Journal of Engineering and Applied Sciences 7 (3): 265-270, 2012
ISSN: 1816-949X

MHD Free Convection Flow Effects on Stoke's Problem Through Porous Medium with Transverse Periodic Suction

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Abstract: This study presents an approximate solution to a 3-dimensional free convection flow of a viscous, incompressible fluid past an impulsively started infinite, vertical porous limiting surface through porous medium with transverse sinusoidal suction in presence of uniform magnetic field. Using perturbation technique, the expressions for the transient velocity in the main flow direction, the temperature, the sinusoidal skin fraction and the rate of heat transfer have been discussed with there respective dependence on the Prandtl number (Pr), Grashoff number (Gr), Magnetic parameter (M) and Porosity parameter (K).

Key words: Free convection, porous medium, heat transfer, incompressible viscous fluid, perturbation, India

INTRODUCTION

The phenomenon of MHD free convection has many importances in technological applications, e.g., in cooling reactors, providing heat sinks in turbine blades, etc. The effects of free stream oscillations on the boundary layer flow of a viscose fluid are also often encountered in engineering applications, e.g., in the aerodynamics of a helicopter rotor or in a variety of bio-engineering problems such as fluttering airfoil, etc. Such a study was initiated by Lighthill (1954). By assuming the oscillatory flow to be super imposed on the steady non zero mean flow, he linearized the momentum equations and solved them by the integral method, Stuart (1955) further extended this idea to study a 2-dimensional flow past an infinite porous plate. Further, Soundalgekar (1973a, b) and Soundalgekar and Pop (1974) analyzed the unsteady flow past an infinite vertical porous plate with constant and variable suction, respectively. In all the study mentioned, the plate is assumed to be stationary.

The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate in its own plane was first studied by Stokes (1851) also known as Rayleigh's problem. Recently, Georgantopoulos (1979) has discussed the free convection effects on the oscillatory flow in the Stoke's problem past an infinite, vertical porous limiting surface with constant suction. Singh analyzed 3-dimensional oscillatory flow past a plate. Most of the researchers have assumed the suction velocity either constant or variable with time thus restricting themselves to the two dimensional flows only. One possible suction distribution is a transverse sinusoidal one which give rise to a cross flow and hence to a 3-dimensional flow over the surface. Gorla and Singh (2005) have been discussed free convection effects on Stoke's problem with transverse periodic suction. Das and Tripathy (2010) have discussed effect of periodic suction on 3-dimensional flow and heat transfer past a vertical porous plate embedded in a porous medium. Therefore, the object of this study is to study the free convection effects on flow of a viscous, incompressible fluid past an impulsively started infinite, vertical porous limiting surface through porous medium with transverse sinusoidal suction in presence of uniform magnetic field.

MATERIALS AND METHODS

Mathematical formulation: Researchers consider 3-dimensional free convection flow of a viscous, incompressible fluid past an impulsively started infinite, vertical porous limiting surface through porous medium with transverse sinusoidal suction in presence of uniform magnetic field which consists of a basic uniform distribution superimposed with a sinusoidal distribution $\varepsilon v_0 \cos\pi\ v_0 z'/v$. A coordinate system is assumed with limiting surface lying vertically on x'-z' plane such that the x' axis is oriented in the direction of the buoyancy force and y'-axis is perpendicular to the plane of the limiting surface. Initially, the limiting surface is at rest but at $t>0$ it starts moving impulsively in its own plane with constant velocity U_0 and its temperature is instantaneously raised or lowered toT_w' which is thereafter maintained constant. Then under the usual Boussinesq's approximation the non-dimensional equations governing the problem are given by:

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$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\frac{1}{4}\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{1}{4}\frac{\partial U}{\partial t}+Gr\theta+\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\left(M+\frac{1}{K}\right)(u-U_0) \tag{2}$$

$$\frac{1}{4}\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{\partial p}{\partial y}+\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\left(M+\frac{1}{K}\right)v \tag{3}$$

$$\frac{1}{4}\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-\left(M+\frac{1}{K}\right)w \tag{4}$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t}+v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right) \tag{5}$$

