Journal of Engineering and Applied Sciences 7 (2): 223-226, 2012
ISSN: 1816-949X

Time History of Concentration Levels for Maximum Disposal of Coal Waste under Acceptable Limits in Damodar River: A Mathematical Model

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Abstract: Damodar river basin is repository of 40% of India coal reserve. Exploitation of coal and related industries in the area has exerted a great impact on the water pollution. In this study, an attempt has been made to develop a mathematical model to obtain time histories of concentration levels under different dumping policies so as to select the policy which disposes maximum waste without exceeding acceptable limits. An analytical solution is obtained to study the concentration levels for different time histories. The model can be useful in guiding engineering and management decision concerned with the efficient utilization of Damodar river water and protect their quality.

Key words: Damodar river, water pollution, coal mining activities, time history, concentration level, India

INTRODUCTION

There are several companies which perform coal mining activities and is located on the banks of Damodar river. In the past, it has disposed of its waste by indiscriminately dumping it into the river, causing high levels of pollution. As a consequences, the local authorities passed new legislation with very high fines if pollution in the river exceed certain specified concentration limits. The problem in this study is to frame a policy for discharging the waste so as to ensure that the concentration levels never exceed the specified limits. Mirbagheri *et al*. (2009) developed mathematical model of water quality in river systems of Jajrood river in Tehran Iran.

The water quality of Danube river and its analysis through mathematical model was studied by Antohe and Stanciu (2009). Agusto and Bamigbola (2007) studied the numerical treatment of the Mathematical models for water pollution. Shukla *et al*. (2008) developed Mathematical models and studied its analysis of the depletion of dissolved oxygen in eutrophied water bodies affected by organic pollutants. The remedy through a new technique for water pollution in river was well studied by Pimpunchat *et al*. (2009). In order to solve the problem of how to keep concentration level of pollutants below prescribed level of pollutants levels, companies need to obtain changes to concentration levels in the river over time under different dumping policies for chemical waste (Murphy, 2011; Chapra, 1997). A policy for dumping is defined by time history of dumping. Researchers assume that the dumping is carried out at discrete time instants. The effect of dumping is increase the concentration level suddenly at the moment it occurs. With the passage of time, the chemical waste diffuses into the water and also gets transported downstream. Depending on the waste, it can either go through a process of transformation so that it ceases to be an undesirable waste or relation its chemical structure or continue to be an undesirable chemical waste.

Assumption: Researchers treat river as a single object with the waste dumped being an input variable (from the system environment) and the water discharge to the sea being an output variable (to the system environment). In the simplest characterization, the river is viewed as a one dimension al channel of constant cross-section A, with $x = 0$ corresponding to the location where the waste is dumped into the river. Let L_1 and L_2 represents the length of the river upstream and downstream from the point where the waste is dumped.

As a result, researcher have $-L_1<x< L_2$ with distances measured downstream being positive and those measured upstream being negative. Let T (t, x) be the concentration level of chemical waste at location x from the point of dumping at time t. The variable changes continuously with t and x. Let v be the velocity of flow which researcher treat as a constant. Since, the river receive clean water from an external source the concentration is zero at $x = -L_1$, hence $T(-L_1, t) = 0$. Resaerchers assume that the sea to which the river discharge is very then the concentration level there is also negligible. Hence, $T(L_2, t) = 0$. The

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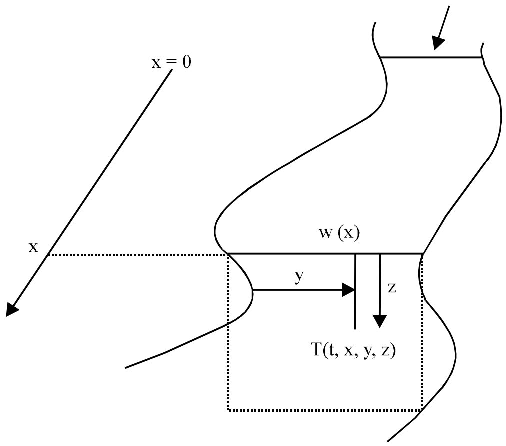

Fig. 1: Coordinate frame for river pollution

diffusion and transportation of the waste is predictable and governed by the law of fluid mechanics. Researchers describe this model as an open deterministic dynamic system. The main objective of this study is to obtain time histories of concentration levels under different dumping policies so as to select the policy which disposes maximum waste without exceeding acceptable limits. In the above characterization, researcher assume the river as a one-dimensional channel with cross-section and ignore the lateral diffusion of the chemical waste across the width. Since, the waste is dumped at a point, rather than along the whole cross-section, the above characterization is over-simplification.

