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Efficient Quadrature Solution for Composite Plate Problems

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Abstract: The efficiency of different moving least square differential quadrature techniques are examined for solving bending plate problems. Based on a transverse shear theory, the governing equations of the problem are derived. The transverse deflection and two rotations of the plate are independently obtained using moving least square approximations. The weighting coefficients for quadrature approximations are derived by three different techniques. For each one the accuracy and efficiency of the obtained results are examined. As well as the obtained results are compared with the previous analytical and numerical ones. Further, a parametric study is introduced to investigate the effects of elastic and geometric characteristics on the values of stress and transverse deflection of the plate.

Key words: Differential quadrature, irregular domains, composites, functionally graded, transverse shear theory

INTRODUCTION

Composite plates have been extensively preferred materials in marine, mechanical, civil, nuclear and aerospace structures. Bending analysis for such composites is one of the most important problems in structural design. Due to the mathematical complexity of such problems, only limited cases can be solved analytically (Wang *et al*., 2001; Zenkour, 2003; Timoshenko and Woinowsky-Krieger, 1959). Numerically, finite difference, finite element, point collocation, boundary element and discrete singular convolution methods have been widely applied for solving, such plate problems (Mukhopadhyay, 1989; Tu *et al*., 2010; Hrabok and Hrudey, 1984; Liew *et al*., 1998; Gupta *et al*., 1995; Tanaka *et al*., 1994; Tanaka and Bercin, 1998; Civalek, 2007, 2009).

The main disadvantage of such techniques is to require a large number of grid points as well as a large computre capacity to attain a considerable accuracy (Vanmaele *et al*., 2007; Belytschko *et al*., 1996a, b; Liew and Huang, 2003; Huang and Li, 2004; Jafari and Eftekhari, 2011; Wang and Wu, 2013).

In seeking, a more efficient technique that requires fewer grid points and achieves acceptable accuracy (Lanhe *et al*., 2007; Liew *et al*., 2002, 2004, 2003, 2011; Zhu *et al*., 2014; Zhang *et al*., 2014) introduced Moving Least Square Differential Quadrature Method (MLSDQM) for solving plate problems. This method own the capability to deal with discontinuty, composite plate, problems as well as irregular domains. Direct application of MLSDQM go through some computational complications with determination of partial derivatives of the field quantities (Bui *et al*., 2011; Ragb *et al*., 2014). Later, Wen and Aliabadi (2008, 2012) proposed two alternatives to overcome this difficulty. The first one indirectly evaluates second and higher order derivatives of the MLS shape functions at field points as much as first derivatives are obtained. The second proposal is to apply MLS approximations over a finite difference grid such that second and higher order derivatives, at the interior nodes can be approximated using central finite difference schemes.

The present research examins accuracy and efficiency of the earlier three mentioned MLSDQ techniques for bending problems of irregular composite plates. The composite is made of a Functionally Graded Material (FGM). The equilibrium equations are written according to a transvers shear theory. The weighting coefficients for the approximated field quantities over the entire domain are obtained using MLSDQ technique. The second and higher order derivatives are approximated using direct, indirect and finite difference schemes. For each selection, the accuracy and efficiency of the obtained results are examined. As well as the obtained results are compared with the previous analytical and numerical ones. Further,

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a parametric study is introduced to investigate the effects of elastic and geometric characteristic of the problem on the values of obtained results.

MATERIALS AND METHODS

Formulation of the problem: Consider a non-homogeneous composite consisting of an isotropic plate bonded (along x-axis) to another one made of a FGM. The elastic characteristics of the composite vary such that:

$$G^f = Ge^{\gamma y},\ E^f = Ee^{\gamma y},\ v^f = v \tag{1}$$

Where:

- G = Shear modulus of the isotropic plate
- E = Young's modulus of the isotropic plate
- v = Poisson's ratio of the isotropic plate
- G^f = Shear modulus of the FG plate
- E^f = Young's modulus of the FG plate
- v^f = Poisson's ratio of the FG plate
- γ = Constant characterizing the composite gradation

Assume that the composite is subjected to a pure bending due to a laterally distributed load q (x, y). Based on a first-order shear deformation theory, the equilibrium equations for such composite thin plate can be written as (Panc, 1975):

$$M_{ik,i} + M_{kj,j} = Q_k, Q_{i,i} + Q_{j,j} = -q, (i = x, j = y), (k = x, y) \tag{2}$$

Where:

- M_{ij} (i, j = x, y) = Bending and twisting moment resultants
- Q_i (i = x, y) = Shearing force resultants

The transverse deflection w (x, y) and the normal rotations φ_x (x, y), φ_y (x, y) are related to the moment and shear resultants through the following constitutive relations (Reddy, 1999):

$$\begin{aligned} M_{ii} &= -D\left(\phi_{i,i} + \nu\phi_{j,j}\right),\ M_{jj} = -D\left(\phi_{j,j} + \nu\phi_{i,i}\right), \\ M_{ij} &= -\frac{1-\nu}{2} D\left(\phi_{i,j} + \nu\phi_{j,i}\right), (i = x, j = y) \end{aligned} \tag{3}$$

$$Q_i = kGh\left(w_i - \varphi_i\right), (i = x, y) \tag{4}$$

where, $D = Eh^3/12\ (1\text{-}v^2)$ and h are the Flexural rigidity and the thickness of the plate. The k is the shear correction factor which is to be taken 5/6 (Reddy, 1997, 1999). On suitable substitution from Eq. 3 and 4 into Eq. 2, the equilibrium equations can be written as:

$$D\left(\begin{aligned} &\frac{\partial^2\phi_x}{\partial x^2} + \frac{(1-\nu)}{2}\frac{\partial^2\phi_x}{\partial y^2} + \frac{(1+\nu)}{2}\frac{\partial^2\varphi_y}{\partial x\partial y} + \\ &\frac{(1-\nu)}{2}\gamma\left[\frac{\partial\varphi_x}{\partial y} + \frac{\partial\varphi_y}{\partial x}\right] \end{aligned}\right) + kGh\left(\frac{\partial w}{\partial x} - \varphi_x\right) = 0 \tag{5}$$

$$D\left(\begin{aligned} &\frac{\partial^2\varphi_y}{\partial y^2} + \frac{(1-\nu)}{2}\frac{\partial^2\varphi_y}{\partial x^2} + \frac{(1+\nu)}{2}\frac{\partial^2\varphi_x}{\partial x\partial y} + \\ &\gamma\left[\frac{\partial\varphi_y}{\partial y} + \nu\frac{\partial\varphi_x}{\partial x}\right] \end{aligned}\right) + kGh\left(\frac{\partial w}{\partial y} - \varphi y\right) = 0 \tag{6}$$

$$kGh\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \gamma\frac{\partial w}{\partial y} - \frac{\partial\varphi_x}{\partial x} - \frac{\partial\varphi_y}{\partial y} - \gamma\varphi_y\right) + qe^{-\gamma y} = 0 \tag{7}$$

Where:

- $\gamma \neq 0$ = FG part
- $\gamma = 0$ = For isotropic one

According to the case of supporting, simply supported and clamped edge, the boundary conditions can be described as:

Simply supported:

$$\text{SS1 } w = 0, \left(\delta_{ik}M_i + (1-\delta_{ik})M_{ij}\right) = 0, (i = n, j = s, k = n, s) \tag{8}$$

$$\text{SS2 } w = 0, \left(\delta_{ik}\varphi_j + (1-\delta_{ik})M_i\right) = 0, (i = n, j = s, k = n, s) \tag{9}$$

Clamped:

$$w = 0, \left(\delta_{ik}\varphi_j + (1-\delta_{ik})\varphi_i\right) = 0, (i = n, j = s, k = n, s) \tag{10}$$

Where:

- n = The subscripts represent the normal and tangent
- s = The subscripts of the directions to the boundary edge
- M_n, M_{ns} and Q_n = Denote the normal bending moment, twisting moment and shear force on the plate edge
- φ_n and φ_s = The normal and tangent rotations about the plate edge

$$\delta_{ik} = \begin{cases} 1 & i = k \\ 0 & i \neq k \end{cases}$$

The continuity conditions (along the interface), must be also satisfied such that:

$$\begin{aligned} w(x,0^-) = w^f(x,0^+), M_{ij}(x,0^-) = M^f_{ij}(x,0^+), \\ (i = x, j = x,y) \end{aligned} \tag{11}$$

Which means that the deflection and moments (along the interface) of isotropic and FG plates must be equaled.

Further, the force resultants and the rotations on the edge can be expressed in terms of the basic unknowns n_x and n_y as follows (Reddy, 1999, 1997):

$$M_{nn} = n_x^2 M_{xx} + 2n_x n_y M_{xy} + n_y^2 M_{yy} \tag{12}$$

$$M_{ns} = (n_x^2 - n_y^2)M_{xy} + n_x n_y (M_{yy} - M_{xx}) \tag{13}$$

$$Q_n = n_x Q_x + n_y Q_y \tag{14}$$

$$\varphi_n = n_x \varphi_x + n_y \varphi_y \tag{15}$$

$$\varphi_s = n_x \varphi_y - n_y \varphi_x \tag{16}$$

where, n_x and n_y are the direction cosines at a point on the boundary edge.

Solution of the problem: The employed MLSDQ techniques can be summarized as follows: descretize the domain of the problem, S, into a finite number of nodes: $\{X_i = (x_i, y_i), i = 1, N\}$. Each node is associated with three nodal unknowns (w, n_x and n_y). The influence domain for each node is determined as shown in Fig. 1. Over each influence domain, (S_4, i = 1, N), the nodal unknowns can be approximated as Liew *et al*. (2002, 2003, 2004):

$$\rho(\mathbf{x}_i) = \rho(x_i, y_i) \cong \rho^h(x_i, y_i) = \sum_{j=1}^{n} \phi_j(x_i, y_i)\rho^j, \quad (\rho = w, \varphi_x, \varphi_y), (i = 1, N) \tag{17}$$

where, n is the number of nodes within the influence domain, S_4. The $D^h = \{w^h, n^h{}_x, n^h{}_y\}$ approximate values for nodal unknowns w, n_x and n_y, respectively. The N_j (x_i, y_i) is defined as the shape function of MLS approximation over the influence domain (S_4, i = 1, N).

The nodal parameters: $\{w^i, n^i{}_x, n^i{}_y\}$ are always not equal to the physical values$\{w(x_i, y_i), n_x(x_i, y_i), n_y(x_i, y_i)\}$, since the MLS shape functions N_j (x_i, y_i) do not satisfy the Kronecker delta condition generally. Apply the MLS technique to approximate $u^h(x)$ to u (x) for any x0S such as (Lancaster and Salkauskas, 1981; Breitkopf *et al*., 2000):

$$u^h(x) = \sum_{i=1}^{m} P_i(x)a_i(x) = P^T(x)a(x) \tag{18}$$

where, a (x) = $\{a_1(x), a_2(x), \ldots a_m(x)\}^T$ is a vector of unknown coefficients. The $P^T(x) = \{p_1(x), p_2(x), \ldots, p_m(x)\}$ is a complete set of monomial basis. The m is the number

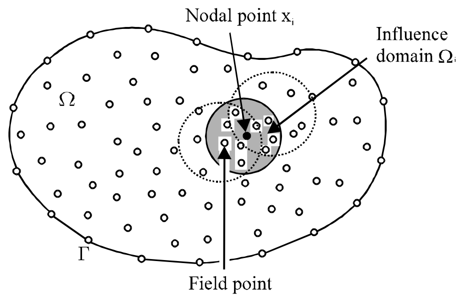

Fig. 1: Domain descretization for moving Least Squares Differential Quadrature Method

of basis terms. The coefficients a_j (x), (j = 1, m) can be obtained at any point x by minimizing the following weighted quadratic form:

$$\Pi(a) = \sum_{i=1}^{n} \varpi(x - x_i)(u^h(x_i) - u_i)^2 = \sum_{i=1}^{n} \varpi(x - x_i)(P^T(x_i)a(x) - u_i)^2 \tag{19}$$

Where:

n = Number of nodes in the neighborhood
x and u_i = Nodal parameter of u (x)

At point x_i§$_i$ (x) = § (x-x_i) is a positive weight function which decreases as $\|x - x_i\|$ increases. It always takes unit value at the sampling point x and vanishes outside the domain of influence of x.