The corresponding boundary conditions are given by:

$$\left.\begin{array}{l} y=0:\ u=1,\ v=-(1+\varepsilon\cos\pi z),\ w=0,\ \theta=-1 \\ y\to\infty:\ u=U(t),\ v=-1,\ w=0,\ p=p_\infty,\ \theta=0 \end{array}\right\} \tag{6}$$

When the amplitude ε<<1, researchers assume the solution in the neighborhood of the limiting surface of the form:

$$\left.\begin{array}{l} u=u_0+\varepsilon u_1+\varepsilon^2 u_2+\ldots \\ v=v_0+\varepsilon v_1+\varepsilon^2 v_2+\ldots \\ w=w_0+\varepsilon w_1+\varepsilon^2 w_2+\ldots \\ p=p_0+\varepsilon p_1+\varepsilon^2 p_2+\ldots \\ \theta=\theta_0+\varepsilon\theta_1+\varepsilon^2\theta_2+\ldots \end{array}\right\} \tag{7}$$

And for the free stream:

$$U=1+\varepsilon e^{i\omega t} \tag{8}$$

Substituting Eq. 7 and 8 in Eq. 1-5 and comparing the coefficient of like power of ε and neglecting those of ε^2. The terms free from ε given as describe a steady two dimensional problem with constant suction at the limiting surface:

$$v_0'=0 \tag{9}$$

$$u_0''-v_0u_0'-nu_0=-n-Gr\theta_0 \tag{10}$$

$$v_0''-v_0v_0'-nv_0=p_0' \tag{11}$$

$$w_0''-v_0w_0'-nw_0=0 \tag{12}$$

$$\theta_0''-Pr\,v_0\theta_0'=0 \tag{13}$$

Where primes denote differentiation with respect to y. The corresponding boundary conditions are:

$$\left.\begin{array}{l} y=0:\ u_0=1,\ v_0=-1,\ w_0=0,\ \theta_0=1 \\ y\to\infty:\ u_0=1,\ v_0=-1,\ w_0=0,\ p_0=p_\infty,\ \theta_0=0 \end{array}\right\} \tag{14}$$

The solution of Eq. 9-13 under the boundary conditions Eq. 14 are:

$$u_0=1+\beta_3\left[e^{-\beta_1 y}-e^{-Pr\,y}\right] \tag{15}$$

$$\theta_0=e^{-Pr\,y} \tag{16}$$

With transverse velocity components v_0 = -1, w_0 = 0 and the pressure p_0 = p_∞. Taking into account the solutions of the transverse velocity components of the above 2-dimensional problem, the terms the coefficient of ε give equations:

$$\frac{\partial v_1}{\partial y}+\frac{\partial w_1}{\partial z}=0 \tag{17}$$

$$\frac{1}{4}\frac{\partial u_1}{\partial t}+v_1\frac{\partial u_0}{\partial y}-\frac{\partial u_1}{\partial y}=\frac{i\omega}{4}e^{i\omega t}+Gr\theta_1+\left(\frac{\partial^2 u_1}{\partial y^2}+\frac{\partial^2 u_1}{\partial z^2}\right)-n\left(u_1-e^{i\omega t}\right) \tag{18}$$

$$\frac{1}{4}\frac{\partial v_1}{\partial t}-\frac{\partial v_1}{\partial y}=-\frac{\partial p_1}{\partial y}+\left(\frac{\partial^2 v_1}{\partial y^2}+\frac{\partial^2 v_1}{\partial z^2}\right)-nv_1 \tag{19}$$

$$\frac{1}{4}\frac{\partial w_1}{\partial t}-\frac{\partial w_1}{\partial y}=-\frac{\partial p_1}{\partial z}+\left(\frac{\partial^2 w_1}{\partial y^2}+\frac{\partial^2 w_1}{\partial z^2}\right)-nw_1 \tag{20}$$

$$\frac{1}{4}\frac{\partial \theta_1}{\partial t}+v_1\frac{\partial \theta_0}{\partial y}-\frac{\partial \theta_1}{\partial y}=\frac{1}{Pr}\left(\frac{\partial^2 \theta_1}{\partial y^2}+\frac{\partial^2 \theta_1}{\partial z^2}\right) \tag{21}$$

