

Hydrodynamic Characterisation of Soils in the Estuaries Coast of Cameroon-Gulf of Guinea

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Abstract: The numerical study of flow in the coastal zones of Cameroon estuary has retained the attention in this research. The soils of this zone are porous. The study has taken in account the Richards model to describe the fluid flow in these porous areas. Researchers have associated to this model that of Brooks and Corey, so as to determine the hydrodynamic parameters for the different types of soils studied. Researchers have resolved Richards equation which is strongly non-linear by taking into account Boussinesq's approximations. Knowledge of initial conditions and the definition of convergence conditions linked to the time scale are indispensable to obtain the objectives. The results obtained define the hydraulic and the hydrodynamic characteristics of soils at the estuaries coast of Cameroon, putting in evidence the parabolic character of the model studied and the presence of capillarity forces in such areas. The curves obtained have permitted to realize the pertinence of these results. It comes out that the pressure gradient and the soil texture of the area studied are indispensable elements in the study of drainage and the infiltration of fluids in porous areas.

Key words: Soils hydrodynamic, estuary of Cameroon, porous areas, gulf of Guinea, modelisation

INTRODUCTION

The determination of hydrodynamics properties of soil of a region is preliminary for the study of fluids behavior. In the aim to master the hydraulics parameters of the soils, many numerical and experimental studies have been done by the technicians of fluids by hydrologists and hydro geologist. Despite these studies, the hydrodynamic behavior of fluids in porous area remains an interesting general study for the environmental sciences, the civil engineering and technical extractions. The gulf of Guinea is a zone where the knowledge of texture and hydrodynamics parameters of the soils is not yet explored or the least remaining is little explored. Many researchers have been using a description in the dimension of Darcy I 2D to determine the dependence of parameters of saturation (water content, hydraulic conductivity) and relative permeability. Hassanizadeh and Gray (1987) have introduced parameters like capillary pressure and the pressure gradient of necessity for the closure system. Principally, the relative set-ups, such as capillarity, the pressure gradient and permeability, at saturation must satisfy the hysteresis phenomenon (Killough, 1976). Manzini and Ferraris (2004) did an

extension on the relation between the capillary pressure and the saturation then they imposed the term which put in evidence the dynamics effects through the pores. At the continuation of the experiences, a description is done on the dynamics coefficients of forces of capillaries pressure. Tomoh and Maslouhi have used the disk infiltrometer, as characterization method of hydrodynamics parameters, through the transitory system of infiltration where by the importance lie on the diminution of time intervals. This study has given rise to the characterization of function $K(h)$ and $\theta(h)$, from 6 soils grouped in 2 geomorphologic different domains. Stauffer (1978), Hassanizadeh and Gray (1987), Di Carlo (2004, 2005), Bottero *et al.* (2006), Mirzaei and Das (2007) and Manthey *et al.* (2005) have introduced some temperature gradients and pressure gradients in the model of Darcy in the case of permanent flows in order to put the effect of dynamics pressure gradients in evidence. These have also been taken into considerations by Barenblatt *et al.* (1990) and their results are similar. The same results have been obtained by Silim and Patzek (2004). Bottero *et al.* (2011) have determined the maximal capillary pressure from some local values and calculated the average saturation from a rigorous method of

calculations. The coastal region of gulf of Guinea and more precisely the cost of Cameroon estuary, presented a great interest due to the discovery of many minerals deposits and gases. On the occasion of the study, researchers are going to load the numerical modelisation by taking into consideration a small space and time to determine the hydrodynamics parameters of the soils of the estuary and establish the influence of forces of capillary pressure on the behavior of fluids in such areas. To arrive the closure of the system takes into considerations, the forces of capillary pressure and the hydraulic conductivity. Thus, researchers leave from Darcys equation and from the mass conservation to the equation of Richards which govern the flow of fluid in porous areas which are homogenous and heterogeneous. This equation (EDP) not being a straight line, researchers have used constitutive relation proposed by Brooks and Corey. Researchers have then transformed Richards line straight by taking into consideration, the approximations of Boussinesq which correspond to the base difference of flow on the total flow which give the disrupt flow. This is the linear form of Richards equation which researchers have used to determine the hydrodynamics parameters of estuaries soils of Cameroon.

