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Effects of Chemical Reaction on MHD Free Convective Flow of Viscous Fluid Through a Porous Medium Bounded by an Oscillating Porous Plate in Slip Flow Regime

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Abstract: Present study is concerned the effects of chemical reaction on free convective flow of viscous, incompressible fluid of small electrical conductivity through a porous medium bounded by an oscillating infinite porous plate in slip flow regime in presence of uniform magnetic field taking into account, the homogeneous chemical reaction of 1st order. The expressions for velocity, temperature, concentration, skin friction and rate of heat transfer are obtained. The effects of various different parameters are discussed graphically.

Key words: MHD, heat transfer, mass transfer, porous medium, chemical reaction parameter, homogeneous

INTRODUCTION

Many researches have been published on the theory of laminar boundary layer in unsteady flow. The study of oscillating flow is important in the paper industry and many other technological fields. Due to this reason, many researchers have paid their attention towards the oscillating flow of viscous incompressible fluid past an infinite plate. Chemical reactions can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single-phase volume reaction. A reaction is said to be of 1st order if the rate of reaction is directly proportional to the concentration itself. In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications, e.g., polymer production, manufacturing of ceramics or glassware and food processing.

Stokes (1901) and Rayleigh (1911) studied the flow of a viscous and incompressible fluid about an infinite flat wall which execute linear harmonic oscillation parallel to itself. Stuart investigated the response of skin friction and temperature of an infinite plate thermometer to fluctuate in the stream with suction at the plate. Ong and Nicholls extended the method to obtained the flow in magnetic field nears an infinite flat wall which oscillate in its own plane. Ahmadi and Manvi (1971) have derived a general equation of motion for viscous flow through a rigid porous medium and applied to some basic flow problems. Yamamoto and Iwamura (1976) investigated the flow with convective acceleration through a porous medium. Gupta and Babu studied the flow of a viscous incompressible fluid through a porous medium near an oscillating infinite porous flat plate in the slip flow regime. Devangana studied MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating plate in the slip flow regime. Jha have studied MHD flow of viscous fluid past an impulsively moving isothermal vertical plate through porous medium with chemical reaction. Recently, Singh and Gupta studied MHD free convective flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer. Koriko (2010) studied approximate solutions of a higher order MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption which resulated from a quadratic reaction.

The aim of present investigation is to study the effect of chemical reaction on free convective flow of viscous, incompressible fluid of small electrical conductivity through a porous medium bounded by an oscillating infinite porous plate in slip flow regime in presence of uniform magnetic field taking into account the homogeneous chemical reaction of 1st order. The velocity, temperature, concentration, skin friction and rate of heat transfer for various parameters are discussed graphically (Koriko, 2010).

MATERIALS AND METHODS

Let us consider free convective flow of viscous, incompressible fluid of small electrical conductivity through a porous medium bounded by an oscillating infinite porous plate in slip flow regime in presence of

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uniform magnetic field taking into account the homogeneous chemical reaction of first order. The velocity components are u, ν in the direction of x, y-axis, respectively. A uniform magnetic field B_0 is acting along the y-axis. Under such conditions, the induced magnetic field due to the flows may be neglected with respect to applied field, the pressure in the fluid is assume constant if V_0 represent the velocity of suction or injection at the plate, the equation of continuity is:

$$\frac{\partial \nu}{\partial y} = 0 \tag{1}$$

With the condition, y = 0, ν leads to the result $\nu = -V_0$, every where. The boundary layer equations are:

$$\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) + g\beta'(C - C_\infty) - \sigma \frac{B_0^2}{\rho} u - \frac{\nu}{K} u \tag{2}$$

$$\frac{\partial T}{\partial t} - V_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial t} - V_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_1 C \tag{4}$$

Where:

ρ	=	The density
g	=	Acceleration due to gravity
β	=	Co-efficient of volume expansion
β'	=	Co-efficient of concentration expansion
ν	=	Kinematic viscosity
T_∞ and C_∞	=	The temperature and concentration of the fluid in the free stream
σ	=	Electric conductivity
B_0	=	Magnetic induction
K	=	Porosity parameter
α	=	Thermal diffusivity
D	=	The concentration diffusivity
K_1	=	The chemical reaction parameter

First order velocity slip boundary condition of the problem when the plate executes liner harmonic oscillation in its own plane are given by:

$$\left.\begin{aligned} &u = U_0 e^{int} + L_1 \frac{\partial u}{\partial y}, \quad T = T_\omega \quad C = C_\omega \quad \text{at} \quad y = 0 \\ &u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty \end{aligned}\right\} \tag{5}$$

Where:

$$L_1 = \frac{(2 - m_1)L}{m_1}$$

and:

$$L = \mu\left(\frac{\pi}{2p\rho}\right)^{\frac{1}{2}}$$

is the mean free path and m_1 is the Maxwell's reflection co-efficient. On introducing the following non-dimensional quantities:

$$y' = U_0 \frac{y}{\nu},\ u' = \frac{u}{U_0},\ \theta' = \frac{(T - T_\infty)}{(T_\omega - T_\infty)},\ \phi' = \frac{(C - C_\infty)}{(C_\omega - C_\infty)},$$

$$t' = U_0^2 \frac{t}{\nu},\ V_0' = \frac{V_0}{U_0},\ n' = \frac{\nu n}{U_0^2}, \qquad K' = \frac{KU_0^2}{\nu},$$

Chemical reaction parameter:

$$\gamma = \frac{K_1 \nu}{U_0^2}$$

Rarefaction parameter:

$$R = U_0 \frac{L_1}{\nu}$$

Schmidt number:

$$Sc = \frac{\nu}{D}$$

Hartmann number:

$$M = \frac{B_0}{U_0}\left(\frac{\nu\sigma}{\rho}\right)^{\frac{1}{2}}$$

Prandtle number:

$$Pr = \frac{\nu}{\alpha}$$

Grashof number for heat transfer:

$$Gr = g\nu\beta \frac{(T_\omega - T_\infty)}{U_0^3}$$

Grashof number for mass transfer:

$$Gc = g\nu\beta' \frac{(C_\omega - C_\infty)}{U_0^3}$$

Equations 2-4 after dropping the dashes can be written as:

$$\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + Gc\phi - Qu \tag{6}$$

$$\Pr\left(\frac{\partial\theta}{\partial t}-V_0\frac{\partial\theta}{\partial y}\right)=\frac{\partial^2\theta}{\partial y^2} \quad (7)$$

$$Sc\left(\frac{\partial\phi}{\partial t}-V_0\frac{\partial\phi}{\partial y}\right)=\frac{\partial^2\phi}{\partial y^2}-Sc\gamma \quad (8)$$

Where:

$$Q=M^2+\frac{1}{K}$$

With the boundary conditions are:

$$\left.\begin{array}{lllll} u=e^{int}+R\frac{\partial u}{\partial y}, & \theta=1, & \phi=1 & \text{at} & y=0 \\ u\to 0, & \theta\to 0, & \phi\to 0 & \text{as} & y\to\infty \end{array}\right\} \quad (9)$$

The solutions of Eq. 6-8 are:

$$u=u_0+u_1e^{int} \quad (10)$$

$$\theta=\theta_0+\theta_1e^{int} \quad (11)$$

$$\phi=\phi_0+\phi_1e^{int} \quad (12)$$

Using Eq. 6-8 in Eq. 10-12 and separating harmonic and non-harmonic terms, we gets:

$$u_0''+V_0u_0'-Qu_0=-Gr\theta_0-Gc\phi_0 \quad (13)$$

$$u_1''+V_0u_1'-(Q+in)u_1=-Gr\theta_1-Gc\phi_1 \quad (14)$$

$$\theta_0''+V_0\Pr\theta_0'=0 \quad (15)$$

$$\theta_1''+V_0\Pr\theta_1'-in\Pr\theta_1=0 \quad (16)$$

$$\phi_0''+V_0Sc\phi_0'-Sc\gamma\phi_0=0 \quad (17)$$

$$\phi_1''+V_0Sc\phi_1'-inSc\phi_1=0 \quad (18)$$

With the boundary conditions:

$$\left.\begin{array}{l} u_0=R\frac{\partial u_0}{\partial y}, u_1=1+\frac{\partial u_1}{\partial y}, \theta_0=1, \theta_1 \\ =0, \phi_0=1,\ \phi_1=0 \text{ at } y=0 \\ u_0\to 0,\quad u_1\to 0, \theta_0\to 0, \theta_1 \\ \to 0, \phi_0\to 0, \phi_1\to 0 \text{ as } y\to\infty \end{array}\right\} \quad (19)$$

The solutions of the Eq. 13-18 applying the boundary conditions (Eq. 19) are:

$$\begin{array}{l} u_0=(A_3Gr+A_4Gc)e^{-m_2y}- \\ A_1Gre^{-\Pr V_0 y}-A_2Gce^{-m_1y} \end{array} \quad (20)$$

$$u_1=(A_7-iA_8)e^{-A_5y}\left[\begin{array}{l} Cos(nt-A_6y)+ \\ iSin(nt-A_6y) \end{array}\right] \quad (21)$$

$$\theta_0(y)=e^{-\Pr V_0 y} \quad (22)$$

$$\theta_1(y)=0 \quad (23)$$

$$\phi_0(y)=e^{-m_1y} \quad (24)$$

$$\phi_1(y)=0 \quad (25)$$

Where:

$$m_1=\frac{V_0Sc+\sqrt{V_0^2Sc^2+4Sc\gamma}}{2},\ m_2=\frac{V_0+\sqrt{V_0^2+4Q}}{2},$$
$$m_3=A_5+iA_6$$

$$A_1=\frac{1}{\Pr^2V_0^2-\Pr V_0^2-Q},\ A_2=\frac{1}{m_1^2-V_0m_1-Q},$$
$$A_3=\frac{A_1(R\Pr V_0+1)}{(1+m_2R)},\quad A_4=\frac{A_2(Rm_1+1)}{(1+m_2R)},$$
$$A_5=\frac{V_0+\alpha_1}{2},\ A_6=\frac{\beta_1}{2},\ A_7=\frac{1+A_5}{(1+A_5)^2+A_6^2},$$
$$A_8=\frac{A_6}{(1+A_5)^2+A_6^2},\ A_9=A_6A_7-A_5A_8,$$
$$A_{10}=A_8A_6+A_5A_7$$

$$\alpha_1=\left[\frac{(V_0^2+4Q)+\sqrt{(V_0^2+4Q)^2+16n^2}}{2}\right]^{\frac{1}{2}}$$

$$\beta_1=\left[\frac{-(V_0^2+4Q)^2+\sqrt{(V_0^2+4Q)^2+16n^2}}{2}\right]^{\frac{1}{2}}$$

Hence, the expression for the transient velocity is given by:

$$u = u_0 + \begin{bmatrix} A_7\text{Cos}(nt - A_6 y) + \\ A_8\text{Sin}(nt - A_6 y) \end{bmatrix} e^{-A_5 y} \tag{26}$$

The expressions for the skin-friction and the rate of heat transfer are given by:

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} = \Pr V_0 \text{Gr} A_1 + m_1 A_2 \text{Gc} - m_2 \left(A_3 \text{Gr} + A_4 \text{Gc}\right) + A_9 \text{Sinnt} - A_{10} \text{Cosnt} \tag{27}$$

$$q_1 = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\Pr V_0 \tag{28}$$

$$q_2 = -\left(\frac{\partial \phi}{\partial y}\right)_{y=0} = -m_1 \tag{29}$$

The velocity distribution Eq. 26, the mass diffusion Eq. 24 and rate of mass transfer Eq. 29 can be adjusted to meet these circumstances if one takes:

- $\gamma > 0$ for the destructive reaction
- $\gamma = 0$ for no reaction
- $\gamma < 0$ for the generative reaction

RESULTS AND DISCUSSION

In Fig. 1, the velocity distribution of boundary layer flow is plotted against y for n = 2, Pr = 0.71, Sc = 0.6, R = 0.2, γ = 2 and nt = $\pi/2$ and different values of Grashoff number Gr, modified Grashoff number Gc, Hartmann number M and porosity parameter K. It is observed that the fluid velocity increases due to increasing Grashoff number Gr, modified Grashoff number Gc and porosity parameter K. But it decreases due to increasing Hartmann number M. In Fig. 2, the velocity distribution of boundary layer flow is plotted against y for n = 2, Gr = 2, Gc = 5, M = 0.5, K= 2, γ = 2 and nt = $\pi/2$ and different values of Prandtl number Pr, Schmidt number Sc and Rarefaction parameter R.

It is observed that the fluid velocity decreases due to increasing Prandtl number Pr, Schmidt number Sc but it increases due to increasing Rarefaction parameter R. In Fig. 3, the velocity distribution of boundary layer flow is plotted against y for Gr = 2, Gc = 5, M = 0.5, K = 2, n = 2, Pr = 0.71, Sc = 0.6, R = 0.2, nt = $\pi/2$ and different values of chemical reaction parameter γ. It is observed that the velocity of fluid decreases due to destructive reaction of chemical and it increases due to generative reaction of chemical. In Fig. 4, the temperature distribution of

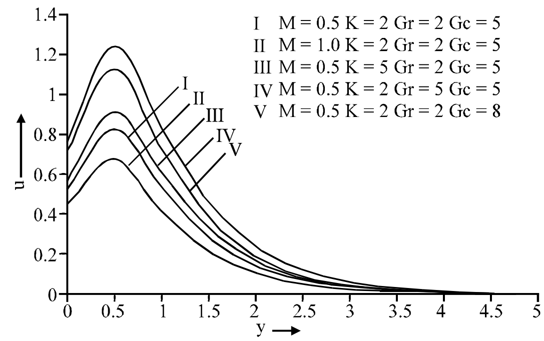

Fig. 1: The velocity distribution for different value of M, K, Gr and Gc

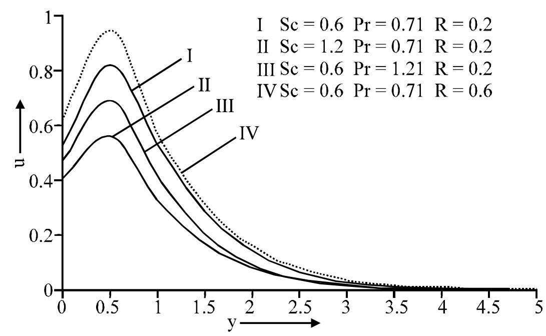

Fig. 2: The velocity distribution for different value of Sc, Pr and R

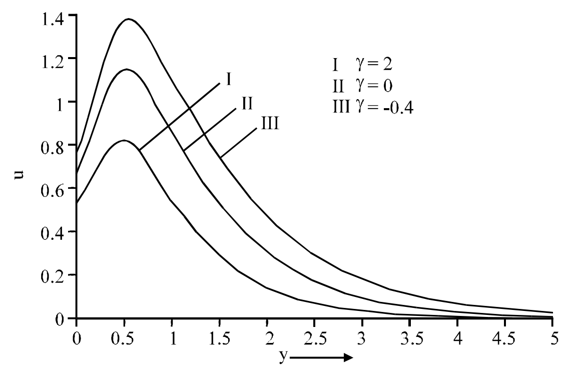

Fig. 3: The velocity distribution for different value of γ

boundary layer flow is plotted against y for Gr = 2, Gc = 5, M = 0.5, K = 2, R = 0.2, Sc = 0.6, n = 2, nt = $\pi/2$, γ = 2 and different values of Prandtl number Pr. It is observed that the temperature decreases continuously with increasing y. It is concluded the fluid temperature decreases with increasing Prandtl number Pr.