With boundary conditions:

$$\left.\begin{array}{l} y=0:\ u_1=0,\ v_1=-\cos\pi z,\ w_1=0,\ \theta_1=0 \\ y\to\infty:\ u_1=e^{i\omega t},\ v_1=0,\ w_1=0,\ p_1=0,\ \theta_1=0 \end{array}\right\} \quad (22)$$

Equation 17-21 describe the 3-dimensional flow. To solve these equations, researchers separate the variables y, z and t as:

$$\left.\begin{array}{l} u_1=u_{11}(y)e^{i\omega t}+u_{12}(y)\cos\pi z \\ v_1=v_{11}(y)e^{i\omega t}+v_{12}(y)\cos\pi z \end{array}\right\} \quad (23)$$

$$\left.\begin{array}{l} w_1=-\left(zv'_{11}(y)e^{i\omega t}+\dfrac{1}{\pi}v'_{12}(y)\sin\pi z\right) \\ p_1=p_{11}(y)e^{i\omega t}+p_{12}(y)\cos\pi z \\ \theta_1=\theta_{11}(y)e^{i\omega t}+\theta_{12}(y)\cos\pi z \end{array}\right\} \quad (24)$$

Expressions for v_1 and w_1 in Eq. 23 and 24 are chosen show that the equation of continuity, i.e., Eq. 17 is satisfied.

Substituting Eq. 23 and 24 in Eq. 18-21 and equating harmonic terms, researchers get the following equations with corresponding boundary conditions:

$$u''_{11}+u'_{11}-\left(\frac{i\omega}{4}+n\right)u_{11}=-\frac{i\omega}{4}-Gr\theta_{11}+v_{11}u'_0-n \quad (25)$$

$$u''_{12}+u'_{12}-\left(\pi^2+n\right)u_{12}=v_{12}u'_0-Gr\theta_{12} \quad (26)$$

$$\left.\begin{array}{l} y=0:\ u_{11}=0,\ u_{12}=0 \\ y\to\infty:\ u_{11}=1,\ u_{12}=0 \end{array}\right\} \quad (27)$$

$$v''_{11}+v'_{11}-\left(\frac{i\omega}{4}+n\right)v_{11}=p'_{11} \quad (28)$$

$$v'''_{11}+v''_{11}-\left(\frac{i\omega}{4}+n\right)v'_{11}=0 \quad (29)$$

$$v''_{12}+v'_{12}-\left(\pi^2+n\right)v_{12}=p'_{12} \quad (30)$$

$$v'''_{12}+v''_{12}-\left(\pi^2+n\right)v'_{12}=\pi^2p_{12} \quad (31)$$

$$\left.\begin{array}{l} y=0:\ v_{11}=0,\ v_{12}=-1,\ v''_{12}=0 \\ y\to\infty:\ v_{11}=0,\ v_{12}=0,\ p_{11}=0,\ p_{12}=0 \end{array}\right\} \quad (32)$$

$$\theta''_{11}+Pr\theta'_{11}-\frac{i\omega}{4}Pr\theta_{11}=Prv_{11}\theta'_0 \quad (33)$$

$$\theta''_{12}+Pr\theta'_{12}-\pi^2\theta_{12}=Prv_{12}\theta'_0 \quad (34)$$

$$\left.\begin{array}{l} y=0:\ \theta_{11}=0,\ \theta_{12}=0 \\ y\to\infty:\ \theta_{11}=0,\ \theta_{12}=0 \end{array}\right\} \quad (35)$$