The stationary value of A (a) with respect to a (x) leads to a linear relation between the coefficient vector a(x) and the vector of fictitious nodal values u such as:

$$A(x)a(x) = B(x)u \tag{20}$$

From which:

$$a(x) = A^{-1}(x)B(x)u \tag{21}$$

Where:

$$A(x) = P(x_i)\varpi_i(x)P^T(x_i) = \sum_{i=1}^{n} \varpi_i(x)P(x_i)P^T(x_i),$$

$$u = \begin{bmatrix} u_1 & u_2 & \cdots & u_n \end{bmatrix}^T,$$

$$B(x) = P(x)\varpi(x) = \begin{bmatrix} \varpi_1(x)P(x_1)\varpi_2(x) \\ P(x_2).....\varpi_n(x)P(x_n) \end{bmatrix}$$

On suitable substitution from Eq. 21 into Eq. 19, u^h (x) can then be expressed in terms of the shape functions as:

$$u^h(x) = P^T(x)a(x) = P^T(x)A^{-1}(x)B(x_i)u = \sum_{i=1}^{n} \phi_i(x)u_i \tag{22}$$

Where the nodal shape function:

$$\phi_i(x) = P^T(x)A^{-1}(x)B_i(x) \tag{23}$$

It should be noted that the MLS shape function and its derivatives are dependent on the weight function and the radius of influence domain. It's also required that n$m in the domain of influence so that the matrix A (x) in Eq. 22 can be inverted (Reddy, 1999, 1997). Determine the MLS shape functions $N_j(x_i, y_i)$ and its partial derivatives as proposed by Belytschko *et al*. (1996a, b) where:

$$\phi_i(x) = P^T(x)A^{-1}(x)B_i(x) = \alpha^T(x)B_i(x) \tag{24}$$

Since, A(x) is a symmetric matrix, then Eq. 24. For the partial derivatives of $N_j(x_i, y_i)$, (one can employ) yields:

$$A(x)\alpha(x) = P(x) \tag{25}$$

Therefore, the problem of determination of the shape function is reduced to solution of Eq. 25. This Equation can be solved using LU decomposition and back-substitution which requires fewer computations than the inversion of A(x). The following techniques:

Direct technique: The first and second order partial derivatives of $N_j(x_i, y_i)$ can be determined as in Liew *et al*. (2003): differentiate Eq. 26 with respect to I, J, (I, = x, y) such as:

$$A(x)\alpha_I(x) = P_I(x) - A_I(x)\alpha(x), (I = x, y) \tag{26}$$

$$\begin{aligned} A(x)\alpha_{IJ}(x) = P_{IJ}(x) - A_{IJ}(x)\alpha(x) - A_I(x)\alpha_J(x) - \\ A_J(x)\alpha_I(x),\ (I, J = x, y) \end{aligned} \tag{27}$$

The first and second order partial derivatives of the shape function can be described as:

$$\begin{aligned} \phi_{j,L}(x_i) = c_j^L(x_i) = \alpha_{j,L}^T(x_i)B_j(x_i) + \\ \alpha_j^T(x_i)B_{j,L}(x_i), (L = x, y) \end{aligned} \tag{28}$$

$$\begin{aligned} \phi_{j,LK}(x_i) = c_j^{LK}(x_i) = \alpha_{j,LK}^T(x_i)B_j(x_i) + \alpha_j^T(x_i)B_{j,LK}(x_i) + \\ \alpha_{j,L}^T(x_i)B_{j,K}(x_i) + \alpha_{j,K}^T(x_i)B_{j,L}(x_i),\ (L, K = x, y) \end{aligned} \tag{29}$$

Indirect technique: The first order partial derivatives of the shape function can be obtained as in Eq. 26 and 28, where:

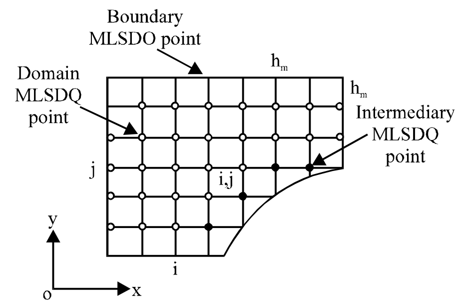

Fig. 2: Grid descretization for the hybrid technique consisting of FDM and MLSDQM

$$\rho_L(x) = c_j^L(x_i) = \sum_{i=1}^{n} \phi_{i,L}(x)\rho_i\ (L = x, y) \tag{30}$$

The second order partial derivatives can be determined, using matrix multiplication approach (Shu, 2000) such that:

$$\begin{aligned} \rho_{,LK}(x) = c_j^{LK}(x_i) = \sum_{i=1}^{n} \phi_{i,LK}(x)\rho_i = \\ \sum_{i=1}^{n} \phi_{i,K}(x_i) \sum_{j=1}^{n} \phi_{j,L}(x_j)\rho_j, (L, K = x, y) \end{aligned} \tag{31}$$

