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An Inventory Model for Deteriorating Items with Time Dependent Demand and Holding Cost under Partial Backlogging

[1]Vinod Kumar Mishra, [2]Lal Sahab Singh and [2]Rakesh Kumar
[1]Department of Computer Science and Engineering, Kumaon Engineering College, Dwarahat, 263653 Almora (Uttarakhand), India
[2]Department of Mathematics and Statistics, Dr. Ram Manohar Lohia Avadh University, 224001 Faizabad (UP), India

Abstract: In this study, researchers considered a deterministic inventory model with time dependent demand and time varying holding cost where deterioration is time proportional. The model considered here allows for shortages and demand is partially backlogged. The model is solved analytically by minimizing the total inventory cost. Result is illustrated with numerical example for the model.

Key words: Inventory model, deteriorating items, demand, time dependent demand, time varying holding cost, India

INTRODUCTION

One of the most unrealistic assumptions in traditional inventory model was that items preserved their physical characteristics while they were kept stored in inventory. However, the deteriorating items are subject to a continuous loss in their masses or utility throughout their life time due to decay, damage, spoilage and penalty of other reasons. Owing to this fact controlling and maintaining inventory of deteriorating items becomes a challenging problem for decision makers.

Harris (1915) developed first inventory model economic order quantity which has generalized by Wilson (1934) and he gave a formula to obtain economic order quantity. Whitin (1957) considered the deterioration of the fashion goods at the end of prescribed shortage period. Ghare and Schrader (1963) developed a model for an exponentially decaying inventory.

Dave and Patel (1981) were the first to study a deteriorating inventory with linear increasing demand when shortages are not allowed. Some of the recent research in this field has been done by Chung studied an inventory model with deteriorating items. Chang and Dye developed an inventory model with time varying demand and partial backlogging. Goyal and Giri (2001) gave recent trends of modeling in deteriorating items inventory. They classified inventory models on the basis of demand variations and various other conditions or constraints. Liang-Yuh *et al.* (2005) developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Chung-Yuan *et al.* (2007) find an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. They assume that a fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases.

Roy (2008) developed a deterministic inventory model when the deterioration rate is time proportional. Demand rate is function of selling price and holding cost is time dependent. Liao (2008) gave an EOQ model with non instantaneous receipt and exponential deteriorating item under two level trade credits. Sarla developed a deterministic inventory model for deteriorating items with salvage value and shortages. Skouri *et al.* (2009) developed an inventory models with ramp type demand rate, partial backlogging and Weibell's deterioration rate. Mishra and Singh (2010) developed a deteriorating inventory model for waiting time partial backlogging when demand and deterioration rate is constant. They made Abad (1996, 2001) more realistic and applicable in practice. Mandal (2010) gave an EOQ inventory model for Weibull distributed deteriorating items under ramp type demand and shortages. Mishra and Singh (2011) gave an inventory model for ramp type demand, time dependent deteriorating items with salvage value and shortages and deteriorating inventory model for time dependent demand and holding cost with partial backlogging. Kuo-Chen (2011) gave an inventory model with

Corresponding Author: Vinod Kumar Mishra, Department of Computer Science and Engineering, Kumaon Engineering College, Dwarahat, 263653 Almora (Uttarakhand), India

generalized type demand, deterioration and backorder rates. In classical inventory models, the demand rate and holding cost is assumed to be constant. In reality demand and holding cost for physical goods may be time dependent.

Time also play an important role in inventory system, therefore in this inventory system we consider demand and holding cost are time dependent. In this study, researchers made the study of Roy (2008) more realistic by considering demand rate and holding cost as a linear function of time and developed an inventory model for deteriorating items where deterioration rate are expressed as linearly increasing function of time. Shortages are allowed and partially backlogged.

