1-4 so that an algebraic solution can be achieved, hence the optimum solution of the model as shown in Table 6, gave 202 giant loaf size, 102 long loaf and 92 small loaf for the optimum process time production batch which also corresponds with the result of the designed BREADPROD simulator as shown in Fig. 5. The result represents 51, 26 and 23% of the total production, respectively and gave a maximum profit of 470333 kobo (₦4703.33). The production mix provides 900 min for mixing, 800 min for matching, 780 min for molding and 800 min for baking. It therefore, becomes imperative that for the bakeries to achieve this proposed production mix there is a need for them to plan for increase in capacity. It is expected that two or three vats will be used for mixing, two or three milling machines for matching and enough hands for molding.

Table 6: Linear programming output summary

Variables	Value	Obj. coeff.	Obj. val. contrib.
x1: Giant loaf	100.00	1500.00	150000.00
x2: Long loaf	200.00	1200.00	240000.00
x3: Small loaf	0.00	500.00	0.00
Constraint	RHS	Slack/surplus+	
1 (<)	900.00	200.00-	
2 (<)	800.00	0.00	
3 (<)	780.00	380.00-	
4 (<)	800.00	0.00	

Sensitive analysis

Variables	Current obj. coeff.	Min obj. coeff.	Max obj. coeff.	Reduced cost
x1: Giant loaf	1500.00	914.29	2400.00	0.00
x2: Long loaf	1200.00	750.00	2250.00	0.00
x3: Small loaf	500.00	-infinity	1012.50	512.50
Constraint	Current RJS	Min RHS	Max RHS	Dual price
1 (<)	900.00	700.00	Infinity	0.00
2 (<)	800.00	533.33	1120.00	262.50
3 (<)	780.00	400.00	Infinity	0.00
4 (<)	800.00	400.00	1200.00	225.00

Model validity and checks: According to Taha (2004), the results are valid if under similar input conditions, the model reproduces past performance.

However, there is no assurance that future performance will continue to duplicate past behaviour, thus in this study, the proposed model appear to be representing a new (nonexisting) system, such that no historical data was available to make the comparism, hence we resorted to the use of simulation as an independent tool for verifying the output of the mathematical model; on this justification therefore the results of the model was ascertained to predict adequately the behaviour of the production processes of industries under study.

The linear programming model adopted gave 202 giant loaf sizes, 102 long loaves and 92 small loaves for the optimum process time per production batch which also corresponds with the result of BREADPROD simulator and thus makes the results intuitively acceptable. Table 6 provides the optimal solution of the model, it recommends production of more giant and long loaf, small loaf is less attractive, not only because it have the smallest objective coefficient (= 500.00) but also because its reduced cost is highest among all the variables (512.00).

The present reduced cost provides the per unit excess cost of consumed resources over the per unit return of small loaf, thus for it to be just profitable, production engineer must either reduce the per unit cost of the resources or increase the per unit return by an amount equal to the reduced cost. This means that the profitability of the small loaf x_3, must be increased by 512.50 kobo for its production to be profitable.

Economic interpretation of production mix model: The LP model hows a resource allocation model in which the objective is to maximize profit associated with small, long and giant loafs, subject to limited time of mixing, matching, molding and backing. Taha (2004) shows duality as an LP defined directly and systematically from the original LP model otherwise known as primal model; duality is therefore utilized in this research as a tool to obtain economic interpretations of the LP resource allocation model. Taha (2004) provides a general presentation of primal and dual problem in which the primal takes the role of a resource allocation model:

Primal	Dual
$\text{Minimize } w = \sum_{i=1}^{m} b_j y_i$	$\text{Maximize } p = \sum_{j=1}^{n} e_j x_i$
Subject to:	Subject to:
$\sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1, 2, ..., m$	$\sum_{i=1}^{n} a_{ij} y_j \ge c_i, i = 1, 2, ..., n$
$yi \ge 0, i = 1, 2, ..., m$	$xj \ge 0, j = 1, 2, ..., n$

The LP model considered has n = 3 economic activities and m = 4 resources. The coefficient c_j in the primal represents the profit per unit of activity j, resource i whose maximum availability is b_i is consumed at the rate a_{ij} units per unit of activity j, y_i represents the worth per unit of resource i.