Finite difference technique: The first and second order partial derivative of the shape function can be approximated over a finite difference grid (Wen and Aliabadi, 2012) with mesh side h_m as shown in Fig. 2.

$$\begin{aligned} &\rho_{,L}(x_i, y_j) = \frac{1}{2h_m}(\rho^{i+1j} - \rho^{i-1j}), \rho_{,LL}(x_i, y_j) = \frac{1}{h_m^2}(\rho^{i+1j} - 2\rho^{ij} + \rho^{i-1j}), \\ &\rho_{,K}(x_i, y_j) = \frac{1}{2h_m}(\rho^{ij+1} - \rho^{ij-1}), \rho_{,KK}(x_i, y_j) = \frac{1}{h_m^2}(\rho^{ij+1} - 2\rho^{ij} + \rho^{ij-1}), \\ &\rho_{,LK}(x_i, y_j) = \frac{1}{4h_m^2}(\rho^{i+1j+1} - \rho^{i+1j-1} + \rho^{i-1j-1} - \rho^{i-1j+1}), \\ &\qquad (\rho = w, \varphi_x, \varphi_y), (L, K = x, y) \text{ and } (i, j = 1, N) \end{aligned} \tag{32}$$

A suitable linear interpolation must be applied to modify Eq. 32 for the intermediate points, (resulting from domain irregularities) as shown in Fig. 2.

On suitable substitution from Eq. 28-31 into the governing Eq. 5-7 the problem can be reduced to the following system of linear algebraic equations:

$$\sum_{j=1}^{n}\left[kGhc_{ij}^{x}w^{j} + D\left(c_{ij}^{xx} + \frac{1-\nu}{2}c_{ij}^{yy} + \frac{1-\nu}{2}\gamma c_{ij}^{y} - kGh\phi_{ij}\right)\varphi_x^j + D\left(\frac{1+\nu}{2}c_{ij}^{xy} + \frac{1-\nu}{2}\gamma c_{ij}^{x}\right)\varphi_y^j\right] = 0, (i = 1, N) \tag{33}$$

$$\sum_{j=1}^{n}\left[kGhc_{ij}^{y}w^{j} + D\left(\frac{1+\nu}{2}c_{ij}^{xy} + \nu\gamma c_{ij}^{x}\right)\varphi_x^j + D\left(c_{ij}^{yy} + \frac{1-\nu}{2}c_{ij}^{xx} + \gamma c_{ij}^{y} - kGh\phi_{ij}\right)\varphi_y^j\right] = 0, (i = 1, N) \tag{34}$$

$$\sum_{j=1}^{n} kGh\left[\left(c_{ij}^{xx}+c_{ij}^{yy}+\gamma c_{ij}^{y}\right)w^{j}-c_{ij}^{x}\varphi_{x}^{j}-\left(c_{ij}^{y}+\gamma c_{ij}\right)\varphi_{y}^{j}\right]=-qe^{(-\gamma y^{j})},(i=1,N) \tag{35}$$

Also on suitable substitution from Eq. 32 into the governing Eq. 5, 6, 7, the problem can be reduced to the following system of linear algebraic equations:

$$\begin{aligned}&2h_m kGh\left(w^{i+1j}-w^{i-1j}\right)+4D\left(\varphi_x^{i+1j}+\varphi_x^{i-1j}+\frac{1-\nu}{2}\left(\varphi_x^{ij+1}+\varphi_x^{ij-1}\right)-2\left(1+\frac{1-\nu}{2}\right)\varphi_x^{ij}+\frac{1-\nu}{2}\gamma\left(\varphi_x^{ij+1}-\varphi_x^{ij-1}\right)\right)\\&-4h_m^2 kGh\sum_{j=1}^{n}\phi_{ij}\varphi_x^{j}+D\frac{1+\nu}{2}\left(\varphi_y^{i+1j+1}-\varphi_y^{i+1j-1}+\varphi_y^{i-1j-1}-\varphi_y^{i-1j+1}\right)+D\frac{1-\nu}{2}\gamma\left(\varphi_y^{i+1j}-\varphi_y^{i-1j}\right)=0,(i=1,N)\end{aligned} \tag{36}$$

$$\begin{aligned}&2h_m kGh\left(w^{ij+1}-w^{ij-1}\right)+D\frac{1+\nu}{2}\left(\varphi_x^{i+1j+1}-\varphi_x^{i+1j-1}+\varphi_x^{i-1j-1}-\varphi_x^{i-1j+1}\right)+D\nu\gamma\left(\varphi_x^{i+1j}-\varphi_x^{i-1j}\right)-4h_m^2 kGh\sum_{j=1}^{n}\phi_{ij}\varphi_y^{j}+\\&4D\left(\varphi_y^{i+1j}+\varphi_y^{i-1j}+\frac{1-\nu}{2}\left(\varphi_y^{ij+1}+\varphi_y^{ij-1}\right)-2\left(1+\frac{1-\nu}{2}\right)\varphi_y^{ij}\right)+D\gamma\left(\varphi_y^{ij+1}-\varphi_y^{ij-1}\right)=0,\quad(i=1,N)\end{aligned} \tag{37}$$

$$\begin{aligned}&\left(w^{i+1j}+w^{i-1j}+w^{ij+1}+w^{ij-1}-4w^{ij}\right)+\gamma\left(w^{i+1j}-w^{i-1j}\right)-2h_m\left(\varphi_x^{i+1j}-\varphi_x^{i-1j}+\varphi_y^{ij+1}-\varphi_y^{ij-1}\right)-\\&2h_m\gamma\sum_{j=1}^{n}\phi_{ij}\varphi_y^{j}=-\frac{4h_m^2 qe^{(-\gamma y^{j})}}{kGh},\quad(i,j=1,N)\end{aligned} \tag{38}$$

The boundary conditions can also be reduced to the following linear algebraic equations.

