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Pressuremeter Identification Procedure Based on Generalised Prager Model

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Abstract: A numerical analysis of soil behaviour around pressuremeter probe, based on the generalised Prager's model associated to the Drucker and Prager Criterion is presented. First of all, the soil behaviour law is described, based on the analytical representation of the stress-strain curves obtained with triaxial tests. This show, how the parameters of the generalised Prager model with a large number of elastoplastic slip elements associated in series can be identified. Secondly, the effect of these parameters on the simulated curve and the principal stresses path around the probe are analysed.

Key words: Pressuremeter probe, elastoplastic-criterion, kinematic hardening, simplex algorithm, Algeria

INTRODUCTION

The pressuremeter is becoming largely used in geotechnical engineering investigation as a measurement apparatus to assess the engineering properties of soils. Both numerical methods and analytical solutions based on different soil models have been proposed to get shear strength characteristics and stress-strain properties of soil from the analysis of the cylindrical cavity expansion induced during the test. The pressuremeter test simulates the expansion of cylindrical cavity. Due to its well defined boundary condition, allows more rigorous theoretical analysis (i.e., cavity expansion theory) than other *in situ* tests. In the present study, a numerical analysis of a pressuremeter test is proposed. The generalised Prager model associated to Drucker and Prager criterion to describe the behaviour of cohesive soil is presented. The model is then introduced in a computer code to identify the model parameters from pressuremeter results. Some examples identification, parameters effect on the simulated curve and the stress path around pressuremeter probe are also presented.

Proposed model: In the numerical theory of plasticity, the Prager model describes the yielding behaviour of a material by a yield surface which gives the state of stress under which yielding first occurs, a work hardening rule which specifies how the yield surface is changed during plastic flow and a flow rule which relates the plastic strain increment to the state of stress and the stress increment. Iwan (1967) proposed a generalisation of collection of yield surfaces in place of Prager's single model surface.

The model is constituted by a chain of nelastoplastic Prager elements connected in series. A kinematic hardening model is supposed. This assumes that during the plastic deformation, the loading surface translates as a rigid body in the stress space keeping the size, shape and orientation of the initial yield surface. The kinematic hardening has the following form for hardening surface:

$$f\left(\sigma_{ij},\varepsilon_{ij}^{p}\right) = F\left(\sigma_{ij} - X_{ij}\right) - k^2 = 0 \quad (1)$$

where, k is a constant and X_{ij} are the coordinates of the center of loading surface which change as the plastic strain continues. The simplest version for determining parameter X_{ij} is to assume a linear dependence of dX_{ij}:

$$dX_{ij} = C d\varepsilon_{ij}^{p} \quad (2)$$

where, C is the work hardening constant, characteristic for given material. Equation 2 may be taken as the definition of the linear work hardening. The plastics train is given by the normality rule (standard material):

$$\left(d\varepsilon_{ij}^{p}\right)^{k} = \left(\frac{\partial f}{\partial \sigma_{ij}}\right) d\lambda_k \quad (3)$$

$$\left(d\varepsilon_{ij}^{p}\right)^{k} = 0 \qquad \text{if} \qquad d\sigma_{ij} n_{ij} \leq 0 \quad (4)$$

The consistency condition means that the yield surface keeps the same radius, this help to determine the plastic proportionality factor $d\lambda$ with:

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$$d\lambda_k = \frac{1}{C_k} \frac{\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij}}{\left(\frac{\partial f}{\partial \sigma_{kl}}\right)\left(\frac{\partial f}{\partial \sigma_{kl}}\right)} > 0 \quad (5)$$

