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Linear and Nonlinear Precoding Techniques for MIMO Broadcast Channel

S. Sobana and K. Meena Alias Jeyanthi
Department of ECE, P.S.N.A. College of Engineering and Technology, Dindigul, Tamil Nadu, India

Abstract: Digital communication using Multiple-Input Multiple-Output (MIMO) has recently emerged, as one of the most significant technical breakthroughs in modern communications. The concept of MIMO systems is used extensively in currently deployed technologies to provide necessary data rates. In the multiuser, MIMO broadcast channel the benefits are enhanced when the transmitter knows the channel. Exploitation of such channel information allows for increasing the channel capacity, improving error performance while reducing hardware complexity. In order to detect the signal at the users terminals without any cooperation between them precoding techniques is required. This study describes the basics of various precoding techniques, examining both theoretical foundations and practical issues. Fundamental measures of the MIMO systems are the capacity and the error exponent, the practical measures are the Pair-wise Error Probability (PEP), Mean Square-Error (MSE), Symbol Error Rate (SER), Bit Error Rate (BER) and the received SNR. A brief comparison is done between the performances of the system with and without precoder, various linear and nonlinear precoding techniques by analyzing the ergodic capacity and BER.

Key words: Multiple-Input Multiple-Output (MIMO), Multiuser MIMO (MU-MIMO), precoding, capacity, channel state information, Bit Error Rate (BER)

INTRODUCTION

In recent wireless communication, MIMO have been used as a most promising technique which employs multiple antennas in the transmitter and/or receiver. The important aspect of the MIMO channel is to achieve correlation between transmit and receive antenna. It depends on the Angle-of-Arrival (AoA) of each multi-path component (Cho *et al*., 2010). MIMO techniques, also enhance link reliability and improve coverage (Vu and Paulraj, 2007). MIMO technology can provide multiplexing gain, if multiple independent data streams are sent simultaneously and in the same frequency band over multiple transmit antennas and recovered at the receiver by using appropriate signal processing techniques. Multi User-MIMO (MU-MIMO) offers a great degree of freedom than SU-MIMO in the spatial dimension because multiple users are multiplexed in the spatial channel. MU-MIMO is aimed at making improvements to the cell average spectral efficiency in a limited feedback environment, developing an effective precoding technique that supports Space Division Multiple Access (SDMA) and implementing with lower computational complexity (She and Chen, 2009). When the receiver knows the communication channels then the benefits of MIMO are realizable, it can be further enhanced if the transmitter also knows the channel. Hence, exploiting transmit channel side information is of great practical interest in MIMO. In this study, researchers assume full channel knowledge at the receiver and study how Channel Side Information at the Transmitter (CSIT) can be used to improve link performance. Precoding is a processing technique that exploits CSIT by operating on the signal before transmission. The CSI can be completely or partially known on the transmitter side. Sometimes, only statistical information on the channel state may be available. Exploitation of such channel information allows for increasing the channel capacity, improving error performance while reducing hardware complexity (Vu and Paulraj, 2007). In the MU-MIMO system, downlink and uplink channels are referred to as Broadcast Channel (BC) and Multiple Access Channel (MAC), respectively (Cho *et al*., 2010).

In BC, the Base Station (BS) is equipped with multiple antennas, simultaneously transmits data to multiple User Equipments (UEs) each with one or more antennas. The coordinated signal detection on the receiver side is not straightforward in BC to eradicate this difficulty interference cancellation at BS is required. The Multi User Interference (MUI), also called as Multiple Access Interference (MAI) can be suppressed by means of transmit beam forming (Khalid and Speidel, 2010). Figure 1 represents a multi-user system in which the

Corresponding Author: S. Sobana, Department of ECE, P.S.N.A. College of Engineering and Technology, Dindigul, Tamil Nadu, India

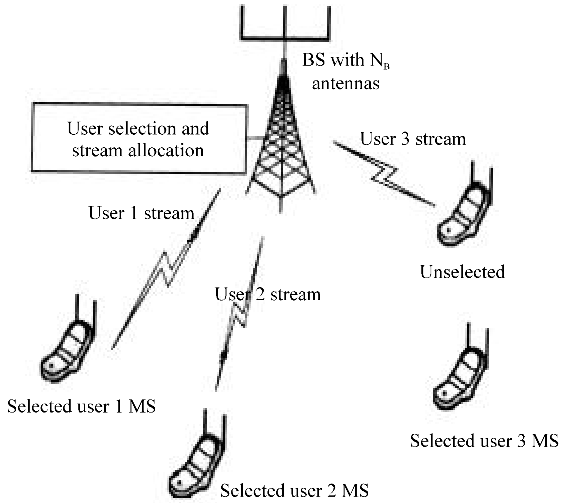

Fig. 1: MU-MIMO communication systems

multiple mobile stations are served by a single base station. CSIT feedback from each user is required for precoding because the type of precoding design depends on the type of CSIT. The performance of MIMO system can be improved in variety of ways with the help of precoding if full CSIT is available. By using CSI resources, such as power and data rates are optimally allocated during transmit and receive processing.

Precoding can also be used to maximize the diversity and coding gain. In this particular form, the linear constellation precoding technique does not even require the knowledge of channel information state. In this study, researchers consider various linear and nonlinear precoding techniques for MU-MIMO downlink systems. The 4 different transmission methods are Channel Inversion (CI), Block Diagonalization (BD), Dirty Paper Coding (DPC) and Tomlinson-Harashima Precoding (THP) (Khalid and Speidel, 2010).