MATERIAL AND METHODS

The model equation of Darcy developed on the base of experiences of the flow meter done on sandy areas can be written on this form:

$$\bar{q}_\alpha = \epsilon_\alpha \bar{V}_\alpha = -\frac{1}{\eta_\alpha} \bar{k} \cdot (\bar{\nabla} P_\alpha - P_\alpha \bar{g}) = -\frac{P_\alpha \bar{g}}{\eta_\alpha} \bar{k} \cdot \bar{\nabla} H_\alpha \quad (1)$$

The pressures are dimensioned by $(\rho_\alpha g)$ in order to be measured in meter of water column: $h_\alpha = P_\alpha / (\rho_\alpha g)$. The hydraulic charge is expressed in the same unit: $H = h_\alpha + Z$ and the velocity units derived directly from the gradient of that charge.

From a dimensional study, researchers simply show that the permeability evolve in function of the square of the dimension characteristic of the area. Else where the evolution of the dependence with the porosity is more delicate. This is naturally very sensible to the geometry of the porous area, the more the solids matrix blocks the passage of the fluid and the more the permeability becomes weak. Researchers restrict the study in this case to a periodic network cylinders, the simulation is limited to a 2D section area. Taking into consideration the linearity of the problem, all the solutions can be decomposed following the 2 principal directions.

When the flows are fast and that the inertia forces become dominant to the pore scale ($Re \geq 1$), the law of

Darcy must be complete. A dimensional analysis of Navier-Stockes equation gives an average dependant pressure gradient with the velocity and the square of the velocity. The model of Forchheimer take into account of these phenomenons in the first order by introducing a new equation partially derived relating the charge and the flow:

$$\tau \frac{\partial q_{\alpha,x}}{\partial t} + q_{\alpha,x} + \beta \rho_\alpha q_{\alpha,x}^2 = -\frac{\rho_\alpha g}{\eta_\alpha} K_x^s \frac{\partial H_\alpha}{\partial x} \quad (2)$$

When the number of Reynolds becomes bigger, the loss in charge induced by the recirculation gives rise to a greater apparent permeability. The convergent of this model to that of Darcy is assured because for small velocities, τ and β become negligible.

The equation of the conservation of mass on a big volume of porous area links the mass variation to the divergent of flux given by the law of Darcy. When the compressibility of the fluid is negligible, $d_\alpha/dP_\alpha = 0$ (for water) and that the mechanical effects bind to the compressibility of rock are negligible, the equation can be written under a zero divergent at the velocity of Darcy:

$$\left\{ \begin{array}{l} \frac{\partial}{\partial t} (P_\alpha \epsilon_\alpha) + \bar{\nabla} \cdot (P_\alpha \bar{q}_\alpha) = 0 \\ \bar{\nabla} \cdot (\bar{K} \cdot \bar{\nabla} H_\alpha) = 0 \end{array} \right\} \quad (3)$$

This last equation presents itself the same, as the equation of diffusion of an unknown scale. The principal problem lies, then in the strong anisotropic and heterogeneity tensor of permeability. Many numerical developments have seen the day these last years in order to treat this case, like the equation of Richards.

The equation of Richards which governs the flows in porous areas is written, as such:

$$\partial_t [\theta(\psi)] - \bar{\nabla} \cdot (K(\psi) \bar{\nabla} (\psi + z)) = 0 \quad (4)$$

If researchers, suppose that the tenure in water is a derived function of the hydraulic charge, it will be possible to rewrite the temporal equation written before by introducing the capillary capacity C which is the tenure derived in water with respect to the charge. The equation of Richards then becomes:

$$C(\psi) \partial_t \psi - \bar{\nabla} \cdot (K(\psi) \bar{\nabla} (\psi + z)) = 0 \quad (5)$$

This can equally have an unknown, such as the tenure in water. This last formulation makes the

intervention of the hydraulic diffusivity D_i which is the relation between the hydraulic conductivity and the capillary capacity:

$$\partial_t \theta - \nabla \cdot (D_i(\theta) \nabla \theta) - \nabla \cdot (K(\theta) \nabla z) = 0 \quad (6)$$

Researchers are going to solve numerically these equations of flow in porous areas being saturated and no saturated because such equations do not have analytical solutions. This phase will help us to explore some tracks which will enable us to frank the utilization of such a model, influenced by different factors: Pressure gradient (h), tenure in water (θ) and the component (θ) h or h (θ). Equation 4-6 are rigorously equivalent in a mathematical point of view. This equations being equivalent, numerous physical considerations bind to the nature of soil giving to each of it a different numerical approach. Ginting (2004), present the advantages and the disadvantages to be used in these lost 2 form.