Where:

k = Used as a subscript denotes the kth element of chain

C_k = The modulus of the kth element of the chain

n_{ij}^k = The normal external vector to the yield surface

The total plastic strain is the sum of the plastic strain of each cell. Some of them may not be strained during loading. In this case, it can be said that they are not activated. The number of activated cells is n^* with $0<n^*<n$. The model is thus defined by the compliance of n elastic elements and their associated n yield surfaces. Eventually, a single linear elastic element can be introduced to represent the initial elastic strain. This, introduce a further compliance noted J_0 and a limit surface of threshold stress S_∞. Thus the model is defined by 2n+2 parameters which can be represented by a discrete spectrum of compliance as shown on Fig. 1. More ever, the components of the tensors X_{ij} which specify the state of the model must be given. In the virgin state, all the residual stresses are equal to zero, hence for each cell, the hardening variables are also equal to zero, X_{ij} (k) = 0. Since each of the n Prager elements will individually obey a linear work hardening law, their combined action leads to a piecewise linear behaviour with a kinematic hardening for the material as a whole.

Modelling of frictional-cohesive soil behaviour assumptions: The material is supposed to be isotropic and normally consolidated or with low value of the Overconsolidation Ratio (OCR). Viscous effects are also supposed to be negligible which justify the use of an elastoplastic model as a first approximation.

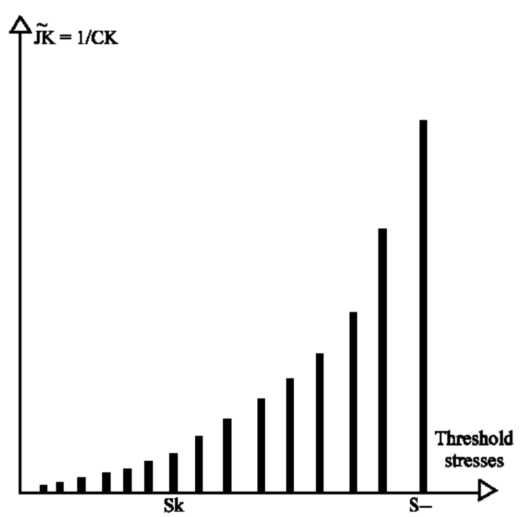

Fig. 1: Compliance spectrum of the model

Yielding criterion: Drucker and Prager criterion is used. This criterion is a generalisation of Von Mises criterion for the frictional-cohesive material (Drucker and Prager, 1952):

$$f = \sqrt{J_2} + \alpha I = K \quad (6)$$

Where the constants α and K, may be related to the coulomb's material constants, C and φ in several ways.

Drucker-Prager material constants: Since the Drucker-Prager criterion has been established as an approximation of the Coulomb criterion, it is natural to determine the material constants α and K in the Drucker-Prager criterion by matching two particular points with those of the Coulomb criterion and thus expressing the two parameter α and K in terms of the given Coulomb constants C and φ. In the three dimensional principal stress, the Drucker-Prager criterion can be matched with the apex of the Coulomb criterion for either point A or B on its π-plane as shown in Fig. 2. In the former case, the cone circumscribes the hexagonal pyramid. Since a line element (compressive meridian) connecting the apex O with the point A contains the same line for both criteria, The relations between α, K and C, φ can be found (Chen and Mizuno, 1990):

$$\alpha = \frac{2\sin\varphi}{\sqrt{3}(3-\sin\varphi)} \quad (7)$$

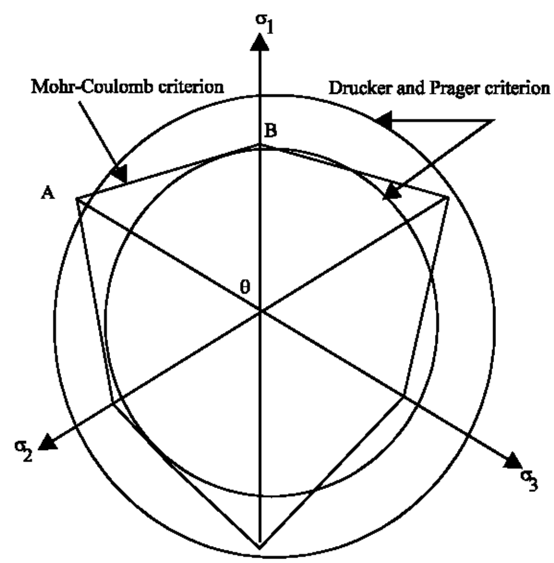

Fig. 2: Shape of yield criteria on the π-plane

$$K = \frac{6 \times C \cos\varphi}{\sqrt{3}(3 - \sin\varphi)} \quad (8)$$

For a tensile meridian:

$$\alpha = \frac{2 \sin\varphi}{\sqrt{3}(3 + \sin\varphi)} \quad (9)$$

$$k = \frac{6 \times C \cos\varphi}{\sqrt{3}(3 + \sin\varphi)} \quad (10)$$

Spectrum compliance: The response of this model on a triaxial path is a polygonal line which can be considered as a discretisation of the experimental curve. Then, if an analytical representation of the experimental triaxial test results is chosen, the parameters of the Prager model can be identified easily. The following relationship proposed by Olivari and Bahar (1993) is used:

$$\varepsilon_d^p = -A\left[\ln(1 - R) + (1 - 2R)\frac{R}{1 - R}\right] \quad (11)$$

$$R = \frac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)_f} \quad (12)$$

Where σ_1 and σ_3 are the principal total stresses, $(\sigma_1\text{-}\sigma_3)_f$ is the asymptotic value of stress difference which is related closely to the strength of the soil and A is a positive parameter defining the curvature. Consequently, the model is totally defined with five parameters; Young's modulus E, Poisson's ratio ν, curvature parameter A, cohesion C and the friction angle φ.

Response on pressuremeter path: The expansion of cylindrical cavity (pressuremeter probe) is assumed to occur in plane strain ($\epsilon_z = 0$), when the cavity is subjected to an internal pressure, a small strain formulation and soil incompressibility hypothesis are assumed. The symmetry of revolution imposes that the stress increments in the directions of r, θ and z are principal (Fig. 3). The boundary conditions can be specified either in displacements or stresses:

- At the wall of the cavity:

$$\Delta U_r = \Delta U_0 \text{ and } \Delta\sigma_r = \Delta\sigma_0 \quad (13a)$$

- At an infinite distance away:

$$\Delta U_r = 0 \text{ and } \Delta\sigma_r = 0 \quad (13b)$$

It can be noted that two concentric annular zones appear around pressuremeter probe:

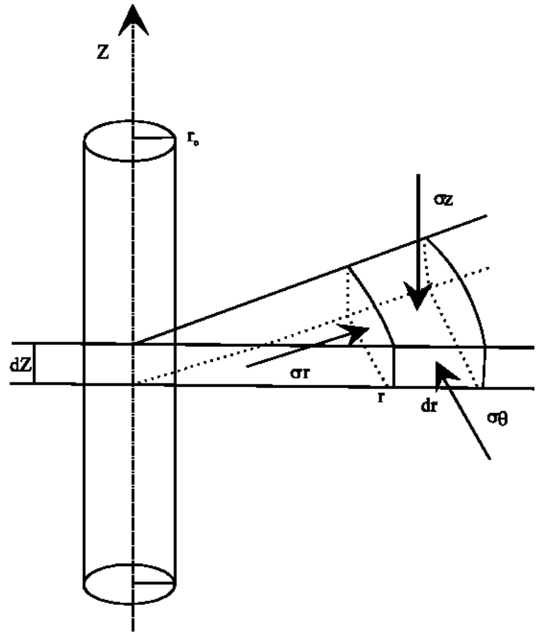

Fig. 3: Equilibrium of a soil element

- In the first zone ($r_0<r<r_e$), the soil undergo an elasto-plastic strains
- In the outside zone ($r>r_e$), the soil behaves elastically

Solution of the problem in elastic zone: Taking into account the radius r_e of the outside elastic zone with a Poisson ratio ν = 0.5, what can be obtained is as follows:

$$\begin{aligned} d\varepsilon_r &= \frac{r_e}{r'}dU_e & d\varepsilon_\theta &= -\frac{r_e}{r'}dU_e & d\varepsilon_z &= 0 \\ d\sigma_r &= \frac{2}{3}E\frac{r_e}{r'}dU_e & d\sigma_\theta &= -\frac{2}{3}E\frac{r_e}{r'}dU_e & d\sigma_z &= 0 \end{aligned} \quad (14)$$

The deformation happens at constant stress without volume changes.