MATERIALS AND METHODS

Assumption and notations: This inventory model is developed on the basis of following assumption and notations:

- Deterioration rate is time proportional
- $\theta(t) = t$, where is the rate of deterioration; $0<\theta<1$
- Demand rate is time dependent and linear, i.e., $D(t) = a+bt$; a, b>0 and are constant
- Shortage are allowed and partially backlogged
- C_2 is shortage cost per unit, per unit time
- β is backlogging rate; $0 \leq \beta \leq 1$
- During time t_1 inventory is depleted due to deterioration and demand of item. At time t_1 inventory becomes zero and shortage start occurring
- There is no repair or replenishment of deteriorating item during the period under consideration
- Replenishment is instantaneous, lead time is zero
- T is length of cycle
- The order quantity of one cycle is q
- Holding cost h(t) per unit time is time dependent and is assumed $h(t) = h+t$ where $\alpha>0$; $h>0$
- C is unit cost of an item
- IB is maximum backorder
- S is lost sale cost per unit

Mathematical formulation and solution: The rate of change of inventory during positive stock period $[0, t_1]$ and shortage period $[t_1, T]$ is governed by the differential equations:

$$\frac{dI_1(t)}{dt} = -D(t) - \theta(t)I_1(t), \quad 0 \leq t \leq t_1 \tag{1}$$

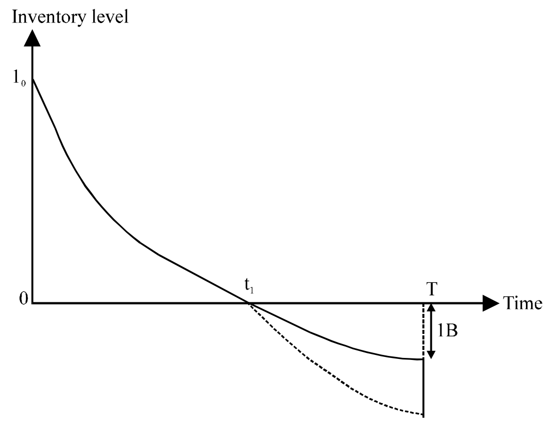

Fig. 1: Graphical representation of inventory system for first cycle

$$\frac{dI_2(t)}{dt} = \beta D(t); \quad t_1 \leq t \leq T \tag{2}$$

The initial inventory level is I_0 unit at time $t = 0$ from $t = 0$ to $t = t_1$ the inventory level reduce. Owing to both demand and deterioration until it reach zero level at time $t = t_1$. At this time shortage are accumulated which are partially backlogged at the rate β.

At the end of the cycle of the inventory reaches a maximum shortage level so as to clear the backlogged and again raise the inventory level to I_0 (Fig. 1). Thus, boundary conditions are as follows:

$$I_1(0) = I_0, \; I_1(t_1) = 0, \; I_2(t_1) = 0$$

The solution of the Eq. 1 and 2 with boundary conditions is:

$$I_1(t) = -e^{-\theta t^2/2}\int_t^{t_1} e^{\theta t^2/2}(a+bt)dt, \; 0 \leq t \leq t_1 \tag{3}$$

$$I_2(t) = -\beta\left[a(T-t) + \frac{b}{2}(T^2 - t_1^2)\right], \; t_1 \leq t \leq T \tag{4}$$

Using Eq. 3, researchers get:

$$I_0 = \int_0^{t_1}(a+bt)e^{\theta t^2/2}dt \tag{5}$$

Inventory is available in the system during the time interval $[0, t_1]$. Hence, the cost for holding inventory in stock is computed for time period $(0, t_1)$ only. Holding cost is:

$$\begin{aligned}H_c &= \int_0^1 (h+\alpha t) I_1(t) \\ &= \int_0^1 (h+\alpha t) e^{-\theta t^2/2} \int_0^1 (a+bu) e^{\theta u^2/2} du dt \\ &= ah\left(\frac{1}{2}t_1^2 + \frac{1}{12}\theta t_1^4 + \frac{1}{90}\theta t_1^6\right) + bh\left(\frac{1}{3}t_1^3 + \frac{1}{15}\theta t_1^5 + \frac{1}{105}\theta t_1^7\right) + \\ &\quad a\alpha\left(\frac{1}{6}t_1^3 + \frac{1}{40}\theta t_1^5 + \frac{1}{136}\theta t_1^7\right) + b\alpha\left(\frac{1}{8}t_1^4 + \frac{1}{48}\theta t_1^6 + \theta\frac{1}{384}t_1^8\right)\end{aligned} \quad (6)$$