The dual prize yields interesting information as shown in Table 6, the zero dual prize means that an increase in minimum requirement in the corresponding activity will not affect the total number of loaf per batch also dual prize of 225 means that a unit increase in the minimum number of allocation in a given batch will increase the total profit per loafs per batch by 225 kobo.

However, these changes are limited by the ranges shown in Table 6. For example, the minimum requirement for baking (constraint 4) can further be increased from 800-1200 without requiring an increase in the total number of loafs per batch. Constraint 1 and 3 have positive slack values which indicates that their resources are abundant as a result, their dual prize (worth per loaf) are zero. Constraint 2 representing matching time has a dual value of 262.50 kobo, indicating that a 1-loaf increase in the production mix is worth 262.50 kobo in net revenue. This information could be available in deciding on the purchase price of each loaf. Since, LP model represents a snapshot of real situation in which the model parameters (objective and constraint coefficients) assume static values. A well-defined optimization algorithm was employed to solve the model; additional information was sourced from selected industries about the behaviour of the optimum solution when the model undergoes some parameter changes, this was necessary to take care of inconsistencies in accuracy of model estimation. We therefore investigate the changes in the optimal solution resulting from making changes in parameter of the LP model.

Taha (2004) outlined all possible cases that can arise in sensitivity analysis together with the actions needed to obtain the new solution:

Condition resulting from the changes	Recommended actions
Current solution remains optimal and feasible	No further action is necessary
Current solution becomes feasible	Use dual simplex to recover feasibility
Current solution becomes non optimal	Use primal simplex to recover optimality
Current solution becomes both non-optimal and infeasible	Use the generalized simplex method to obtain a new solution

Availability of raw materials for bread production has a very important role to play in the functionality of the developed LP production mix model hence we attempt to determine the range for which the current solution remains feasible. A subtle assumption made in this formulation is that the industries will continue baking three sizes of loafs considered as it meets the need of the moment. So, we now consider only effects of changing the availability of resources (i.e., the-right hand-side vector). Table 6 shows that the current basic solution remains feasible for ($700 \leq T_1 \geq \infty$), ($533.33 \leq T_2 \geq 1120$), ($400 \leq T_3 \geq \infty$), ($400 \leq T_4 \geq 1200$).

RESULTS AND DISCUSSION

A summary of the results as regards the production mix and optimum profit for Premier bread industry, Stephens bread industry and recommended optimum model is shown in Table 7. The analysis of the data above shows that the production mix of Premier bread industry is 35% giant loaves, 41% long loaves and 24% small loaves. This production mix yielded a total profit of ₦2,415.00.

Table 7: Summary of optimal solutions

Facotors	Loaves	Percentage	Profit
Premier			
Giant	82	35	₦2,415.00
Long	96	41	
Small	55	24	
Stephens			
Giant	115	43	₦2,755.00
Long	40	15	
Small	110	42	
Optimum			
Giant	202	51	₦4,703.33
Long	102	26	
Small	92	23	

In the same vein, the production mix of Stephens bread industry is 43% giant loaf, 15% long loaf and 42% small loaf giving a total profit of ₦2755.00 kobo. The profits made by the two bakeries are very close although Stephens bakery operates at a high profit margin. Both bakeries produced more giant loaves since, they make higher profit from it. Based on the data for both factories, a best production mix was proposed and confirmed with the simulator.

Production mix of 51% giant loaf, 26% long loaf and 23% small loaf. It also gave a total profit of ₦4703.33 kobo which is almost double of the profit made by the studied bakeries. The two bread industry studied sale at the same price yet with different profits margin. Therefore, the proposed production mix is a good attempt to optimize the available capacities in the bakeries and increase productivity.

It equally positions the bakeries to meet with their daily order from their various customers. The bar chart of Fig. 6 shows graphical representation of the number of loaves produced per sizes of bread for the two studied bread industries and the optimum recommended production mix.