A more redefined characterization includes the spatial variations. For example, researchers can treat the depth as being nearly constant and researcher represent this by a parameter h and the width of the river is changing with x. Let w(x) represent the width of the river at a distance x from the point of dumping along the river then the cross-sectional area A(x) changes with x. The concentration level is given by a variable T(t, x, y, z), the concentration at a point whose coordinates are given by (x, y, z) as shown in Fig. 1 at time, t. The limits for y and z are given by $0<z<h$ and $0<y<w(x)$. Since, the flow volume must be constant for all x, the velocity of flow U(x) is a function of x.

MATERIALS AND METHODS

Model 1: Let T(x, t) represent the concentration level of pollution at a distance x from the point of discharge at time, t. From the system characterization researcher have:

$$-L_1 \leq x \leq L_2 \text{ and } 0 \leq t < \infty \tag{1}$$

The concentration level changes due to diffusion and transportation. Let D be the coefficient of diffusion and U be the velocity of the river which is assumed to be constant. The diffusion is governed by Fick's law of diffusion current vector J is given by:

$$J = -D\frac{\partial T(x,t)}{\partial x} \tag{2}$$

From the equation of continuity, for the diffusing substance researchers have:

$$\frac{\partial^2 T(x,t)}{\partial x} = -U\frac{\partial T(x,t)}{\partial x} + D\frac{\partial^2 T(x,t)}{\partial x^2} \tag{3}$$

This is obtained by considering a small volume centered at x at time t and evaluating the net inflow and outflow of pollution in a small time δt. The net change in the small volume can be related to the net inflow and net outflow and in the limit $t \to 0$ and $\delta x \to 0$. Researchers have the above equation. Clearly, Eq. 3 involves a parabolic partial differential equation. Researchers have the following initial and boundary conditions:

$$IC: T(x,0) = G(x), \ -L_1 \leq x \leq L_2 \tag{4}$$

Indicating the concentration levels along the river at t = 0 and BC:

$$T(-L_1, t) = 0, \ T(L_2, t) = 0, < t < \infty \tag{5}$$

Researchers slightly modify the model by taking into account the discharge of the pollution at x = 0. In Eq. 3, researcher include an extra term F(x, t) which is of the form:

$$F(x, t) = \alpha g(t)\delta(x) \tag{6}$$

Where, g(t) is the time history of dumping and $\delta(x)$ is the Dirac delta function.

Model 2 and its analytical solution: Assume that the velocity of flow is negligible hence, U = 0. As a result, the model for pollution concentration is given by:

$$\frac{\partial^2 T(x,t)}{\partial x^2} = \frac{1}{D}\frac{\partial T(x,t)}{\partial t} \tag{7}$$

With the BC and IC given by:

BC:

$$T(-L_1, t) = 0 = T(L_2, t), \ 0 \leq t < \infty$$

IC:

$$T(x,0) = G(x) \tag{8}$$

Researchers use the method of separation of variables. This method assumes a solution of the form:

$$T(x,t) = W(x).V(t) \tag{9}$$

Substituting Eq. 9 in Eq. 7, yields:

$$\frac{1}{D}W(x)\frac{dV(t)}{dt} = V(t)\frac{d^2W(x)}{dx^2} \tag{10}$$

$$\Rightarrow \frac{1}{W(x)}\frac{d^2W(x)}{dx^2} = \frac{1}{DV(t)}\frac{dV(t)}{dt} = -\lambda^2 \tag{11}$$