Overview of precoding technique: To maximize the signal power at the receiver output in conventional single stream beam forming the same signal is emitted from each of the transmit antennas with appropriate weighting. Single stream beam forming may not be sufficient to maximize the signal level if the receiver has multiple antennas. It is necessary to use multi-stream beam forming to maximize the throughput in multiple receive antenna systems (Srinivasan, 2012). In precoding technique, the transmitter will get the information about the channel by using a feedback filter and sends the coded information to the receiver depending upon the pre-knowledge of the channel. The receiver is a simple detector, such as matched filter and does not have to know the channel side information. Figure 2 illustrates the block diagram of a precoder with feedback. For K users, the preprocessed transmit symbol is represented as $x = [x_1, x_2, ..., x_k]^T$ and it will pass through the channel H. The complex valued elements h_{kl} of the channel matrix H, describe the couplings between the transmission path l→k, i.e., the cross talk of the user I onto the user k. The ideal channel matrix H without couplings is a diagonal matrix, preferably an identity matrix. The channel matrix can be estimated by various known methods with back channel or incase of duplexing with time division multiplex and gaussian noise n. The received signal at receiver front-end will be, $y = xH+n$ and the elements of vector $y = [y_1, y_2, ..., y_k]^T$ are the receive symbols at the individual receivers (Stankovic and Haardt, 2006). The receiver is designed with known h and n. Similar to equalization precoding is also a preprocessing technique but the channel equalization aims to minimize channel errors, precoder aims to minimize the error in the receiver output. Once transmit channel information is obtained at the receiver this information is fed back to the transmit side and precoding matrix is obtained from transmit channel matrix. The precoder matrix can be a time varying matrix or it may be a time-invariant matrix.

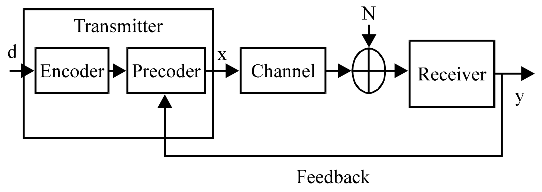

Fig. 2: Block diagram of precoder with feedback

Capacity of downlink channel: In this study, researchers discuss an achievable downlink channel capacity of a system by considering number of base station antennas (N_B) = 2, number of mobile station antennas (N_M) = 1 and number of independent users (K) = 2 (Costa, 1983). The received signal is generally expressed as:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_k \end{bmatrix} x + \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_k \end{bmatrix} \qquad (1)$$

Where, $x_u \varepsilon C^{N_B \times 1}$ is the transmitted signal from BS, $y_u \varepsilon C^{N_M \times 1}$ is the received signal from uth user, u = 1, 2, ..., K and $H_u \varepsilon C^{N_M \times N_B}$ is the channel gain between the BS and uth user. Figure 3 shows the downlink (broadcast) channel model of a MIMO system. The earlier equation is rewritten, as for the given specifications as follows:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} x + \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad (2)$$

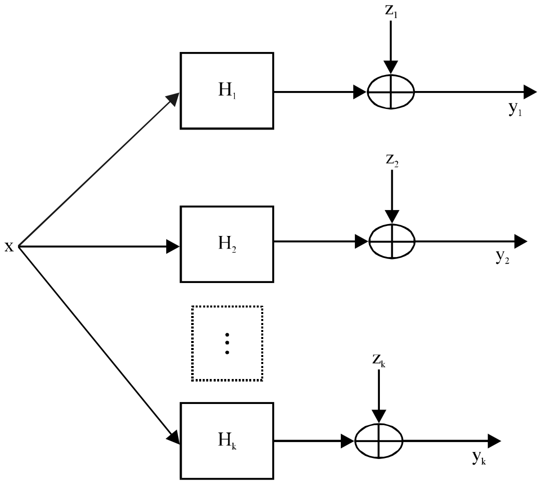

Fig. 3: Downlink model for MU-MIMO

If the channel information is completely available at BS, the overall channel can be decomposed as:

$$H = \begin{bmatrix} l_{11} & 0 \\ l_{21} & l_{22} \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \tag{3}$$

Where:

$l_{11} = \|H_1\|$
$q_1 = 1/l_{11}H_1$
$l_{21} = q_1.(H_2)^H$
$l_{22} = \|H_2 - l_{21}q_1\|$
$q_2 = 1/l_{22}(H_2 - l_{22}q_1)$

In Eq. 3 $\{q_i\}_{i=1}^{2}$ are the orthonormal row vectors. Based on the channel information the transmitted signal can be precoded as:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = Q^H \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 - \frac{1}{l_{22}} l_{21}\tilde{x}_1 \end{bmatrix} \tag{4}$$

Then, the received signal is given as:

$$y_{BC} = Hx + z = \begin{bmatrix} \|H_1\| & 0 \\ 0 & \|H_2 - l_{21}q_1\| \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + z \tag{5}$$

Assume that the total power P is divided into β P and (1-β) P for the 1st and 2nd users, respectively that is:

$$E\left\{|x_1|^2\right\} = E\left\{|x_1|^2\right\} = \beta P$$

And:

$$E\left\{|x_2|^2\right\} = E\left\{\left|x_2 - \frac{l_{21}}{l_{22}}x_1\right|^2\right\} = (1-\beta)P;\ \beta \in [0, 1]$$

Then, the capacities for the 1st and 2nd user are, respectively given as:

$$R_1 = \log\left(1 + \|H_1\|^2 \frac{\beta P}{\sigma_z^2}\right) \tag{6}$$

$$R_2 = \log_2\left(1 + \|H_2 - l_{21}q_1\|^2 \frac{(1-\beta)P}{\sigma_z^2}\right) \tag{7}$$

If $l_{21} = 0$ then the earlier equation can be rewritten as follows:

$$R_2 = \log_2\left(1 + \|H_2\|^2 \frac{(1-\beta)P}{\sigma_z^2}\right) \tag{8}$$

From Weingarten *et al.* (2006), the duality of the uplink and downlink channel capacities was used to derive the capacity of broadcast channel. In Aktas *et al.* (2004), it was shown that the sum rate capacity is proportional to min (N_B, K×N_M).