From these equations with the help of Eq. 23 and 24, the solutions for u_1, v_1, w_1, p_1 and θ_1 are obtained as:

$$u_1=\left[1-e^{-N_1y}\right]e^{i\omega t}+\begin{bmatrix} \beta_{26}e^{-\beta_4y}+\beta_{21}e^{-\beta_{17}y}- \\ \beta_{22}e^{-\beta_{18}y}-\beta_{23}e^{-\beta_{19}y}+ \\ \beta_{24}e^{-\beta_{20}y}-\beta_{25}e^{-\beta_5y} \end{bmatrix}\cos\pi z \quad (36)$$

$$v_1=\left[n_1n_2e^{-\pi y}-n_4e^{-\beta_4y}\right]\cos\pi z \quad (37)$$

$$w_1=-\frac{1}{\pi}\left[\beta_4n_4e^{-\beta_4y}-\pi n_1n_2e^{-\pi y}\right]\sin\pi z \quad (38)$$

$$p_1=n_2e^{-\pi y}\cos\pi z \quad (39)$$

$$\theta_1=\left[\beta_9e^{-\beta_5y}-\beta_8e^{-\beta_{17}y}+\beta_7e^{-\beta_{18}y}\right]\cos\pi z \quad (40)$$

Substituting Eq. 15, 16 and 36-40 in expressions for u and θ in Eq. 7, researchers get the expression for the main flow velocity and temperature field as:

$$u(y,z,t)=u_0(y)+\varepsilon u_1(y,z,t) \quad (41)$$

$$\theta(y,z)=\theta_0(y)+\varepsilon\theta_1(y,z) \quad (42)$$

The main flow velocity can now be expressed in terms of the unsteady fluctuating parts as:

$$u(y,z,t)=u_0(y)+\varepsilon\left[U_r\cos\omega t-U_i\sin\omega t+u_{12}\cos\pi z\right] \quad (43)$$

Where:

$$U_r+iU_i=1-e^{-N_1y} \quad (44)$$

Hence, the expression for the transient velocity for $\omega t=\pi/2$ is given by:

$$u(y,z,\frac{\pi}{2\omega})=u_0(y)+\varepsilon\left[u_{12}\cos\pi z-e^{-A_2y}\sin B_2y\right] \quad (45)$$

From the velocity components u and w, researchers calculate the skin friction in the main flow direction and in the direction perpendicular to the main flow, respectively in the non-dimensional form as:

$$\tau_x = \frac{\tau'_x}{\rho U_0 v_0} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = u'_0 + \varepsilon u'_1 \quad \text{at} \quad y = 0 \qquad (46)$$

$$\tau_z = \frac{\tau'_z}{\rho v_0^2} = \left(\frac{\partial w}{\partial y}\right)_{y=0} = -\frac{\varepsilon}{\pi}\left[\pi^2 n_1 n_2 - \beta_4^2 n_3\right]\sin\pi z \qquad (47)$$

In terms of the amplitude and the phase of skin friction Eq. 46 can be written as:

$$\tau_x = \tau_{zx} + \varepsilon|m|\cos(\omega t + \alpha) \qquad (48)$$

Where:

$$\tau_{zx} = (\beta_{11} - \beta_{10}) + \varepsilon\left[\beta_{27}\cos\pi z - B_2\right]$$

$$m = m_r + i m_i, \quad \tan\alpha = \frac{m_i}{m_r}$$

From the expression for the temperature field, researchers can calculate q, the rate of heat transfer as:

$$q = -\frac{q'v}{v_0 k(T' - T'_\infty)} = \left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \theta'_0 + \varepsilon\theta_1 \quad \text{at } y = 0$$

$$q = -\Pr + \varepsilon\beta_{28}\cos\pi z$$

Where:

$$n = M + \frac{1}{K},\; n_1 = \frac{\pi}{(\pi + n)},\; n_2 = \frac{\beta_4^2}{n_1(\pi^2 - \beta_4^2)},\; n_3 = (1 + n_1 n_2)$$

$$n_4 = \frac{\Pr n_1 n_2}{\pi},\; N_1 = A_2 + iB_2,\; N = (\pi^2 + n),\; n_5 = \frac{\Pr^2(n_1 n_2 - 1)}{\beta_4^2 + \Pr\beta_4 - \pi^2}$$

$$\beta_1 = \frac{1 + \sqrt{1 + 4n}}{2},\; \beta_2 = \Pr^2 - \Pr - n,\; \beta_3 = \frac{Gr}{\beta_2},\; \beta_4 = \frac{1 + \sqrt{1 + 4N}}{2},\; \beta_5 = \frac{\Pr + \sqrt{\Pr^2 + 4\pi^2}}{2},\; \beta_6 = \beta_4^2 + \beta_4\Pr - \pi^2,$$

