Journal of Engineering and Applied Sciences 7 (2): 165-169, 2012
ISSN: 1816-949X

Chaotic Behaviours in Dynamical Systems with Application to Secure Communication

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Abstract: Attention has recently been focused on irregular aspects of the behaviour of dynamical systems. One of such behavioural modes is chaos which has been widely reported as having a great potential in communications. This study aimed at demonstrating the major concepts in non-linear dynamics and chaos and the application of these properties in communication. The behavioural modes of logistic map which is a simple dynamical system were investigated with different values of system parameters and initial conditions by computer simulation using Microsoft Excel. The simulation results indicated the sensitivities of the logistic map to parameter changes as well as variations in the initial conditions. The noise-like chaotic trajectory obtained for certain parameter value was employed in masking a message signal. The resulting signal confirmed the potential application of chaos in secure communication.

Key words: Dynamical systems, chaos, logistic map, simulations, secure communication, Ibadan

INTRODUCTION

A dynamical system is a set of functions that specify how variables change over time (Clayton, 1997). It is a non-linear system which may be described by a differential equation or a map (difference equation) (Salam *et al*., 1983; Drazin, 1992). Dynamical systems can exhibit a great variety of behavioural modes some of which are of complex nature (Freire *et al*., 1984). Over the years, there has been a lot of analysis of the regular motion of dynamical systems like the solar system. Attention has recently been focused on irregular aspects of the behaviour. Indeed, most dynamical systems such as weather are highly irregular and notoriously difficult to predict (Harrel II, 1992).

Chaos refers to the unpredictable, seemingly random, motion of trajectories of a dynamical system which was first observed in the two-dimensional Van Der Pol equation with a forcing term (Salam *et al*., 1983). Chaos is mathematically defined as randomness generated by a simple deterministic system (Pecora and Carroll, 1990). This randomness is a result of the sensitivity of chaotic system to initial conditions. If any two identical chaotic systems have slightly different initial conditions, they will diverge from each other to give uncorrelated outputs. However because the systems are deterministic, chaos sometimes has some orderliness inherent in it. Two identical chaotic systems driven by the same signal will produce the same output even though, the output is unpredictable. Thus, chaos can essentially be thought of as deterministic noise. Chaotic dynamics have interested chemical engineers with respect to fluids applications and biotechnology engineers as regards periodic beat patterns in a rabbit heart (Liao and Huang, 1997). However, chaos, most of the time has undesirable effect in mechanical and electrical engineering design and must be averted. Certain effects blamed on noise in electrical systems are examples of chaotic behaviour of a completely deterministic nature (Freire *et al*., 1984). The investigation of the synchronization of chaotic systems began only recently but it already has some fruitful return, especially in the areas of communication and signal processing (Chen, 1996). The synchronization problem consists of making two systems oscillate in a synchronized manner.

Theory: A measure of behaviour of a dynamical system as a function of time is often referred to as time series. An attractor is the status that the system eventually settles down to. It is a set of values in the phase space to which a system migrates over time or iterations. A phase space or state space is an abstract space used to represent the behaviours of a system and its dimensions are the variables of the system. It is essentially a graph in which each axis is associated with one dynamic variable. When a system settles down to a point, the attractor is called a one-point attractor (Clayton, 1997). When a system settles down to alternating between two points, its attractor is referred to as a two-point attractor. The change from a one-point attractor to a two-point attractor is referred to as a bifurcation or period doubling. Bifurcation is a qualitative change in the behaviour (attractor) of a dynamical system associated with a change in a control

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parameter (Drazin, 1992; Clayton, 1997). A dynamical system having an infinite-point attractor often called a strange attractor is known as Chaotic System. The Logistic Map or Logistic Difference Equation is a model often used to introduce chaos (Clayton, 1997). Although it is simple, it displays the major chaotic concepts. As an example of the logistic map, consider a Limited Growth (Verhulst) Model which was used in 1845 by Verhulst to predict American population from census data (Harrel II, 1992). This model is expressed as (Clayton, 1997):

$$x_{n+1} = rx_n[1 - x_n] \quad (1)$$

Where:

- x_{n+1} = The new population
- x_n = The old population
- r = The rate of population growth
- n = The time interval

In Eq. 1, x_{n+1}, x_n is the variables while r is the parameter. Chaotic attractors being noise-like in appearance can be used to hide information from an unwanted receiver. To recover the information, it is necessary to have another copy of the chaotic system with exact parameters which are used in the transmitter. The chaotic system at the receiver end must synchronize with that at the transmitter end if the exact information transmitted is to be recovered at the receiver end. In general, two periodic systems are referred to as being synchronized if either their phases or frequencies are locked. For Chaotic System, however the notion of frequency and phase are in general not well defined and can thus not be used in characterizing synchronization (Parlitz *et al*., 1999). A slight variation in the initial conditions causes identical chaotic systems to diverge and fall out of synchronism with each other. Also, any slight change in the parameters results in a change in the trajectory and this will affect synchronization between the transmitter and the receiver and it then becomes impossible to recover the information at the receiver end. Various sychronization techniques have been widely reported in the literature (Pecora and Carroll, 1990; Yang, 2004; Dedieu *et al*., 1993, Parlitz *et al*., 1999).