Channel State Information (CSI): In wireless communications, the known channel property of a communication link is referred, as Channel State Information (CSI). CSI describes how a signal propagates from the transmitter to the receiver and represents the combined effect of, for example scattering, fading and power decay with distance. In multi-antenna systems, it is crucial to achieve reliable communication with high data rates because the CSI adapt transmissions to current channel conditions. CSI needs to be estimated at the receiver and usually quantized and fed back to the transmitter. Therefore, the transmitter and receiver can have different CSI. The CSI at the transmitter and the CSI at the receiver are sometimes referred to as CSIT and CSIR, respectively. Basically, the levels of CSI are named, as instantaneous CSI and statistical CSI.

By knowing the impulse response of a digital filter the current channel conditions are known and this process is referred, as instantaneous CSI (or short-term CSI). This gives an opportunity to adapt the transmitted signal to the impulse response and thereby optimize the received signal for spatial multiplexing or to achieve low bit error rates. Statistical CSI (or long-term CSI) means that a statistical characterization of the channel is known. This description can include, for example the type of fading distribution, the average channel gain, the line-of-sight component and the spatial correlation. As with instantaneous CSI, this information can be used for transmission optimization. Practically, the CSI acquisition is limited by how fast the channel conditions are changing. Statistical CSI is reasonable in fast fading systems where channel conditions vary rapidly under the transmission of a single information symbol. On the other hand, instantaneous CSI is reasonable in slow fading systems where CSI can be estimated with reasonable accuracy and used for transmission adaptation for some time before being outdated. The available CSI often lies in between these 2 levels in practical systems instantaneous CSI with some estimation/quantization error is combined with statistical information.

Bit Error Rate (BER): The Bit Error Rate or Bit Error Ratio (BER) is defined, as the ratio of the number of error-bits received to the total number of transferred bits during a studied time interval. BER is a unit less performance measure; it is usually expressed as a percentage or negative power of ten. In digital transmission, it can be defined as the number of received bits of a data stream over communication channels that have been altered due to noise, interference or bit synchronization errors. The BER may be improved by choosing strong signal strength, a slow and robust modulation scheme or line coding scheme and by applying channel coding schemes, such as redundant forward error correction codes. The transmission BER is the number of detected bits that are incorrect before error correction, divided by the total number of transferred bits (including redundant error codes). BER in a noisy channel, is often expressed as a function of the normalized Signal-to-Noise Ratio (SNR) measure denoted E_b/N_0 (energy per bit to noise power spectral density ratio) or E_s/N_0 (energy per modulation symbol to noise spectral density). For example in the case of QPSK modulation and AWGN channel, the BER as function of E_b/N_0 is given by:

$$\mathrm{BER} = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

The functionality of a digital communication system is described by plotting BER vs. SNR (Proakis and Salehi, 2007).

MATERIALS AND METHODS

Linear precoding: In linear precoding, users are assigned different precoding matrices at the transmitter. The precoders are designed based on any number of designs including Zero-Forcing (ZF) and Minimum Mean Square Error (MMSE). From a practical point of view, the relevant criteria are error probability and sum rate, maximizing SINR, etc. (Gesbert *et al*., 2007).

In Zero-Forcing (ZF), the Self-Interference (SI) can be completely eliminated by restoring the orthogonality between the parallel data streams with an equalization coefficient $G_{1,1}$, l = 1, .., L:

$$G_{1,1} = \frac{H^*_{1,1}}{\left|H_{1,1}\right|^2} \tag{9}$$

Where, $H_{1,1}$, l = 1, …, L are the diagonal components August 11, 2014 of H. Based on the received sample values, a set of coefficients can be determined to force all but the center tap of the filtered response to 0. Hence for small amplitudes of $H_{1,1}$, the equalizer enhances the noise n_1 in such a way that the SNR may go to zero on some sub carriers. In ZF technique is the solution to the set of equations is reduced to a simple matrix inversion, this is the main advantage of this technique. The drawback of ZF is that the channel response may often exhibit attenuation at high frequencies, hence it applies high gain to these upper frequencies which tends to exaggerate noise (Smalley, 1994).

For Minimum Mean Square Error (MMSE), the diagonal elements of equalization coefficient G is:

$$G_{1,1} = \frac{H^*_{1,1}}{\left|H_{1,1}\right|^2 + \sigma^2} \tag{10}$$

The computation of G requires an estimate of the actual variance of the noise σ^2. The drawback of MMSE is due to the estimation of σ^2 the complexity is increased. To overcome this additional complexity, suboptimal MMSE equalization is introduced (Gesbert *et al*., 2007). In suboptimal MMSE equalization, the variance σ^2 is set equal to a threshold λ at which the optimal MMSE equalization guarantees the maximum acceptable BER. The equalization coefficient with suboptimal MMSE equalization results in new G:

$$G_{1,1} = \frac{H^*_{1,1}}{\left|H_{1,1}\right|^2 + \lambda} \tag{11}$$

and requires only information about $H_{1,1}$. The value λ has to be determined during the system design (Kaiser, 2002).