The model of Brooks and Corey define the expressions which describe the evolution in function of C , S_e , K , θ and the pressure gradient H. These expressions needs the knowledge of values like θ_s et θ_r which are the tenure in water at saturation and the tenure in water of residual areas, respectively. The numbers without dimensions (a, m, n and l) are the area characteristics. This approach equally takes into consideration the pressure of air in saturated soils ($H_p \geq -1/\alpha$) and the unsaturated soils ($H_p < -1/\alpha$). To obtain the unique or only solution to the problem, it is important to define the initial conditions and the limited conditions. For this, researchers suppose initially that the pressure gradient on all the first line/surface of the stitch 2D is uniform. The limited conditions are fixed, thus presented in Table 1.

This model allows the efficient utilization of limited conditions and the soil parameters to calculate and verify the effective saturation in order to do a comparative study on graph obtained. The parameters values of form and the limited conditions proposed by the model of Brook and Corey are noted in Table 2.

The general form of the equation of Richards submitted to the study is written as:

$$C(H_p) \frac{\partial H_p}{\partial t} = \nabla \cdot [K(H_p) \nabla (H_p + Z)] \quad (7)$$

This being non linear, it is judicious to put in a linear form before the process of discretisation.

In an equilibril situation where the initial hydraulic charge is H_0 after a small potential variation of the charge due to a small variation of the water column, the coefficients (C (H_p) et K (H_p)) are written as:

Table 1: Limited conditions chosen for the simulation in infiltration

Parameters	Values
Height of column (cm)	15, 30, 45
Homogenous soils	Argile, limon
Variable initial conditions	HBS = -0.06 cm
Lower limited conditions	HBI = -40 cm
Period of simulation (h)	2, 4
Stitch heights tested (Δz)	0.0025 (ref.) and 0.005
Time scale (Δt)	0.01 sec

Table 2: Parameters of form proposed by the model of Brooks and Corey

Parameters	Values
g_r	9.2
ρ_f	10^3
X_p	10^{-8}
X_r	$4.4e^{-10}$
K_s	$5.83e^{-5}$
θ_s	0.417
θ_r	0.02
A	13
N	0.592
M	n/a
L	1

$$\Leftrightarrow \begin{cases} C(H_0 + h) = a_1 \times \frac{1}{1 + b_1 h} \\ K(H_0 + h) = a_2 \times \frac{1}{1 + b_2 h} \end{cases} \quad (8)$$

A limited development of Eq. 8 by the formula of Taylor and Mc Lauren (Eq. 9), near the position H_0 , leads to the Eq. 10. Taking into consideration this relation and the approximations of Boussinesq by rewriting the terms of convection and diffusion, the equation of Richards becomes linear and written itself by Eq. 11, this govern the flow in porous areas:

$$f(h) = \sum_{i=0}^n h^i \frac{f^{(i)}(H_0)}{i!} \quad (9)$$

$$\begin{cases} C(h) = C_1 h + A_1 \\ \text{et} \\ K(h) = C_2 h + A_2 \end{cases} \quad (10)$$

$$A_1 \frac{\partial(h)}{\partial t} = C_2 \frac{\partial(h)}{\partial z} + A_2 \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} \right) \quad (11)$$

By using the method of finite volume, researchers obtain a discretization 2D of this equation under the Eq. 12.

$$a_p h_p = a_E h_E + a_w h_w + a_N h_N + a_s h_s + b \quad (12)$$

Where:

$$a_p = a_E + a_w + a_N + a_s + a_p^\circ + \frac{1}{2} \Delta F \quad (13)$$



Fig. 1: Map of Cameroon estuary, microsoft encarta (2008)