$$\beta_7 = \frac{\Pr^2 n_3}{\beta_6},\; \beta_8 = \frac{\Pr n_1 n_2}{\pi},\; \beta_9 = \beta_8 - \beta_7,\; \beta_{10} = \beta_1\beta_3,\; \beta_{11} = \beta_3\Pr,\; \beta_{12} = n_1 n_2\beta_{11} + Gr\beta_8,\; \beta_{13} = n_4\beta_{11} + Gr\beta_7,$$

$$\beta_{14} = n_1 n_2\beta_{10},\; \beta_{15} = n_4\beta_{10},\; \beta_{16} = Gr\beta_9,\; \beta_{17} = \Pr + \pi,\; \beta_{18} = \beta_4 + \Pr,\; \beta_{19} = \pi + \beta_1,\; \beta_{20} = \beta_1 + \beta_4,$$

$$\beta_{21} = \frac{\beta_{12}}{\beta_{17}^2 - \beta_{17} - N},\; \beta_{22} = \frac{\beta_{13}}{\beta_{18}^2 - \beta_{18} - N},\; \beta_{23} = \frac{\beta_{14}}{\beta_{19}^2 - \beta_{19} - N},\; \beta_{24} = \frac{\beta_{15}}{\beta_{20}^2 - \beta_{20} - N},\; \beta_{25} = \frac{\beta_{16}}{\beta_5^2 - \beta_5 - N},$$

$$\beta_{26} = \beta_{25} - \beta_{24} + \beta_{23} + \beta_{22} - \beta_{21},\; \beta_{27} = \beta_{19}\beta_{23} - \beta_{20}\beta_{24} + \beta_{25}\beta_5 - \beta_{26}\beta_4 - \beta_{21}\beta_{17} + \beta_{22}\beta_{18},$$

$$A_1 = \left[\frac{(1 + 4n) + \sqrt{(1 + 4n)^2 + \omega^2}}{2}\right]^{\frac{1}{2}},\; B_1 = \left[\frac{-(1 + 4n) + \sqrt{(1 + 4n)^2 + \omega^2}}{2}\right]^{\frac{1}{2}},\; A_2 = \frac{1 + A_1}{2},\; B_2 = \frac{B_1}{2}$$

RESULTS AND DISCUSSION

In order to get a physical insight into the problem and for the purpose of discussing the results obtained, numerical calculations have been carried out for the transient velocity, the temperature, the rate of heat transfer and the sinusoidal skin-friction. In this study, researchers study the effects of the Grashoff number (Gr), Prandtl number (Pr), Magnetic parameter (M) and porosity parameter (K).

The transient velocity profiles are plotted against y shown in the Fig. 1 when the suction effects is maximum (z = 0) for ω = 5, ε = 0.2, M = 0.2 and K = 2. It is observed that transient velocity increases sharply till y = 1.2, after it transient velocity decreases continuously with increasing in y^-. It is also observed that the transient velocity increases with increasing Grashoff number Gr but it decreases with increasing Prandtl number (Pr).

The transient velocity profiles are plotted against y shown in the Fig. 2 when the suction effects is maximum (z = 0) for ω = 5, ε = 0.2, Gr = 5 and Pr = 0.71. It is observed that transient velocity increases sharply till y = 1.2, after it transient velocity decreases continuously with increasing in y^-. It is also observed that the transient velocity decreases with increasing Magnetic parameter (M) but it increases with increasing porosity parameter K.

The temperature profiles are plotted against y shown in the Fig. 3 when the suction effects is maximum (z = 0) for ω = 5, ε = 0.2, M = 0.2, K = 2 and Gr = 5. It is observed that the temperature decreases with increasing Prandtl number (Pr). The rate of heat transfer is plotted against z shown in the Fig. 4 for different value of Pr. It is observed that the rate of heat transfer decreases with increasing Prandtl number (Pr).