Solution of the problem in elasto-plastic zone: The behaviour law can be written by:

$$\Delta\varepsilon_{ij} = A_{ijkl} \times \Delta\sigma_{kl} \quad (15)$$

Where A_{ijkl} describes the soil behaviour for a loading increment. This matrix is function of the stress state and the loading path. It is written as follows:

$$A_{ij} = -\frac{\nu}{E} + \sum_{k=1}^{n^*}\frac{J_k}{S_k^2}\left(\alpha + \frac{1}{4J_2}\right)(S_i - X_i(k))(S_j - X_j(k)) \quad (16)$$

Numerical code: From the analysis of the pressuremeter test results, a computer code baptised PRESSIO-IDENT is developed in order to identify automatically the parameters of the proposed behaviour law. The simplex algorithm is used to evaluate the objective function (in this case, minimising the area between the experimental curve and the simulated curve). The proposed code can be exploited in two ways:

Direct use: Knowing the model parameters, the code allows the determination of the simulated curve.

Indirect use: From the experimental curve, the code allows the identification of the model parameters.

Analysis of the parameters effect on simulated curve: The procedure followed in order to show the effect of the model parameterson the simulated curve was by considering the sensitivity of every parameter. For this, the use of the numerical code (computation of the pressuremeter curve knowing the parameters of the model) allows to get a simulated pressuremeter curve. To carry out this analysis, the experimental pressure meter curve carried out on the doughy chalk of Nogent (France) at the depth of 12.50 m has been used.

The parameters for this analysis are shown in Table 1. Every parameter has been varied by ±50% of its initial value keeping constant the three others. Table 2 shows the obtained results as well as the area corresponding to the variation of every parameter. This area is delimited by the curve deduced from initial parameters and the curve deduced from the varied parameters.

Young modulus effect: The strain parameter E (Young modulus) has an influence mainly in the first part of the curve. Nevertheless, what can be seen is that the great strains part of the simulated curve are slightly influenced by this parameter (Fig. 4).

Curvature parameter effect: The parameter which describes the curvature of the simulated curve has almost no influence on the curve particularly on its starting part (Fig. 5).

Table 1: Parameters of the doughy chalk of Nogent (France) at the depth of 12.50 m

P_0 (kPa)	V_0 (cm^3)	E (kPa)	A	C (kPa)	φ (°)
104	84	12000	0,030	150	20

Table 2: Effect of the model parameters on the simulated curve

	Variation of the generated area surface (%)			
Parameter variation	E	A	C	φ
- 50%	-33.70	-2.80	-37.88	-31.21
+ 50%	+12.43	+3.90	+20.79	+20.75

Failure parameters effect: The two failure parameters, the cohesion (C) and the friction angle (ö) representing the behaviour in great strain have a negligible influence on the initial part of curve. However, this influences seem to be more pronounced within the last part of the pressure meter curve where the failure occurs (Fig. 6 and 7). This analysis shows that none of the three parameters of the model can be set with an average value; the three parameters have to be determined by the proposed optimization method.

Initial values of parameters to identify: The knowledge of the experimental curve allows evaluating the model approached parameters using the classical methods

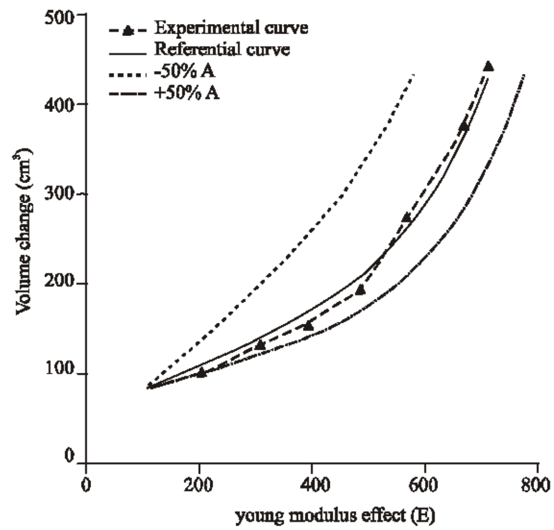

Fig. 4: Young modulus effect (E)