Simply supported:

$$\begin{aligned}&SS1\, w^{i}=0,\ \sum_{j=1}^{n}\left[\left((n_x^2+\nu n_y^2)c_{ij}^{x}+(1-\nu)n_x n_y c_{ij}^{y}\right)\varphi_x^{j}+\left((\nu n_x^2+n_y^2)c_{ij}^{y}+(1-\nu)n_x n_y c_{ij}^{x}\right)\varphi_y^{j}\right]=0,\\&-\frac{1-\nu}{2}D\sum_{j=1}^{n}\left[\left((n_x^2-n_y^2)c_{ij}^{y}-2n_x n_y c_{ij}^{x}\right)\varphi_x^{j}+\left((n_x^2-n_y^2)c_{ij}^{x}+2n_x n_y c_{ij}^{y}\right)\varphi_y^{j}\right]=0,\quad(i=1,N)\end{aligned} \tag{39}$$

$$\begin{aligned}&SS2\, w^{i}=0,\sum_{j=1}^{n}\phi_{ij}\left(n_x\varphi_y^{j}-n_y\varphi_x^{j}\right)=0,\\&\sum_{j=1}^{n}\left[\left((n_x^2+\nu n_y^2)c_{ij}^{x}+(1-\nu)n_x n_y c_{ij}^{y}\right)\varphi_x^{j}+\left((\nu n_x^2+n_y^2)c_{ij}^{y}+(1-\nu)n_x n_y c_{ij}^{x}\right)\varphi_y^{j}\right]=0,(i=1,N)\end{aligned} \tag{40}$$

Clamped:

$$w^{i}=0,\quad \sum_{j=1}^{n}\phi_{ij}\left(n_x\varphi_y^{j}-n_y\varphi_x^{j}\right)=0,\quad \sum_{j=1}^{n}\phi_{ij}\left(n_x\varphi_x^{j}+n_y\varphi_y^{j}\right)=0,\quad(i=1,N) \tag{41}$$

Along, the interface the following algebraic equation must also be considered:

$$w(x_i,y_i)=w^{f}(x_i,y_i),M_{xx}(x_i,y_i)=M_{xx}^{f}(x_i,y_i),M_{xy}(x_i,y_i)=M_{xy}^{f}(x_i,y_i),(i=1,N_1) \tag{42}$$

Where, N_1 is the number of nodes along the interface.

RESULTS AND DISCUSSION

For the present results, Gaussian weight function with a circular influence domain is adopted for the MLS approximation such as:

$$w_i(x)=\begin{cases}\dfrac{\exp(-(d_i/c)^2)-\exp(-(r/c)^2)}{1-\exp(-(r/c)^2)} & d_i\le r\\ 0 & d_i>r\end{cases} \tag{43}$$

where, $d_i=\sqrt{(x-x_i)^2+(y-y_i)^2}$ is the distance from a nodal

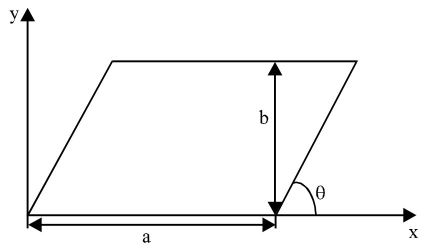

Fig. 3: Skew rhombic plate

x_i to a field x in the influence domain of x_i. The r is the radius of support domain and c is the dilation parameter In the present research, the dilation parameter is selected such as: c = r/4. Also, a scaling factor d_{max} is defined as:

$$d_{max} = \frac{r}{h_m} \tag{44}$$

where, h_m is the grid size which can be regarded as the distance between the nodal point x_i and the second nearest neighboring field nodes. For regular node arrangement spaced by $, h_m can be taken as:

$$h_m = \sqrt{2}\beta$$

For squared plates, the numerical results are normalized such as Liew *et al*. (2003):

$$\overline{w} = 100wD^s/(qa^4), \overline{M_{ij}} = 10M_{ij}/(qa^2), \bar{\sigma}_{ij} = \sigma_{ij}/E, (i, j = x, y) \tag{45}$$

As well as for skew rhombic plate shown in Fig. 3, the numerical results are normalized such as Liew *et al*. (2003):

$$\overline{w} = 1600wD^s/(qa^4), \overline{M_{ij}} = 40M_{ij}/(qa^2), \bar{\sigma}_{ij} = \sigma_{ij}/E, (i, j = x, y) \tag{46}$$

Where:

$\overline{w}, \bar{M}_{ij}, \bar{\sigma}_{ij}$ = Normalized deflection, moments and stresses
a = Composite width
D^s = Flexural rigidity of the isotropic plate

A numerical scheme is designed to investigate the influence of computational characteristics, (radius of support domain r, completeness order of the basis functions N_c and scaling factors d_{max}) on accuracy of the obtained results. The boundary conditions (28-31) are directly substituted into equilibrium ones (26 and 27). The reduced system is solved using MATLAB. The problem is solved over a regular grid with N_1 = (7, 25). For different boundary conditions, Table 1 shows that the results for N_1 = 11 are nearly the same, as those corresponding to N_1 = (13, 25). Therefore, the parametric study is introduced over grid 11×11 nodes.