MATERIALS AND METHODS

To observe the various behavioural modes of the logistic map, the function is simulated Microsoft Excel using different values of parameter, r and initial conditions. The time series and phase portraits were obtained by plotting x_{n+1} against n and x_{n+1} against x_1, respectively. To demonstrate the potential application of chaos in communication, a chaotic signal was added to a message signal (a pure sinusoid) to hide the information from unwanted receiver.

RESULTS AND DISCUSSION

Plotting x_{n+1} against n for $0<r<1$ and $x_1 = 0.5$ gave the time series graph shown in Fig. 1a, b is a phase portrait for r = 0.5. Figure 2-5 shows the time series and phase portraits for $x_1 = 0.5$, $1<r<3$; $x_1 = 0.5$, r = 3.2; $x_1 = 0.5$, r = 3.54 and $x_1 = 0.5$, r = 3.99, respectively.

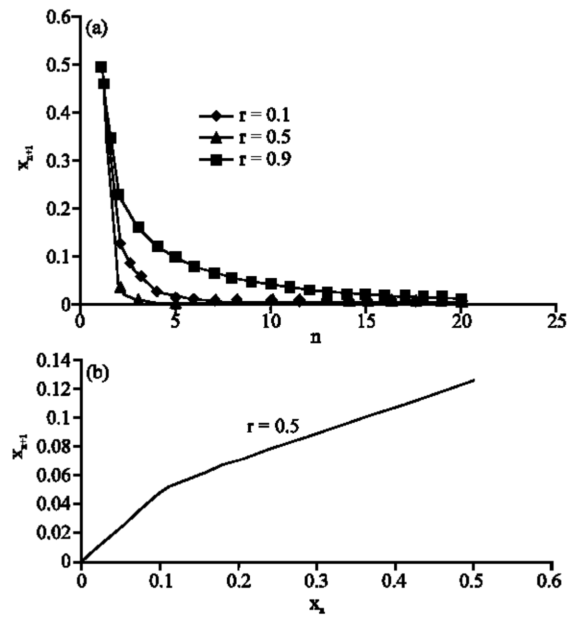

Fig. 1: Time series graph

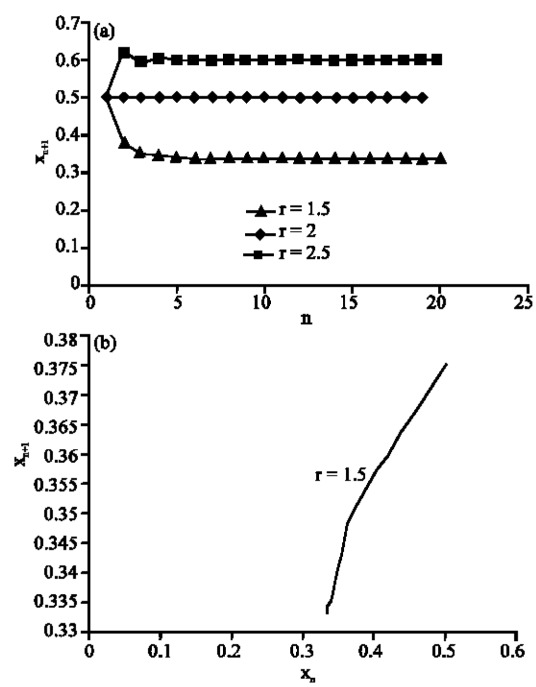

Fig. 2: Time series and phase prtrait for I<r<3, x_1= 0.5; a) Time series and b) Phase portrait