Shortage due to stock out are accumulated in the system during the interval $[t_1, T]$. The optimum level of shortage are present at $t = T$, therefore total shortage cost during this time period:

$$\begin{aligned}S_c &= C_2 \int_t^T -I_2(t) dt \\ &= \beta a C_2 (T-t_1)^2 + \frac{1}{2}\beta b C_2 (T-t_1)^2 (T=t_1)\end{aligned} \quad (7)$$

Due to stock out during (t_1, T) shortage are accumulated but not all customers are willing to wait for the next lot size to arrive. Hence, this result in some loss of sale which account to loss in profit. Lost sale cost is calculated as:

$$\begin{aligned}LSC &= S\int_{t_1}^T (1-\beta) D(t) dt \\ LSC &= S(1-\beta)\left[a(T-t_1) + \frac{1}{2}b(T^2 - t_1^2\right]\end{aligned} \quad (8)$$

Purchase cost is:

$$\begin{aligned}PC &= C\left(I_0 + \int_{t_1}^T \beta D(t) dt\right) \\ PC &= CI_0 + C\beta a(T-t_1) + \frac{1}{2}C\beta a(T^2 - t_1^2)\end{aligned} \quad (9)$$

Total cost:

$$TC = OC + PC + HC + SC + LSC$$

$$TC = \left|\begin{array}{l} A + CI_0 + \beta C + \left[a(T-t_1) + \frac{b}{2}(T^2 - t_1^2)\right] + \\ ah\left[\frac{1}{2}t_1^2 + \frac{1}{12}\theta t_1^4 + \frac{1}{90}\theta t_1^6\right] + bh\left[\frac{1}{3}t_1^3 + \frac{1}{15}\theta t_1^5 + \frac{1}{105}\theta t_1^7\right] + \\ \alpha a\left[\frac{1}{6}t_1^3 + \frac{1}{40}\theta t_1^5 + \frac{1}{136}\theta t_1^7\right] + \alpha b\left[\frac{1}{8}t_1^4 + \frac{1}{48}\theta t_1^6 + \frac{1}{136}\theta t_1^8\right] + \\ \beta a C_2 (T-t_1) + \frac{1}{2}\beta b C_2 (T-t_1)^2 (T+t_1) + \\ S(1-\beta)\left[a(T-t_1) + \frac{1}{2}b(T^2 - t_1^2)\right] \end{array}\right| \quad (10)$$

Differentiates the Eq. 10 with respect to t_1 and T then researchers get:

$$\frac{\partial TC}{\partial t_1} \text{ and } \frac{\partial TC}{\partial T}$$

To minimize the total cost TC (t_1, T) per unit time the optimal value of T and t_1 can be obtained by solving the following equations:

$$\frac{\partial TC}{\partial t_1} = 0 \text{ and } \frac{\partial TC}{\partial T} = 0 \quad (11)$$

Provided Eq. 17 satisfies the following condition:

$$\left(\frac{\partial^2 TC}{\partial t_1^2}\right)\left(\frac{\partial^2 TC}{\partial T^2}\right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T}\right) > 0 \text{ and } \frac{\partial^2 TC}{\partial t_1^2} > 0 \quad (12)$$

By solving Eq. 11, the value of T and t_1 can be obtained and with the use of these optimal value, Eq. 10 provides minimum total inventory cost per unit time of the inventory system.

Since, the nature of the cost function is highly non linear thus the convexity of the function is discussed in the study.