Channel Inversion (CI): It is a linear precoding technique for MU-MIMO downlink system where each UE is equipped with a single receive antenna. Figure 4 represents the structure of MIMO system with channel inversion. In Channel Inversion (CI) technique, the

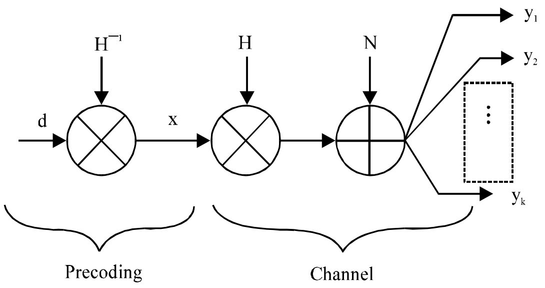

Fig. 4: MIMO system with channel inversion

inter-user interference cancelation is performed by pre multiplying the signal aimed at the terminals by a precoding matrix. The precoding matrix can be designed, so as to eliminate all the interference among the users streams. This technique is, also referred to as Zero-Forcing (ZF) precoding and utilizes the inverse of the channel matrix as the transmission filter. In MU-MIMO scenario, the process of channel inversion is same, as the ZF pre-equalization (Khalid and Speidel, 2010).

Assuming that the number of receive antenna must be less than or equal to number of transmit antenna $N_M \le N_B$. The $N_B \times 1$ transmitted signal vector is given by:

$$x = H^{+} \times d = H^{H}\left(HH^{H}\right)^{-1}.d \tag{12}$$

Where:

d = The data vector

H^+ = The pseudo inverse of the $N_M \times N_B$ channel matrix H

The ith column of the precoding matrix is given by:

$$P_{zf} = \frac{h_i^{i+j}}{\sqrt{\left\|h_i^{i+j}\right\|^2}} \tag{13}$$

Where, h_i^{i+j} is the ith column of H^+. For each user different SNRs can be obtained by weighting the columns of H^+, depending on their given rate requirement (Spencer *et al*., 2004a).

For $N_M = 1$ and $K = N_B$. Let $\tilde{x}_u \in c^{N_{M,u}\times 1}$ denote the uth user signal, $H_u = C^{1\times K}$ denotes the channel matrix between BS and the uth user, u = 1, 2 ... K. Then, the received signal $y_u \in c^{N_{M,u}\times 1}$ of all the users can be expressed as:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_k \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_k \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_k \end{bmatrix} \quad u = 1, 2...k \tag{14}$$

Channel inversion (ZF precoding) is a good solution for low-noise or high-power situations. The power of the precoded signal is increased significantly, if the channel is ill-conditioned which results in a poor Signal-to-Noise Ratio (SNR) at the receivers. One way to overcome, this noise is to allow for some interference among the users by regularizing the channel inverse. MMSE precoding also known, as Regularized Channel Inversion (RCI) provides a better alternative especially at low SNR. In this design, the precoding matrix following a wiener filter approach in such a way that the Mean Square-Error (MSE) is minimized. Having a lower MSE, this increases the performance in terms of Bit Error Rate (BER) performance. The transmitted signal vector for RCI is given by:

$$x = H^{H}\left(HH^{H} + \alpha I\right)^{-1} d \tag{15}$$

Where, α is the loading factor. For a MU-MIMO downlink system with total transmit power P_T and K simultaneous users, $\alpha = K/P_T$ maximizes the SNR at the receivers (Khalid and Speidel, 2010).

Block diagonalization: The channel inversion and regularized channel inversion, uses the concept of the multiple users with a single antenna where the signals other than target signal x_u is considered, as interference and are canceled from the received signal (y_u) using precoding. The same method can be related to multiple users, each with multiple antennas. Since, the inter-antenna interference in its own signal, as well as other user interference are mitigated in the channel inversion processes, noise enhancement becomes more severe from the perspective of the target user (Cho *et al*., 2010). Block Diagonalization (BD) method is more suitable in this situation. BD is a generalization of channel inversion method for multi-antenna user equipments. This method only cancels the interference from other user signals during the process of precoding. Then by using various signal detection methods, such as ZF signal detection or MMSE signal detection, the inter-antenna interference for each user can be canceled (Spencer *et al*., 2004b).

For BD, the total number of receive antenna (N_M) must be less than or equal to the number of transmit antennas (N_B), i.e., $N_M \le N_B$. For K simultaneous users each with N_M receive antennas for i = 1, 2, ..., K, the total number of receive antennas (Khalid and Speidel, 2010):

$$N_M = \sum_{i=1}^{K} N_{Mi}$$

Let, $N_{M,u}$ denote the number of antennas for uth user, u = 1, 2, ..., K. For the uth user signal ($\tilde{x}_u$) the received signal (y_u) is given as:

$$y_u = H_u \sum_{k=1}^{K} P_k \tilde{x}_k + z_u = H_u P_u \tilde{x}_u + \sum_{k=1, k\neq u}^{k} H_k P_k \tilde{x}_k + z_k \tag{16}$$

Where:

H_u = The channel matrix between BS and the uth user with dimension $N_{M,u} \times N_B$