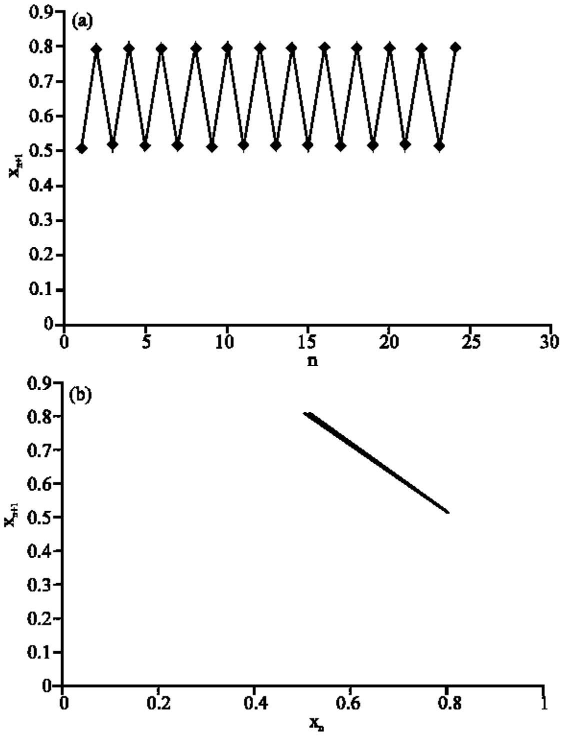

Fig. 3: For r = 3.2; a) trangular wave oscillator and b) bifurcation or period doubling

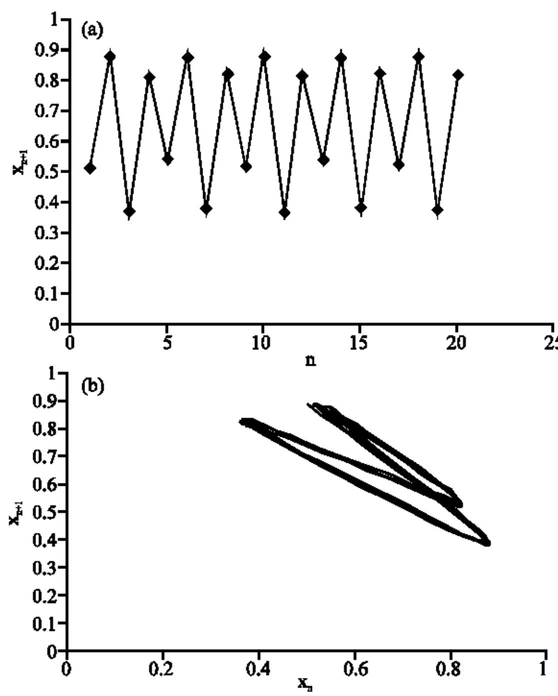

Fig. 4: Time series and phase portrait for r = 3.54, x_1 = 0.5; a) Time series and b) Phase portrait

The time series in Fig. 6 showed entirely different trajectories for initial conditions x_1 = 0.3 and x_1 = 0.301. The

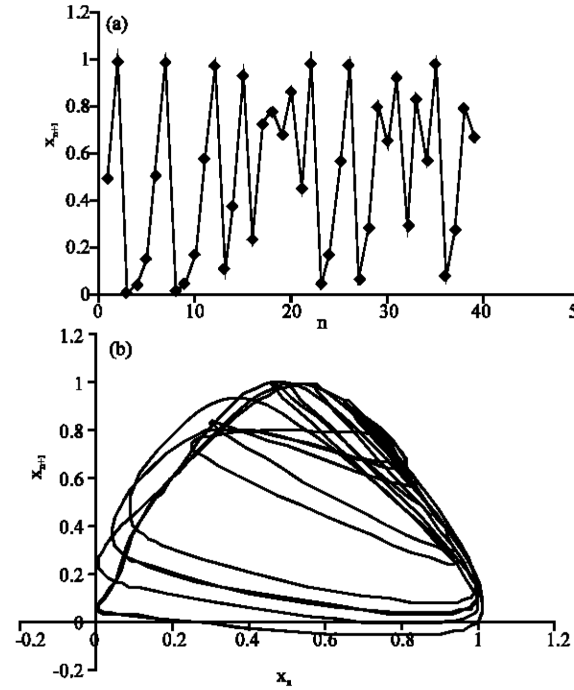

Fig. 5: At r = 3.99; a) Strange attractor and b) Choes

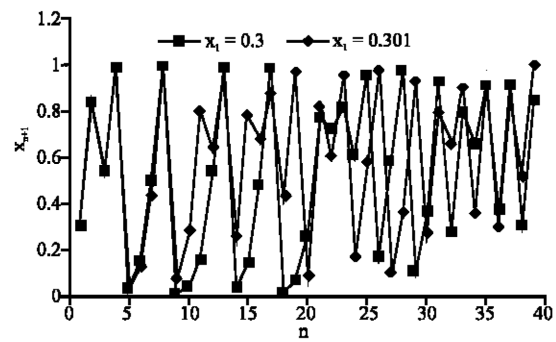

Fig. 6: Time series for r = 3.99; x_1 = 0.3 and r = 3.99; x_1 = 0.301

trajectory of the signal was obtained by adding the message signal in Fig. 7 and the chaotic signal in Fig. 8. The graph in Fig. 1 in which x_{n+1} goes towards zero as n increases is usually referred to as a one-point attractor. The system shown in Fig. 1 behaves like an RC circuit where the curve represents an exponentially decaying current or an RL circuit where the curve represents an exponentially decaying voltage. Figure 2 shows that regardless of the starting value, the system displays non-zero one-point attractors. The system illustrated in Fig. 2 behaves like an amplifier with almost constant gain. For r = 3.2 as shown in Fig. 3, the system settles down to alternating between two points. This is a two-point attractor which is also a periodic attractor.