RESULTS AND DISCUSSION

The following numerical values of the parameter in proper unit has considered as input for numerical and graphical analysis of the model:

$$A = 2500,\ a = 10,\ b = 50, C = 10,\ C_2 = 4,$$
$$h = 0.5,\ \theta = 0.8,\ \alpha = 20,\ \beta = 0.10,\ S = 8$$

The output of the model by using maple mathematical software (the optimal value of total cost, the time when inventory level reaches zero and the time when maximum shortages occur) is as follows: TC = 2500.158284, t_1 = 0.12659904 and T = 0.12826694. The graph of TC with respect to T, TC with respect to t1 and TC with respect to t_1 and T are as follows:

The observation from Fig. 2-4 is that the total cost function is strictly convex function. Thus the optimum value of T and t_1 can be obtained with the help of total cost function of the model provided the total inventory cost per unit time of the inventory system is minimum.

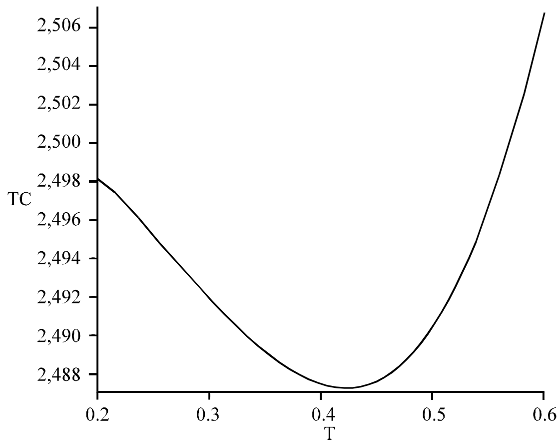

Fig. 2: Total cost vs. T at t_1 = 0.1265

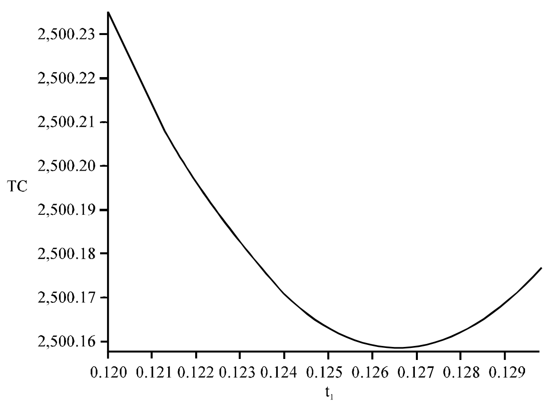

Fig. 3: Total cost vs. t_1 at T = 0.1282

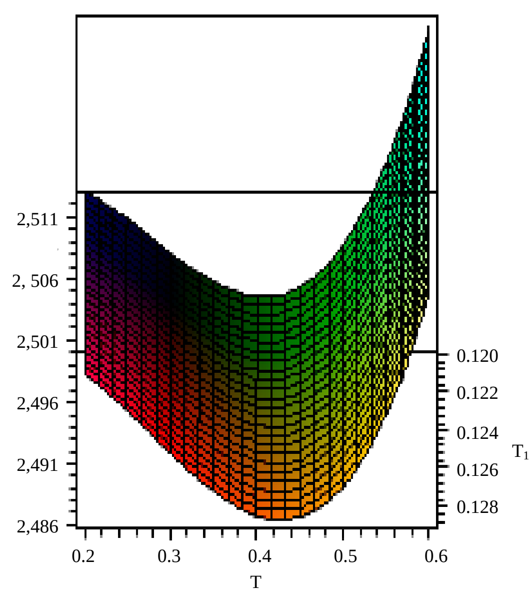

Fig. 4: Total cost vs. t_1 and T

CONCLUSION

This study presents an inventory model of direct application to the business enterprises that consider the fact that the storage item is deteriorated during storage periods and in which the demand rate and holding cost depends upon the time. In this study, researchers developed a deterministic inventory model with time dependent demand and time varying holding cost where deterioration is time proportional. The model allows for shortages and demand is partially backlogged. The model is solved analytically by minimizing the total inventory cost. Finally, the proposed model has been verified by a numerical and graphical analysis. The obtained results indicate the validity and stability of the model. The proposed model can further be enriched by taking more realistic assumptions such as finite replenishment rate, probabilistic demand rate, variable deterioration rate, etc.

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