Table 3: Parameters of soils of the cameroon estuaries, gulf of Guinea

Variables	Limoneux soils	Argileux soils
θ_s	0.3658	0.4658
θ_r	0.0286	0.106
A	0.028	0.0104
K_s	$6026 e^{-6}$	$152 e^6$
N	2.239	1.3954
L	0.5	0.45
h_p	0.5	2

Table 4: Conditions imposed for the simulations in the case of infiltration

Conditions	Values
Height of the column and parallelepiped dimensions	100 cm, 100×1000×100 cm ³
Homogenous soils	Table 1
Water height	H = 20 cm
Higher limited conditions	HBS = -0.06 cm
Conditions a la limite inferieure	HBI = -90 cm
Period of simulation (h)	2, 4
Stitch heights tested	0.0025 cm
($\Delta X, \Delta Z$) (cm)	0.005, 0.01
Time scale (Δt)	0.01sec (reference), 1 sec, 1 min

And:

$$b = a_p h_p^\circ$$

The solution of the form discretisated of the equation of Richards by the Tri Diagonal Matrix Algorithm (TDMA), gives the pressure gradient in each point of the stitch by using Eq. 14:

$$h_j = A_j h_{j+1} + C'_j \quad (14)$$

Where:

$$A_j = \frac{\alpha_j}{D_j - \beta_j A_{j-1}} \text{ et } C'_j = \frac{\beta_j C'_{j-1} + c_j}{D_j - \beta_j A_{j-1}}$$

Before proceeding to numerical tests, researchers are going to define the different types of soil textures of the Cameroon estuary. It is a zone of approximatively 20 km²

of surface, situated in the loop of the gulf of Guinea as shown in Fig. 1. It takes over many towns of Cameroon including the economic capital, Douala. This zone presents essentially the limoneux and sandy-argilo soils. The parameters used are those proposed by Belfort and Lehmann (2005), presented in Table 3.

The limited conditions are defined by taking into account the test cases, reserved exclusively to the infiltration in a column and in a parallelepiped rectangle, the flux of water (initial charges) being known. At the limit of study domains, these flux and the fluids velocities are zero. Table 4 present a summary of these conditions imposed to the study domain.

RESULTS AND DISCUSSION

The functioning of the hydrodynamic of a soil is controlled by 2 macroscopic characterizing, depending at the same time on its texture and structure:

- The hybrid retention graph which binds the tenure in water volume (θ) to the pressure potential (h) and which expressed the capacity of soil to retain water to a given energy state
- The hybrid conductivity graph which expressive soil capacity at water transmission in function to its saturation state measured by the pressure gradient (h) or by the tenure in (θ)

Thus for different soils studied, researchers are going to present the different graphs indicating the evolution of the pressure gradient in function in function to the time in the different areas, the graph showing the distribution of water in different pores, the graphs of hydraulic retention and the graphs of hydraulic

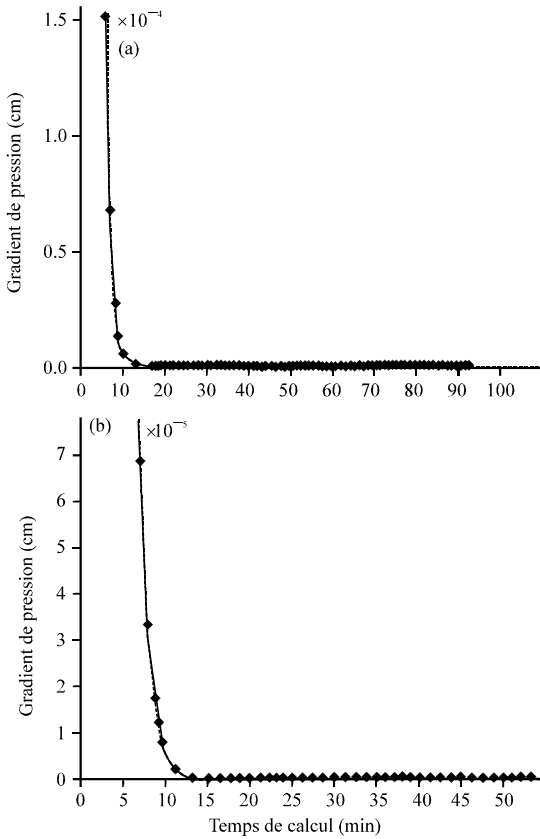


Fig. 2: The pressure gradient in function to time for soils of: a) Limonous type; b) Clays

conductivity. The following Fig. 2a and b, represent the pressure gradient in function of time, respectively for the types of soils limonous and clays.