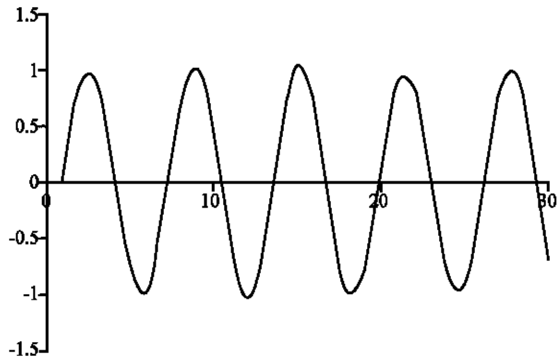

Fig. 7: Sinusoidal message signal

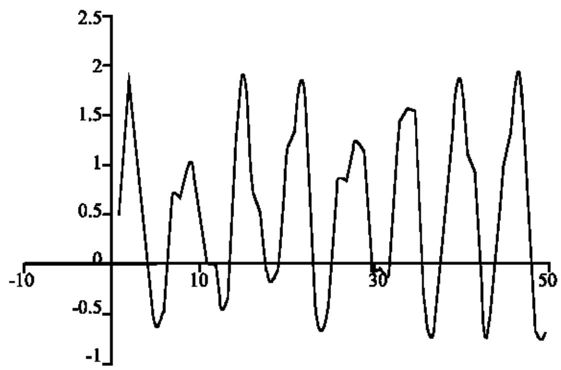

Fig. 8: Choatic signal waveform

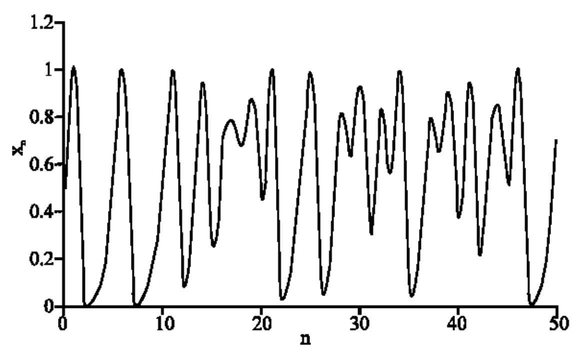

Fig. 9: Secured message signal (original message signal plus choatic signal)

The system shown in Fig. 3 behaves like a triangular wave oscillator. The change from a one-point attractor to a two-point attractor is referred to as a bifurcation or period doubling. Figure 4 shows another bifurcation from a two-point attractor to a four-point attractor with r = 3.54. The system in Fig. 4 also behaves like an oscillator but with its frequency halved (or period doubled). At r = 3.99 as shown in Fig. 5, the system has infinite-point attractor which is referred to as a strange attractor. At this point, the system has an unpredictable trajectory indicating that chaos has set in. While the behaviours of the systems shown in Fig. 1-4 are predictable because they all have a finite number of attractors, the system shown in Fig. 5 has an infinite number of attractors and so it is chaotic and its behaviour is unpredictable.

This system would produce an unpredictable output no matter the input signal. Another important feature of a Chaotic System is its sensitivity to initial conditions (Clayton, 1997). Any two identical chaotic systems having slightly different initial states will diverge from each other. Consider the time series for r = 3.99; x_1 = 0.3 and r = 3.99; x_1 = 0.301 and using the first 40 iterations as shown in Fig 6. It is observed from Fig. 6 that even a small change in the initial condition as low as 0.001 causes the time series to diverge from one another. The chaotic signal effectively concealed the message signal as shown in Fig. 9 from unwanted receiver, demonstrating the application of chaos in secure communication. Although, the dynamical system modelled in this research is abstract in nature, it is not impossible to build a practical system that has the properties of the system. Thus, a Chaotic System may be used to mask an information signal such that an unwanted interceptor would not be able to make any meaning of it. The information is therefore made secure by employing chaos (Kennedy *et al*., 2000). Practical chaos-based secure communication systems make use of higher dimension systems such as the Lorenz System and the Chua oscillator which are more complex and hence, provide higher security of information being transmitted.

CONCLUSION

The major behavioural modes of a dynamical system including chaos have been discussed in this research and illustrated using computer simulations. The simulation results show the sensitivity of dynamical systems to both changes in system parameters as well as the initial conditions. The applications of the different behavioural modes of the modelled dynamical system were discussed with a special attention paid to the application of the chaotic behaviour of the dynamical system in secure communication.

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