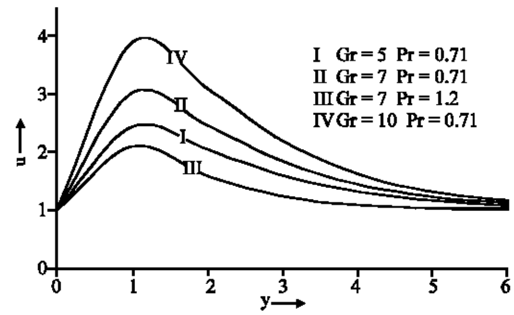

Fig. 1: Transient velocity profiles for different value of Pr and Gr

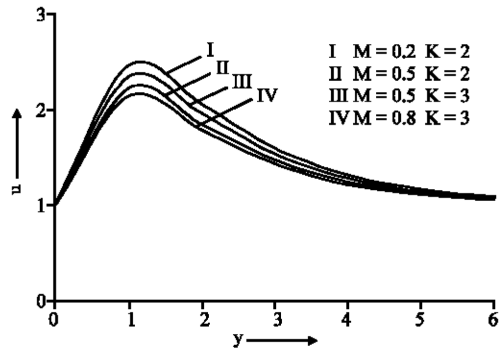

Fig. 2: Transient velocity for different value of M and K

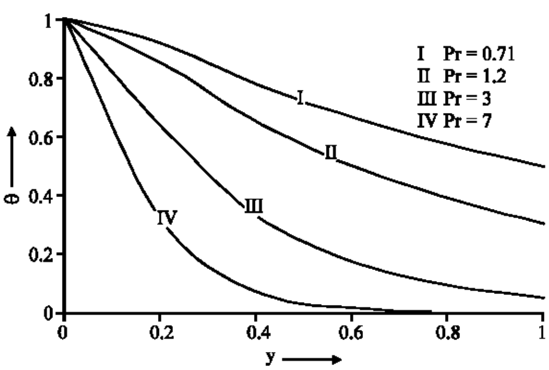

Fig. 3: Temperature profile for different value of Pr

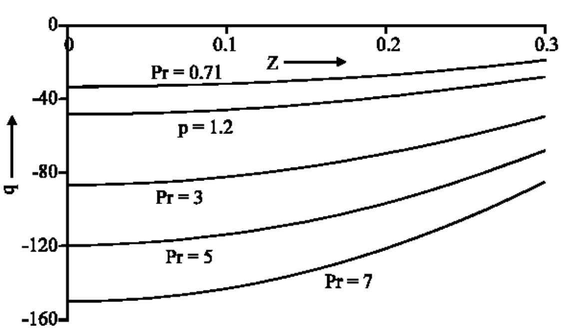

Fig. 4: Rate of heat transfer for different value of Pr

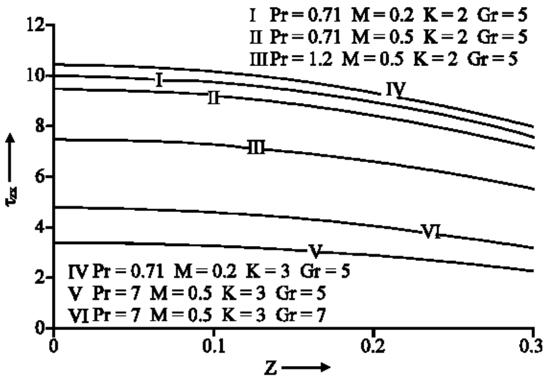

Fig. 5: Sinusoidal skin friction for different value of Pr, M, K and Gr

The sinusoidal skin friction is plotted against z shown in Fig. 5, for different value of Pr, M, K and Gr. It is observed that the sinusoidal skin friction decreases with increasing Prandtl number (Pr) and Magnetic parameter (M) but it increases with increasing Grashoff number (Gr) and porosity parameter (K).

CONCLUSION

The object of this study is to study the free convection effects on flow of a viscous, incompressible fluid past an impulsively started infinite, vertical porous limiting surface through porous medium with transverse sinusoidal suction in presence of uniform magnetic field.

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