$P_u \in C^{N_B \times N_{M,u}}$ = The precoding matrix of uth user

z_u = The noise vector

The interference-free transmission will be warranted, as long as the effective channel matrix can be block-diagonalized that is:

$$H_u Pk = 0_{N_{M,u}} \times N_{m,u} \forall \neq k \tag{17}$$

Channel matrix that contains the channel gains of all users except for the uth useris given as:

$$\tilde{H}_u = \left[H_1^T \cdots H_{i-1}^T H_{i+1}^T \cdots H_K^T\right]^T \tag{18}$$

The total numbers of antennas used by all active users are the same as the number of BS antennas. To design the precoders that satisfies Eq. 17. The dimension of the matrix $\tilde{H}_u \in C\left(N_{M,total} - N_{M,u}\right) \times N_B$ is less than min ($N_{M,\,total}$ - $N_{M,\,u}$, N_B). For $N_{M,\,total} = N_B$, min ($N_{M,\,total}$ - $N_{M,\,u}$, N_B) = N_B - $N_{M,\,u...}$. The unitary column vectors for the channel are obtained by the Singular Value Decomposition (SVD) of the $\tilde{H}_u$. It can be expressed as:

$$\tilde{H}_u = \tilde{U}_u \tilde{D}_u \left[\tilde{V}_u^{non\text{-}zero} \quad \tilde{V}_u^{zero}\right]^H \tag{19}$$

Where $\tilde{v}_u^{non-zero}$ and $\tilde{v}_u^{zero}$ correspond to non-zero and zero singular values, respectively. The following relationship is obtained by multiplying $\tilde{H}_u$ with $\tilde{v}_u$:

$$\tilde{H}_u \tilde{V}_u = \tilde{U}_u \left[\tilde{D}_u \, 0\right] \begin{bmatrix} \tilde{V}_u^{non\text{-}zero^H} \\ \tilde{V}_u^{zero^H} \end{bmatrix} \tilde{V}_u^{zero} = 0 \tag{20}$$

When a signal is transmitted in the direction of $\tilde{v}_u^0$ all but the uth user receives no signal at all. Thus, P_u = $\tilde{v}_u$ can be used for precoding the uth user signal. For $N_B = 4$, K = 2 and $N_{M,\,1} = N_{M,\,2} = 2$, the channel vectors are given as follows:

$$\begin{gathered} \tilde{H}_1 = \tilde{U}_1 \tilde{D}_1 \left[\tilde{V}_1^{non\text{-}zero} \quad \tilde{V}_1^{zero}\right]^H = \left[\tilde{U}_{11} \tilde{U}_{12}\right] \\ \begin{bmatrix} \tilde{\lambda}_{11} & 0 & 0 & 0 \\ 0 & \tilde{\lambda}_{12} & 0 & 0 \end{bmatrix} \left[\tilde{V}_{11} \tilde{V}_{12} \tilde{V}_{13} \tilde{V}_{14}\right]^H \end{gathered} \tag{21}$$

$$\begin{gathered} \tilde{H}_2 = \tilde{U}_2 \tilde{D}_2 \left[\tilde{V}_2^{non\text{-}zero} \quad \tilde{V}_2^{zero}\right]^H = \left[\tilde{U}_{21} \tilde{U}_{22}\right] \\ \begin{bmatrix} \tilde{\lambda}_{21} & 0 & 0 & 0 \\ 0 & \tilde{\lambda}_{22} & 0 & 0 \end{bmatrix} \left[\tilde{V}_{21} \tilde{V}_{22} \tilde{V}_{23} \tilde{V}_{24}\right]^H \end{gathered} \tag{22}$$

Then, the precoding matrix $P_u \epsilon C^{4\times 2}$ for u = 1, 2 is given as:

$$P_1 = \tilde{V}_1^{zero} = \left[\tilde{V}_{13} \tilde{V}_{14}\right]; P_2 = \tilde{V}_2^{zero} = \left[\tilde{V}_{23} \tilde{V}_{24}\right]$$

The transmitted signal is then given, as $X = P_1 \tilde{x}_1 + P_2 \tilde{x}_2$. The receiver signal of user 1 and 2 is obtained by using the fact that $H_1 = \tilde{H}_2$ and $H_2 = \tilde{H}_1$:

$$\begin{aligned} y_1 &= H_1 x + z_1 = H_1 \left(P_1 x_1 + P_2 x_2\right) + z_1 \\ &= H_1 \tilde{v}_1^{zero} \tilde{x}_1 + z_1 = H_2 \tilde{v}_1^{zero} \tilde{x}_1 + z_1 \end{aligned} \tag{23}$$

$$\begin{aligned} y_2 &= H_2 x + z_2 = H_2 \left(P_1 x_1 + P_2 x_2\right) + z_2 \\ &= H_2 \tilde{v}_2^{zero} \tilde{x}_2 + z_2 = H_1 \tilde{v}_2^{zero} \tilde{x}_2 + z_2 \end{aligned} \tag{24}$$

Equation 23 and 24 shows that the received signal is composed by desired signal only (Cho *et al*., 2010).

Nonlinear precoding: The precoding techniques can be classified depends upon the amount of the MUI they allow (as zero or non-zero MUI techniques) and their linearity (as linear and nonlinear techniques). The advantage of linear precoding technique is that they require no overhead to provide the demodulation information but they are less computationally expensive and less sensitive to the channel estimation errors at the transmitter than nonlinear counter parts. Since, linear precoding techniques typically suffer from more noise than nonlinear precoding and hence are not used in most wireless applications. Nonlinear precoding involves additional transmit signal processing to improve error rate performance. The precoders are designed jointly based on CSI of all the users, based on the equalization techniques such as Decision Feedback Equalization (DFE).