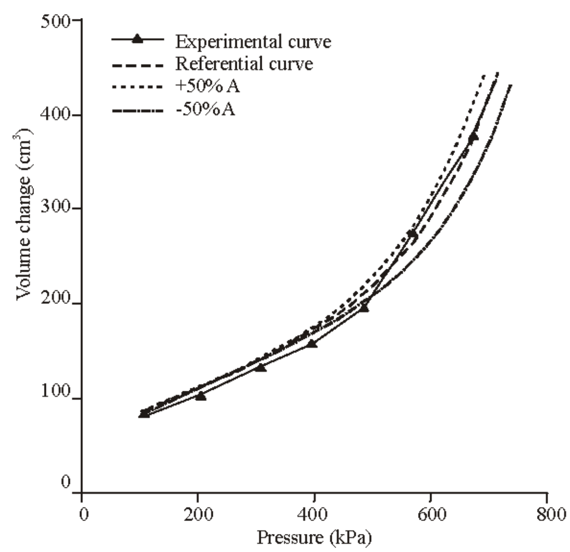

Fig. 5: Curvature parameter effect (A)

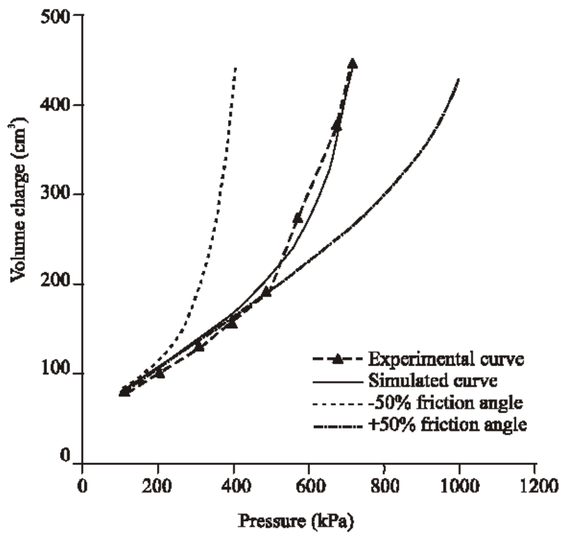

Fig. 6: Friction angle effect

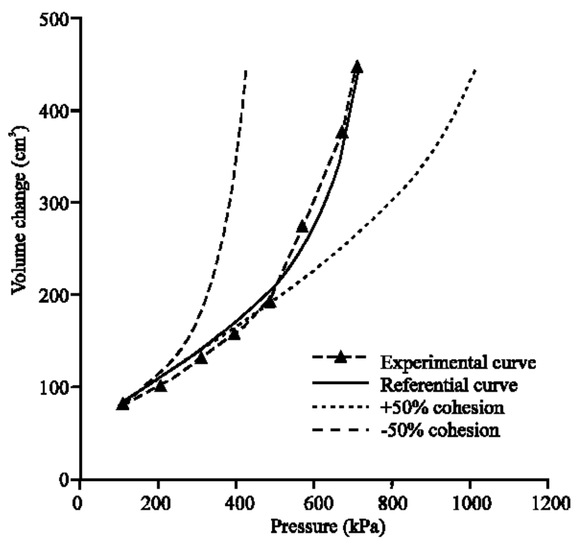

Fig. 7: Cohesion effect

(Salencon, 1966). These parameters allow starting the iteration process which will lead to the finale parameters. Figure 8 shows an identification example of the model parameters. The simulated curve deduced from the injected initial parametersis very close to this final one. This shows the importance of the initial solution in the proposed process of identification.