As a result, W(x) is given by:

$$W(x) = \sum_{\lambda}\{C_\lambda \sin\lambda(x+L_1) + D_\lambda \cos\lambda(x+L_1)\} \tag{12}$$

and V(t) is:

$$V(t) = e^{\gamma t} \tag{13}$$

Since, the BC has to be satisfied, we have $D_\lambda = 0$ and λ must satisfy:

$$\lambda(L_1+L_2) = n\Pi;\ n = 0, 1, 2.... \tag{14}$$

or;

$$\lambda = \frac{n\prod}{L_1+L_2},\ n = 0, 1, 2... \tag{15}$$

As a result, the solution is given by:

$$T(x,t) = \sum_{n=0}^{\infty}\alpha_n e^{-\gamma t}\sin\left\{\frac{n\prod(x+L_1)}{L_1+L_2}\right\} \tag{16}$$

$$\gamma = \frac{Dn^2\prod^2}{(L_1+L_2)^2} \tag{17}$$

and α_n to be selected to satisfy the IC From Eq. 16 researcher have:

$$T(x,0) = \sum_{n=0}^{x}\alpha_n \sin\left\{\frac{n\prod(x+L_1)}{L_1+L_2}\right\} \tag{18}$$

and for this to equal G(x) researcher must have:

$$\alpha_n = \int_{-L1}^{L2} G(x)\sin\{\frac{n\prod(x+L_1)}{L_1+L_2}\}dx \tag{19}$$

Thus, the analytical solution is given by Eq. 16 with α_n evaluated from Eq. 19.

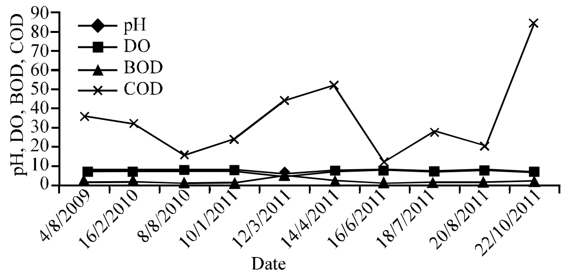

Fig. 2: Time histories of different biochemical deposition in Damodar river at Dhanbad region

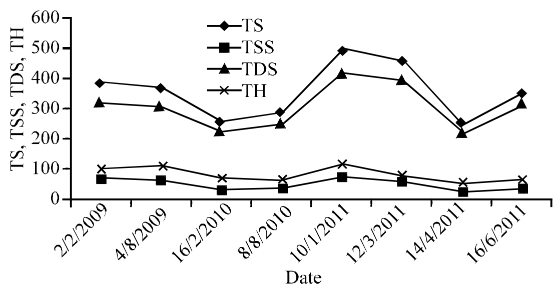

Fig. 3: Time histories of different bio gradient deposition in Damodar river at Dhanbad region

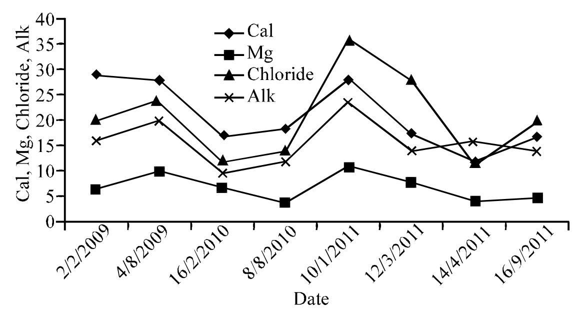

Fig. 4: Time histories of different chemical deposition in Damodar river at Dhanbad region

Data on pollutants dumped in the Damodar river at different times: Time histories of different biochemical, chemical and bio-gradients waste being deposited in Damodar river at Dhanbad region is collected. Their chemical composition representing pH, DO, BOD, COD, TS, TSS, TDS, TH, Cal, Mg, Chloride and Alkanality (mg L^{-1}) were well studied in the sample collected for the last 3 years that is January, 2009 till October, 2011. An analysis of all these chemical compositions is shown in Fig. 2-4. Chemical Oxygen Demand (COD) has a very high variation and has exceeded all its limits in October, 2011 which is shown in Fig. 2 an immediate attention has to be paid for this wide variation. Disposal of Chloride is also varying too much and has reached a limit in the month of March, 2011which needs an immediate attention (Fig. 4).

CONCLUSION

Based on the time histories of dumping pollutants (chemical, bio chemical and bio gradient) in the Damodar river at Dhanbad region, researchers developed two different Mathematical models. The time histories of concentration levels under different dumping policies were taken into consideration so as to select the policy which disposes maximum waste without exceeding acceptable limits. As Damodar river at Dhanbad passes through coal reserve areas, the concentration level of pollutants is very high and also the disposal of coal extracts into the river is not at a single point, the developed models will be useful in guiding engineering and management decision concerned with the efficient utilization of Damodar river water and protect their quality.

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