The basic idea of the DFE is to cancel the ISI contributed by the symbols by subtracting past symbol values with appropriate weighting from the equalizer output with the assumption that the values of the symbols already detected are correct. The feedback filter is used to reshape the received signal such that the ISI at the output of the feed forward filter is causal and the current pulse height is as high as possible with respect to the residual trailing ISI which is then subtracted in the feedback part without noise enhancement. If an incorrect feedback decision is taken, this will cause multiple errors following the first one. This is considered, as a major

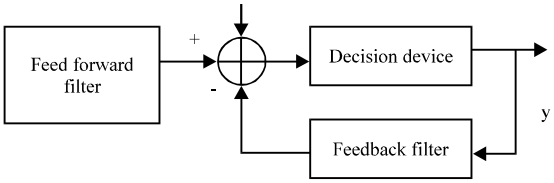

Fig. 5: Structure of DFE

disadvantage of the system and it is named as propagation error. Figuer 5 shows the structure of DFE.

Moreover, since the reliable detected decisions are only available after a delay, channel coding cannot be combined with DFE in a straightforward manner (Kuo *et al*., 2008).

Dirty Paper Coding (DPC): Dirty Paper Coding (DPC) is a promising new transmission technique that allows a base station to efficiently transmit data to multiple users at the same time. DPC allows a transmitter to send information, so that each user sees no interference from other users. It is a type of nonlinear precoding technique where the AWGN channel is modified by adding known interference at the transmitter side (Cho *et al*., 2010). In telecommunications, DPC is a technique for efficient transmission of digital data through a channel subjected to some interference known to the transmitter. By using dirty paper coding, researchers can increase the channel capacity without power penalty and without requiring the receiver to gain knowledge of the interference state. The technique consists of precoding the data in order to cancel the effect caused by the interference and it can be realized by subtracting the potential interferences before transmission. It should be implemented when channel gains are completely known on the transmitter side. That is, the interferences due to the first up to (k-1)th user signals are canceled in the course of precoding the kth user signal. Dirty paper coding is applied to eliminate the interference from the preceding users and this known interference (Cho *et al*., 2010). DPC techniques based on vector precoding is used to increase the sum capacity of the MU-MIMO downlink channel which is defined, as the maximum system throughput achieved by maximizing the sum of the information rates of all users. DPC on the transmitter side is very similar to Decision Feedback Equalization (DFE) on the receiver side (Khalid and Speidel, 2010).

In DPC vector precoding, the desired signal vector w is offset by a vector l of integer values and this operation is followed by channel inversion, resulting in the transmitted signal x, given by Khalid and Speidel (2010). The DPC technique is shown in Fig. 6:

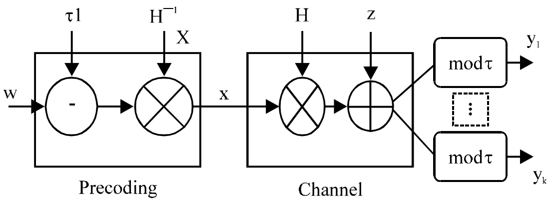

Fig. 6: Dirty paper coding

$$x = H^{-1}(w + \tau l) \tag{25}$$

Where, the vector l is chosen to minimize the power of x, i.e.,

$$I = \arg\min \left\| H^{-1}(w + \tau l') \right\| i \tag{26}$$

Then, the received signal is represented as:

$$y_k = w_k + \tau l_k + z_k \tag{27}$$

Considering $N_B = 3$, $K = 3$ and N_M; $u = 1$, $u = 1, 2, 3$, the uth user signal is given by $\tilde{x}_u = c$, then the received signal is given as (Cho *et al*., 2010):

$$\begin{aligned}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} &= \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \\ &= \begin{bmatrix} l_{11} & 0 & 0 \\ 0 & l_{22} & 0 \\ 0 & 0 & l_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}\end{aligned} \tag{28}$$

The main drawback, of the DPC technique is that it is a highly nonlinear technique which makes its implementation a very challenging problem which has yet to be implemented in practical systems (Jindal and Goldsmith, 2005). Other nonlinear transmission schemes try to reach the capacity bound set by DPC by using a simplified design.

Tomlinson-Harashima Precoding (THP): To reduce the Peak or Average Power (PAP) in the Decision Feedback Equalizer (DFE) which suffers from error propagation Tomlinson-Harashima Precoding (THP) was invented. THP is equivalent to the combination of DPC with symmetric modulo operation (Cho *et al*., 2010). Figure 7 represents the communication channel over AWGN channel. THP was designed to mitigate the effect of Inter Symbol Interference (ISI). By means of a feedback filter the interference between the streams is cancelled in a sequential fashion in THP. In this approach, a modul