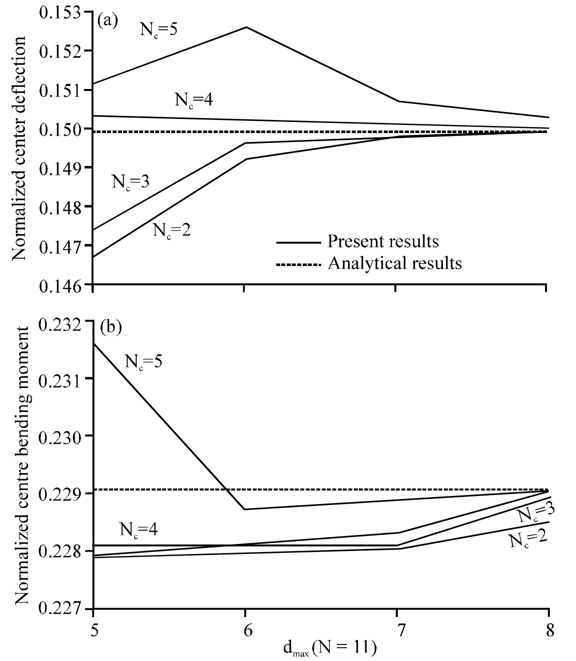

Fig. 4: Variation of the results with the radius of support domain and completeness order for a regular descretized clamped-clamped plate (N_1 = 11); (Redddy, 1999, 2002; Timoshenko and Woinowsky-Krieger, 1959)

Table 1: Comparison between the obtained bending moment and the previous analytical ones at the centre of a simply supported squared plate

No. of grid nodes	Exact*	Obtained results (Support radius) r = 5.5	r = 6	r = 6.5	r = 7
6×6	0.47886	0.51503	0.520080	0.509190	0.49623
11×11	0.47886	0.47767	0.479290	0.477135	0.47845
16×16	0.47886	0.47902	0.476785	0.478440	0.47819
21×21	0.47886	0.47784	0.478620	0.478940	0.47786

*Reddy (2002) and Timoshenko and Woinowsky-Krieger (1959)

Also, the completeness order N_c of the interpolation basis ranges from 2-7 with various scaling factors d_{max} from 2-10 as shown Fig. 4-6. It is found that d_{max}\$$N_c$+0.5 is required for reasonable numerical solutions. This was previously recorded by Liew *et al*. (2004). For irregular discretizations, the discrete nodes inside the plate are randomly generated while the boundary nodes are still equally spaced. For regular and irregular discretizations, Fig. 4-6 show that the accuracy of the obtained results increases with increasing both of N_c and the radius of support domain while it decreases with increasing of the grid size h_m for different values of plate thickness h. This result exactly agrees with that recorded in Liew and Han (1998).

For regular discretizations, Fig. 4 shows that one can select N_c = 4 and d_{max}\$5 to obtain accurate results.

Figure 5 and 6 show that for irregular discretizations, one can select $N_c \geq 5$ and $d_{max} \geq 7$ to obtain accurate results.

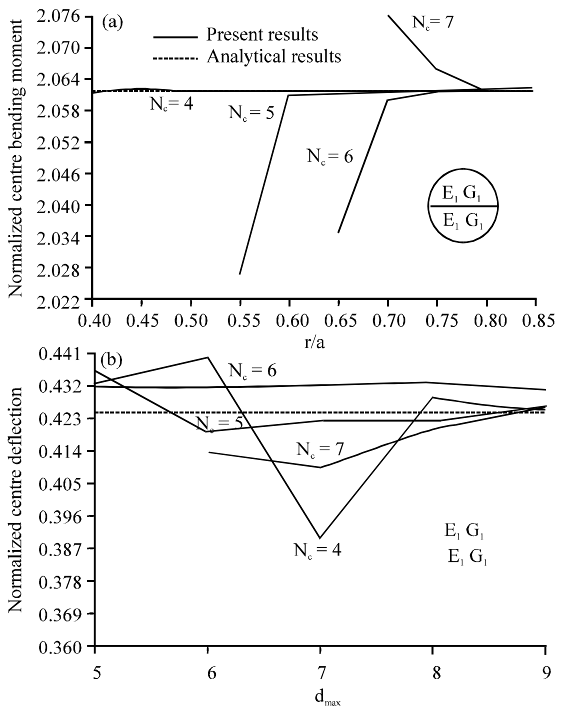

Fig. 5: Variation of the results with the radius of support domain and completeness order for an irregular descretization simply supported plate; (Redddy, 1999; Timoshenko and Woinowsky-Krieger, 1959)

To examine the validity of the obtained results, the bending problem of regular and irregular isotropic plates is solved and compared with the previous analytical ones by Timoshenko and Woinowsky-Krieger (1959); Reddy (1999); Liew and Han (1997) and Sengupta (1995). Table 2-5 show a very good agreement between the obtained results and the previous analytical solutions. For clamped plates, the error between obtained results and the previous exact ones by Timoshenko and Woinowsky-Krieger (1959) and Reddy (1999) is $\leq qa^4/D\ 10^{-7}$. As well as Table 3-5 show that indirect moving least squares differential quadrature method is more efficient and accurate than the other DQ techniques.

Further, a parametric study is derived to investigate behavior of the composite due to Young's modulus gradation ratio (E_2/E_1), shear modulus gradation ratio (G_2/G_1), Poisson's ratio (v_2/v_1), graduation factor (γ), stress distribution, σ_{yy}, aspect ratio a/b and the interface location (a_1/a_2) where a and b are the width and length of the rectangular plate $a = a_1+a_2$. Figure 7-8 and 9 shows

Table 2: Comparison between the obtained bending moment and the previous analytical ones at the centre of a clamped circular plate

		Obtained results			
Support size	Exact*	$N_c = 4$	$N_c = 5$	$N_c = 6$	$N_c = 7$
r = 0.7a	0.8125	0.8125	0.7301	0.8161	0.3536
r = 0.8a	0.8125	0.8125	0.8125	0.8124	0.8090
r = 0.9a	0.8125	0.8125	0.8125	0.8125	0.8111
r = a	0.8125	0.8125	0.8125	0.8125	0.8125
r = 1.1a	0.8125	0.8125	0.8125	0.8125	0.8125

*Reddy (1999) and Timoshenko and Woinowsky-Krieger (1959)