However, it will be observed from Fig. 6 that the optimum production mix gave a higher output as regards the number of breads produced per sizes of loaves which clearly shows an increase in productivity as compared with the two studied bread industries. It is clear that the optimum production mix yielded the highest profit of ₦4,703.33 as against ₦2,415.00 for Premier bread industry and ₦2,755.00 for Stephens bread industry which is almost double of the profit made by the two studied bread

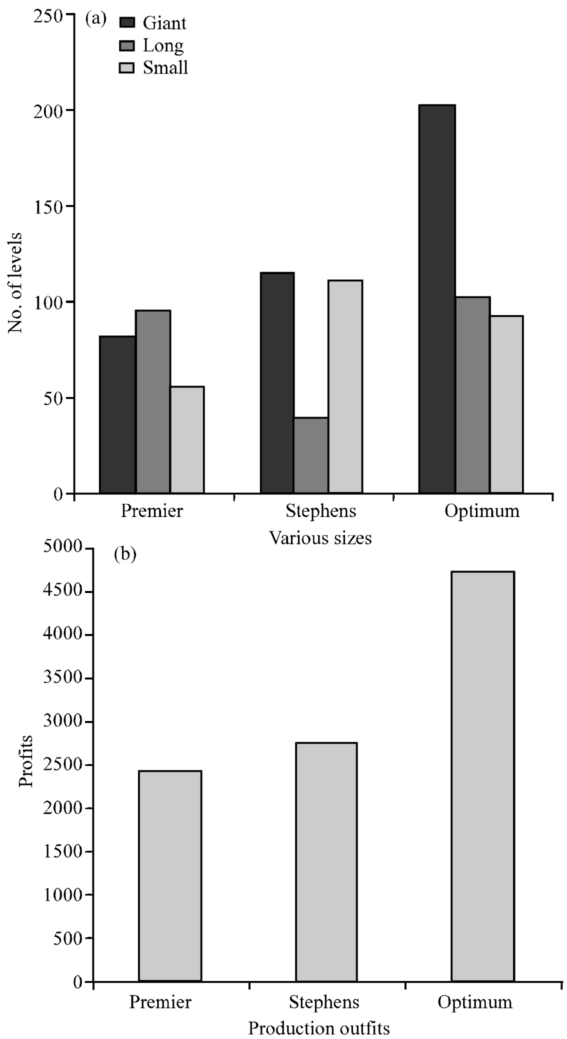

Fig. 6: Graphical representation of the results of simulation and optimization

industries. Thus, the proposed production mix is a good attempt to optimize production processes in bread industries for increase in productivity using linear programming as a model.

CONCLUSION

In this study, it was desired to develop a production mix which will give maximum profit and an attempt was made to analyze various process time and profit margin for two bread industries in Eastern Nigeria namely, Premier bread industry and Stephens bread industry. Linear programming was employed in determining production mix and associated total profit for the two bread industries. A simulator designed in MATLAB™ GUIDE WINDOW was used to validate the production mix and accrued total profit for the two bread industries. The study further gave a production mix of 51% giant loaf, 26% long loaf and 23% small loaf with a profit margin of over 100%. The analysis of LP model and the simulation which operates on GUIDE window of MATLAB and runs on run on either cost minimization or profit maximization will help any bread industry to estimate the quantity of bread to be produced in order to maximize profit and minimize production cost. The simulator can equally be used by industries to know their production standard and consequently improve to the optimum within their constraint and capacity. The simulator can also be adopted by service providers who handle more than one type of service or operate multichannel or different outlets method.

REFERENCES

Azadivar, F. and Y.H. Lee, 1988. Optimization of discrete variable stochastic systems by computer simulation. Mathe. Comput. Simulation, 30: 331-345.

Bender, E., 2000. An Introduction to Mathematical Modeling. Dover Publications, New York.

Carson, Y. and A. Maria, 1997. Simulation optimization: Methods and applications. Proceedings of the 1997 Winter Simulation Conference, Dec. 7-10, Atlanta, GA, USA., pp: 118-126.

Fu, M. and K. Healy. 1992. Simulation optimization of (SS) inventory systems. Proceedings of the 24th Conference on Winter Simulation, Arlington, VA, USA., Dec. 13-16, ACM Press, pp: 506-514.

Hall, J.D., R.O. Bowden and J.M. Usher, 1996. Using evolution strategies and simulation to optimize a pull production system. J. Mater. Process. Technol., 61: 47-52.

Lutz, C.M., 1995. Determination of buffer size and location in scheduling systems buffer management, inventory profiles. Ph.D. Thises, University of Georgia.

Mustapha, S., 1998. Wheat production in Nigeria: Past, present and future prospect. Proceedings of the National Wheat Production Workshop, Feb. 17-18, Nigeria-Prospects and Constraints, pp: 14-24.

Taha, H., 2004. Operation Research: An Introduction. Prentice-Hall of India Pvt. Ltd., New Delhi.