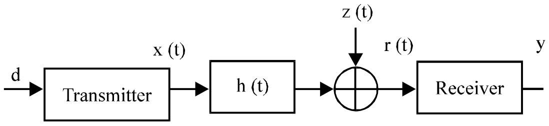

Fig. 7: Communication over AWGN channel

operation is inserted at both the transmitter and the receiver to reduce the unscaled transmit power. Since, the complete knowledge of the channel impulse response is only available by a feedback from the receiver for time-invariant channel. This idea is used to cancel the post-cursor ISI in the transmitter where the past transmit symbols are known without possibility of errors. TH precoding consider the precoding in the one dimensional case, hence the data symbol x is drawn from the M-ary PAM constellation:

$$\{-(A-1),-(A-3),\ldots,-3,-1,1,3,\ldots,(A-3),(A-1)\}$$

Where, A = $\sqrt{M}$. An expanded symbol c can be defined by adding 2 A•m to the data symbol x where m is an integer (Cho *et al.*, 2010). Where:

$$c = x + 2A \bullet m \tag{29}$$

The discrete time channel impulse response is given by hl. Let us assume without loss of generality, $h_0 = 1$. The channel sampled outputs are given by:

$$r_i = \sum_{l=0}^{L-1} h_l x_{i-l} \tag{30}$$

Researchers see that the symbols x_i, x_{i-1} . . . x_{i-L+1} will interfere with each other. If the transmitter knows the channel impulse response, the Inter Symbol Interference (ISI) effect can be overcome by a precoder with a transfer function equal to the inverse of the transfer function of the channel. When channel transfer function value is close to zero, the output of this precoder may diverge to infinity. To overcome this, a nonlinear block with transfer function T (D) is inserted before the h-1 (D) block, as shown in Fig. 8 (Kuo *et al.*, 2008).

In Eq. 29 m must be chosen to minimize the magnitude of the expanded symbol c in the transmitter which will reduce the peak or average power. From Cho *et al.* (2010), the original data symbol x recovered from the expanded symbol c by the symmetric modulo operation can be defined as:

$$x = \mathrm{mod}_A(C) \triangleq C - 2A\lfloor (C+2A)/2A \rfloor \tag{31}$$

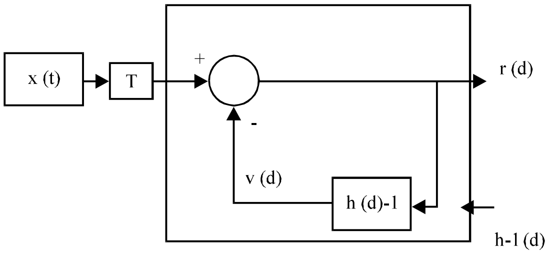

Fig. 8: Block diagram of Tomlinson-Harashima precoder

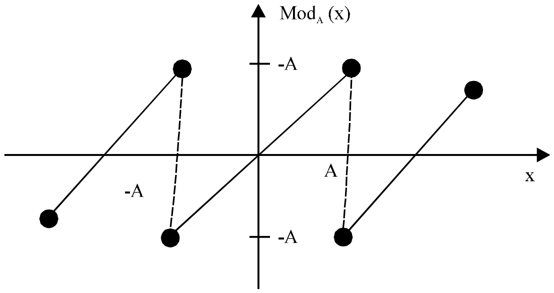

Fig. 9: Illustration of symmetric modulo operation (real part of x)

Where, symmetric modulo operation is defined as:

$$\mathrm{mod}_A(x) = x - 2A\lfloor x + A + jA)/2A \rfloor \tag{32}$$

The symmetric modulo operation is illustrated in Fig. 9. MMSE THP is an iterative precoding technique used to eliminate the MUI below the main diagonal of the equivalent combined channel matrix by combining MMSE precoding and THP. The users are 1st arranged according to some optimal ordering criterion and the precoding matrix P is calculated column by column starting from the last user K. The received signal for K = N_B = 3 is given as:

$$\begin{aligned}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} &= \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} Q^H x^{TH} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \\ &= \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1^{TH} \\ x_2^{TH} \\ x_3^{TH} \end{bmatrix} + \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}\end{aligned} \tag{33}$$

Where, $x_1^{TH} = \tilde{x}_1$. When the channel state information is available in the transmitter the Tomlinson-Harashima precoding can be regarded an attractive alternative to the ZF-DFE, it has the same performance for high SNR values over any ISI channel as ZF-DFE. THP can be viewed, as a DFE whose feedback section has moved to the transmitter. Since, the equalization is performed at the transmitter, the transmitting signals are perfectly known and the error propagation phenomenon never occurs, therefore it does not have the disadvantages of DFE. Channel coding can be combined easily with TH precoding because it does not require immediate decision (Kuo *et al*., 2008). The use of THP precoding results in improved diversity and substantial capacity gains, especially at low SNRs.

RESULTS AND DISCUSSION

In this study, researchers provide some numerical examples to illustrate the performance of the MIMO system and precoding techniques discussed in earlier studies. For the analysis of any system like SISO, MISO and MIMO the parameters, such as ergodic capacity, BER and sum-rate throughput are necessary. In this study, ergodic capacity and BER rate for a MIMO system with four users with and without precoding is analyzed and also compared the BER performance of linear and nonlinear precoding techniques.

Figure 10 compares the ergodic capacity of MIMO system with and without precoding. The ergodic capacity is plotted, as a function of the SNR in the independent rayleigh fading channel is shown in a downlink channel with 4 transmit and receive antennas. For SNR = 10 dB the ergadic capacity of a MIMO system without precoding is about 11 bps Hz^{-1} for a system with precoding 14 bps Hz^{-1} it shows that the MIMO system designed with precoder provides more capacity than the system without precoder.

Figure 11 presents the comparison of BER of a MIMO system with and without precoding. The BER is plotted, as a function of the SNR in the independent rayleigh fading channel with N_B and N_M = 4. At SNR = 10 dB, the BER of the MIMO system with precoder is approximately 0.002 and for the system without precoder is approximately 0.02. This shows that the BER of the MIMO system with precoder outperforms without precoding by approximately 10%.