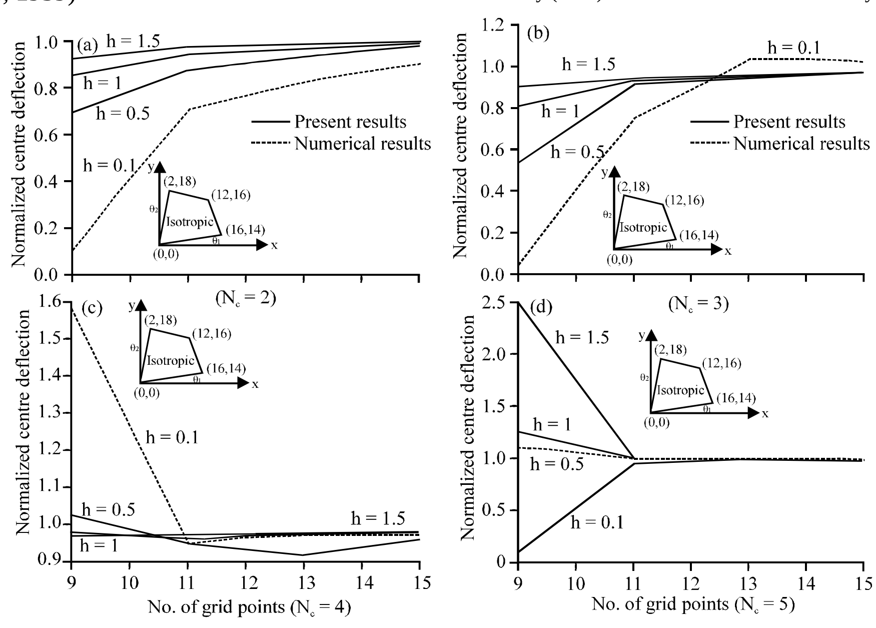

Fig. 6: Variation of the results with the completeness order N_c and the number of grid points N_1 for an irregular quadrilateral plate with various thickness h ($d_{max} = 8$) (Liew and Han, 1998)

Table 3: Comparison between the obtained deflection by different methods and the previous analytical ones for simply supported skew rhombic plates at the centre (2 = 45°)

	Exact		MLSDQM		MLSDQM+FDM		IMLSDQM	
Support size	1	2	$N_c = 4$	$N_c = 5$	$N_c = 4$	$N_c = 5$	$N_c = 4$	$N_c = 5$
r = 0.5a	2.1193	2.1285	1.4257	1.7483	1.42575	1.74834	1.42592	1.75012
r = 0.6a	2.1193	2.1285	1.9384	1.9394	1.93843	1.93945	1.93884	1.94226
r = 0.7a	2.1193	2.1285	2.0541	2.0109	2.05412	2.01095	2.05495	2.01134
r = 0.8a	2.1193	2.1285	2.0888	2.0354	2.08884	2.03544	2.08888	2.03753
Execution time (sec)			4.317165		3.497017		3.237787	

1 = Liew and Han (1997) and 2 = Sengupta (1995)

Table 4: Comparison between the obtained deflection by different methods and the previous analytical ones for clamped circular plate at the centre

Support size	Exact*	MLSDQM	MLSDQM + FDM	IMLSDQM
r = 0.7a	1.6339	1.392360	1.392440	1.395200
r = 0.8a	1.6339	1.633900	1.633900	1.633900
r = 0.9a	1.6339	1.633900	1.633900	1.633900
r = a	1.6339	1.633900	1.633900	1.633900
r = 1.1a	1.6339	1.633900	1.633900	1.633900
Execution time (sec)		4.265994	3.245480	3.050844

Table 5: Comparison between the obtained deflection by different methods and the previous analytical ones, (*qa^4/D) for clamped rectangular plates at the centre

	b/a						
W (0,0)	1	1.1	1.2	1.3	1.4	1.5	Execution time (sec)
Exact*	0.00126000	0.00150000	0.00172000	0.00191000	0.0020700	0.00220000	-
MLSDQM	0.00126100	0.00150040	0.00172140	0.00191160	0.0020692	0.00220250	3.576989
MLSDQM + FDM	0.00126054	0.00150028	0.00171960	0.00190970	0.0020702	0.00220030	2.529735
IMLSDQM	0.00126035	0.00150013	0.00171987	0.00190985	0.00207011	0.00220021	2.474488

	b/a					
W (0,0)	1.6	1.7	1.8	1.9	2	Execution time (sec)
Exact*	0.002300000	0.00238000	0.00245000	0.00249000	0.00254000	-
MLSDQM	0.002296000	0.00237800	0.00245202	0.00248900	0.00254104	3.576989
MLSDQM + FDM	0.002230100	0.00238030	0.00244950	0.00248970	0.00254078	2.529735
IMLSDQM	0.002230095	0.00238015	0.00244976	0.00248981	0.00254052	2.474488

*Reddy (1999) and Timoshenko and Woinowsky-Krieger (1959)

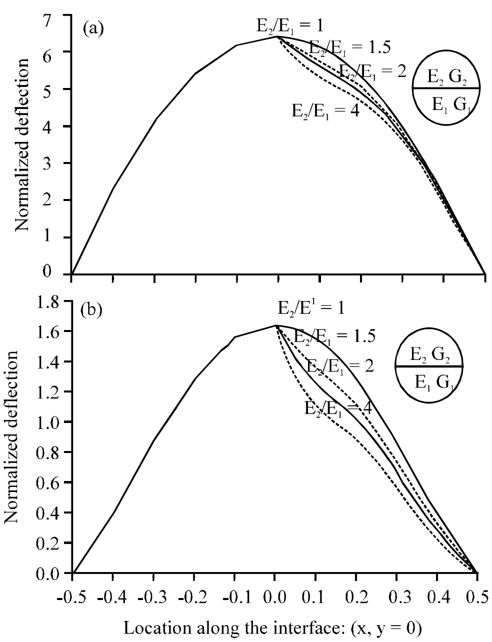

Fig. 7: Variation of the normalized deflection with Young's modulie for composite circular plates: a) Simply supported; b) Clamped-clamped ($G_2 = G_1$; $v_2 = v_1$)