Stresses path around pressuremeter: The proposed method allows visualising stresses path in all the point of a chosen discretisation. Figure 9 shows the distribution of the main stresses at the end of loading for the pre-boring pressuremeter test undertaken on the Nogent doughy chalk in France (Pre Boring Pressuremeter probe; R = 8 cm

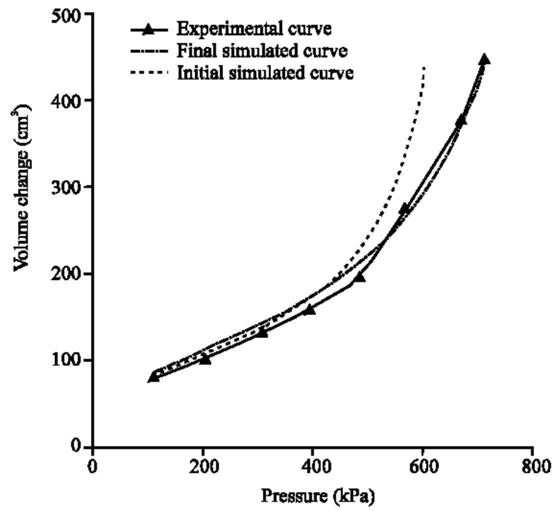

Fig. 8: Final simulated curve compared to the initila simulated curve

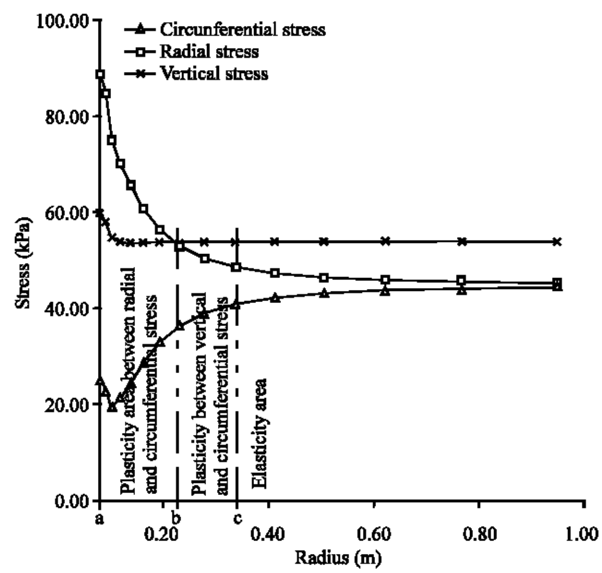

Fig. 9: Distribution of main stresses at end of loading

and L = 32 cm). Three different areas of soil from the borehole wall to the infinite radius are considered. Plasticity appears between the radial stress σ_r and circumferential stress σ_θ in the horizontal plane. This first plastic area extends between the radius a (borehole wall) and b (external radius of the plastic area).

As shown by Wood and Wroth (1977) and Monnet (2007), plasticity may also appear in the vertical plane between the vertical stress σ_z and circumferential stress σ_θ in an area between the radius b and radius c (external radius of both plastic areas). An elastic area extends beyond the radius b.

CONCLUSION

The proposed model derived from the Prager model, developed in its original version to describe the behaviour of clays is generalized to the coherent soil. Its generalisation to the coherent soil can be carried out by using the Drucker and Prager criteria. Because of the limited number of parameters introduced, this model can practically do well with a more or less random choice of other parameters intervening in other behaviour laws which the pressuremeter test cannot identify. The strain parameter E (Young modulus) has an influence within the first part of the curve (left small strain). Nevertheless, It can be noticed that the great strains part of the simulated curve are slightly influenced by this parameter. The parameter which describes the curvature of the simulated curve has a negligible influence on thecurve particularly on its first part. The two failure parameters have an important effect on the simulated curve in particular on the great strains part of curve. The offered model is currently in development status to be able to describe cycles of unloading re-loading, thing which is going to allow us derive the initial elastic modulus. The distribution of main stresses at the end of loading shows that the plasticity may occur between between the radial stress σ_r and the circumferential stress σ_θ and between the vertical stress σ_z and the circumferential stress σ_θ.

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