Table 1 will give a brief idea about the overall performance the linear and nonlinear precoding techniques. Researchers considered the 4×4 MIMO system and the range of SNR is from 0-18 dB.

Finally in Fig. 12, researchers have compared all the linear and nonlinear precoding techniques, such as channel inversion, block diagonalization, dirty paper coding, Tomlinson-Harashima for a system with 4 transmit and receive antennas in which 4 users with the highest channel norm values are selected out of K = 20. As an example, if the SNR = 10 dB the BER for varies types of precoding techniques are obtained as follows for channel inversion BER = 0.13, regularized channel inversion

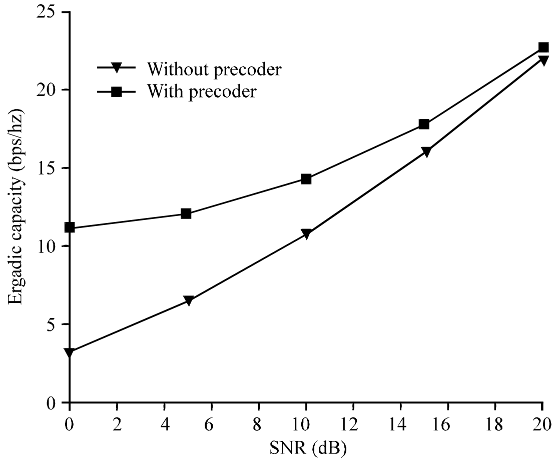

Fig. 10: Capacity comparison of MIMO system with and without precoding

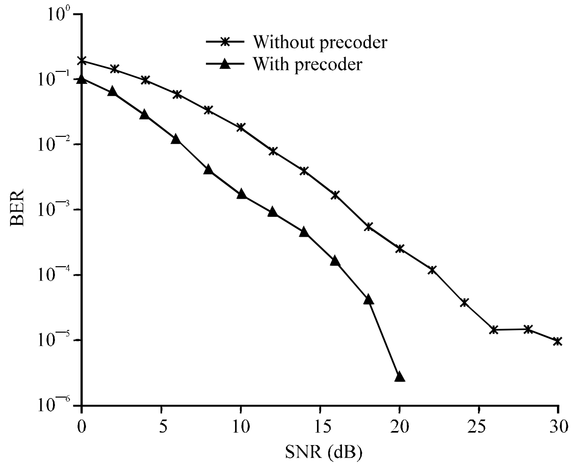

Fig. 11: Performance comparison of MIMO system with and without precoder

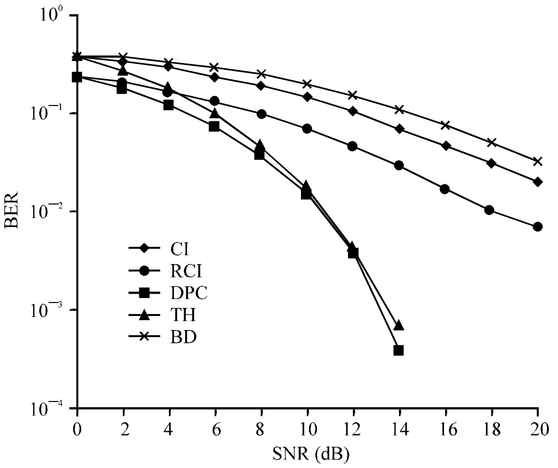

Fig. 12: BER performance comparison of different precoding techniques

Table 1: BER performance of precoding techniques

Precoding techniques/ SNR (dB)	BER Channel Inversion (CI)	Regularized Channel Inversion (RCI)	Block Diagonalization (BD)	Dirty Paper Coding (DPC)	Tomlinson-Harashima (TH)
0	0.353625	0.23425	0.387875	0.255188	0.39475
2	0.319688	0.196	0.356375	0.20475	0.319938
4	0.279125	0.1605	0.3235	0.154188	0.227938
6	0.2335	0.126188	0.283375	0.106188	0.149438
8	0.187375	0.095125	0.2455	0.062563	0.082625
10	0.13975	0.06675	0.199625	0.030938	0.038375
12	0.1	0.044625	0.15075	0.013125	0.0145
14	0.068	0.028938	0.114125	0.004313	0.0015
16	0.045625	0.017	0.078875	0.0015	0.0015
18	0.030313	0.010125	0.051875	0.00025	0.00025

BER = 0.06, block diagonalization BER = 0.19, dirty paper coding BER = 0.030, Tomlinson-Harashima BER = 0.038. From Fig. 12, researchers can understand that the nonlinear precoding techniques (DPC and TH) outperform the linear precoding techniques (RCI, CI and BD).

CONCLUSION

In this study, researchers have provided an overview of precoding techniques, such as linear and nonlinear precoding techniques for exploiting CSIT MIMO wireless systems. It discusses principles and methods for acquiring the CSIT, related issues and complexity. The evolution of MIMO system is reviewed along with its many advantages. Information theory concepts concerning capacity are presented with derivations for corresponding capacity of a MIMO system. For the MU-MIMO downlink, the nonlinear dirty paper coding techniques are capable of achieving the sum capacity of Gaussian multiuser channels with single-antenna user. Transmission schemes based on partial CSI which require minimal CSI feedback from the users represent the most suitable choice for practical implementation.

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