In Fig. 5, the concentration distribution of boundary layer flow is plotted against y for Gr = 2, Gc = 5, M = 0.5, K = 2, R = 0.2, n = 2, nt = $\pi/2$, Pr = 0.71 and different values of Schmidt number Sc and chemical reaction parameter γ.

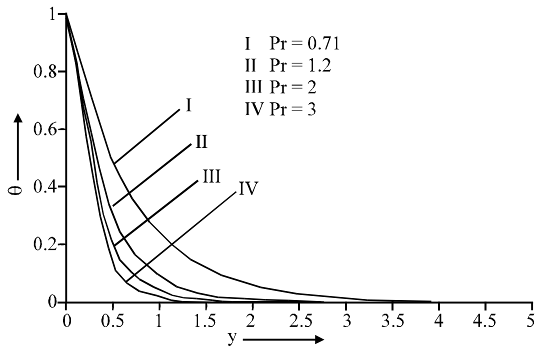

Fig. 4: The temperature distribution for different value of Pr

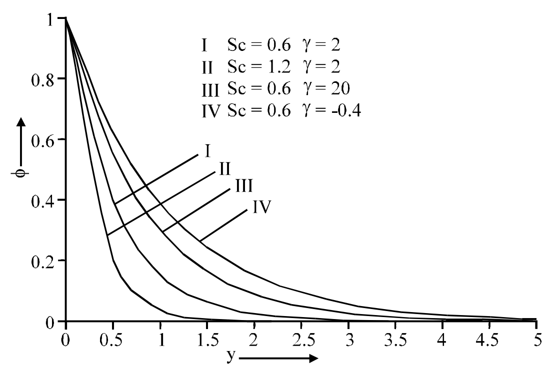

Fig. 5: The concentration distribution for different value of Sc and γ

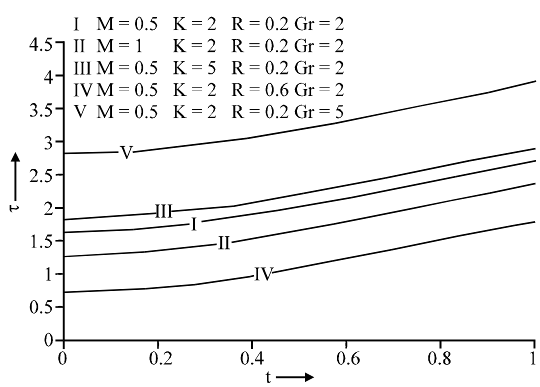

Fig. 6: Skin friction distribution for different value of M, K, R and Gr

It is observed that the concentration of fluid decreases due to increasing Schmidt number Sc and for destructive reaction of chemical and it increases due to generative reaction of chemical.

In Fig. 6, the skin friction is plotted against t for n = 2, Pr = 0.71, Sc = 0.6, , γ = 2, Gc = 5 and for different values of Grashoff number Gr, Hartmann number M, Rarefaction parameter R and porosity parameter K. It is observed that the skin friction decreases due to increasing Hartmann number M and Rarefaction parameter R but it increases due to increasing porosity parameter K and Grashoff number Gr.

The rate of heat transfer and mass transfer are tabulated in following data for different value of Prandtl number Pr, Schmidt number Sc and chemical reaction parameter γ at V_0 = 2.

Pr	Sc	γ	q_1	q_2
0.71	0.6	2.0	-1.420	-1.84900
1.20	1.2	2.0	-2.400	-3.15959
1.60	0.6	0.0	-3.200	-1.20000
2.00	0.6	-0.4	-4.000	-0.94641

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