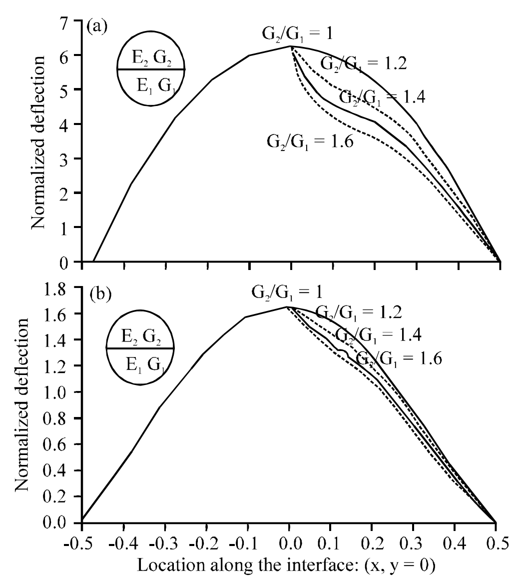

Fig. 8: Variation of the normalized deflection with shear modulie for composite circular plates: a) Simply supported; b) Clamped-clamped ($E_2 = E_1$; $v_2 = v_1$)

that the values of normalized deflection decrease with increasing the Young's modulus, the shear modulus and

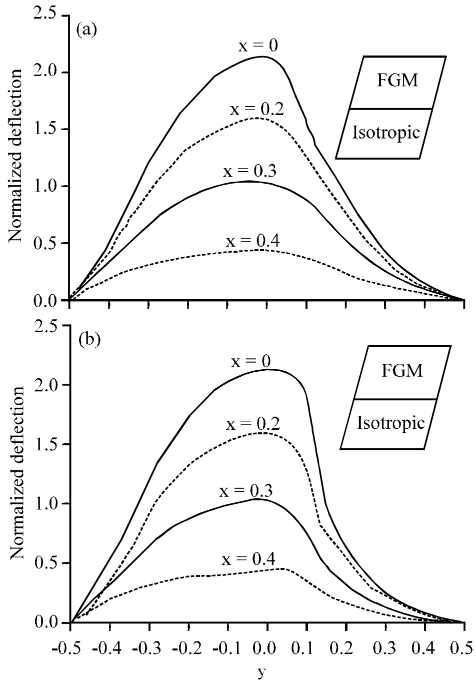

Fig. 9: Normalized deflection distribution through different locations for FG simply supported skew plate: a) (= 1; b) (= 10

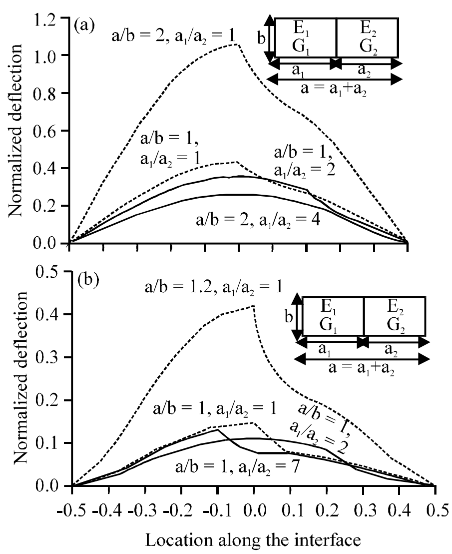

Fig. 10: Variation of the normalized deflection with aspect ratio for composite rectangular plates: a) Simply supported; b) Clamped-clamped (E_2/E_1 = 2; G_2 = G_1; v_2 = v_1)

the graduation factor (. While, these values increase with increasing of the aspect ratio (a/b) as shown in Fig. 10. Figure 11 show stress distribution, F_{yy}, through the

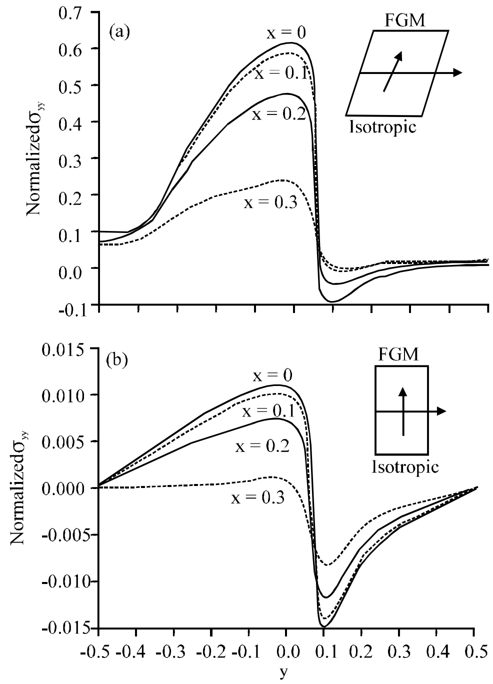

Fig. 11: Normalized stress distribution through different locations for FG; a) Simply supported; b) Clamped-clamped ((= 10, h = 0.1)

composite. This may be investigated through the stiffness concepts. Also, the computations declare that the results do not affect significantly by the Poisson's ratio v_1, v_2. Further, Fig. 9 insist the advantages of FG composites that treat the interfacial discontinuity problems.

CONCLUSION

The efficiency and accuracy of moving least square differential quadrature techniques, are examined for solving composite plate problems. All of these methods lead to accurate results but indirect technique gives more the efficiency and accuracy. Also, a parametric study is introduced to investigate the effects of computational, geometric and elastic characteristics of the problem on the values of the obtained results. Further, this research can be considered as an extension for quadrature solutions of composite plate problems.

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