Figure 2 shows that for an imposed flux, the pressure gradient evolution in saturated variable areas is very slow. Figure 2a shows that for 1 h 35 min, the pressure gradient evolution has reduced to 0.015 min. This pressure gradient evolution has taken place during the first 15 min and has stayed constant for the rest of the time which shows that porous areas are the saturated soils in some places. This conclusion is in agreement with that observed by Zaadnoordlik. This slow evolution of water is the proof of the optimization necessity of flux extracted from a fluid of such areas. Figure 2b, presents the same realities. In this last case, researchers notice an evolution of the fluid more slowly as considered of it weak porosity of the area less significant than the one defining limonous types of soils. It shows that during the 61st min, the pressure gradient decreased to 0.0078 mm.

Figure 3a and b, represents the hydraulic conductivity of water, respectively in soils of limonous and clays soils types. They show that the hydraulic conductivity at saturation in the limonous areas is of the

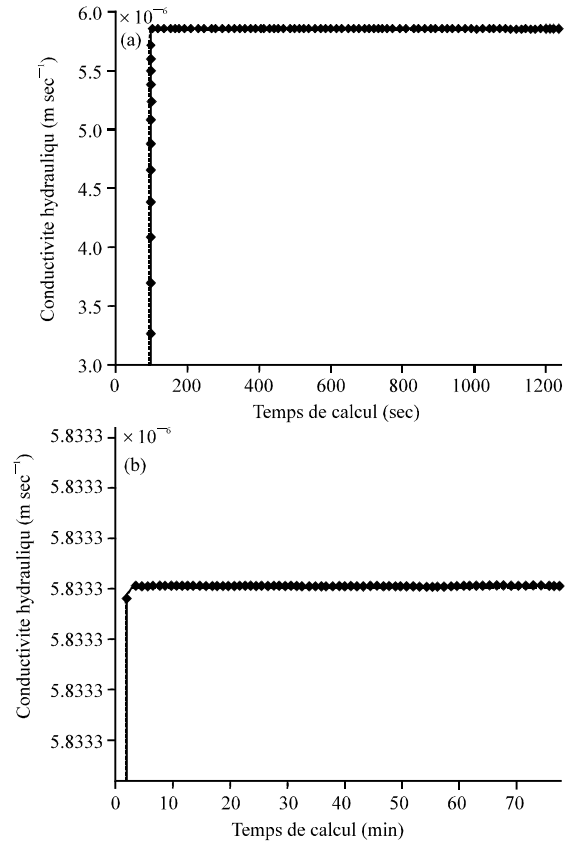


Fig. 3: The evolution of the hydraulic conductivity in: a) Limonous soils; b) Clays

order 10^{-6} and particularly $5.8 \times 10^{-6} \text{ m sec}^{-1}$, in the limonous soils types of the estuary zone of Cameroon, gulf of Guinea. This help to understand that the track travelled by water in the column between many consecutives knots, may be the more reduced as possible. Figure 3b shows that the hydraulic conductivity in clays types soils of the estuary zone of Cameroon gulf of Guinea is $0.972 \times 10^{-6} \text{ m sec}^{-1}$.

The characterization of a soil leads to know one of its characteristic elements, such as the tenure in water/porosity. The aim of such a research is to predict, the spatiotemporal evolution of the humidity in the superficial layer of a soil, through a numerical simulation of transfers.

The variation of tenure in water in the columns in function to time is translated by the kinetic humidification of water columns. Figure 4a and b after show that the tenure in water sample of the soil high develops with the time. Researchers remark that for a height of 100 cm of soil, the humidification of the soil column stays 100 sec for soils limonous type and more than a 10th of minutes for clays types. Referring to the researchs of Ouedraogo who has worked on the kinetic drying of soil,

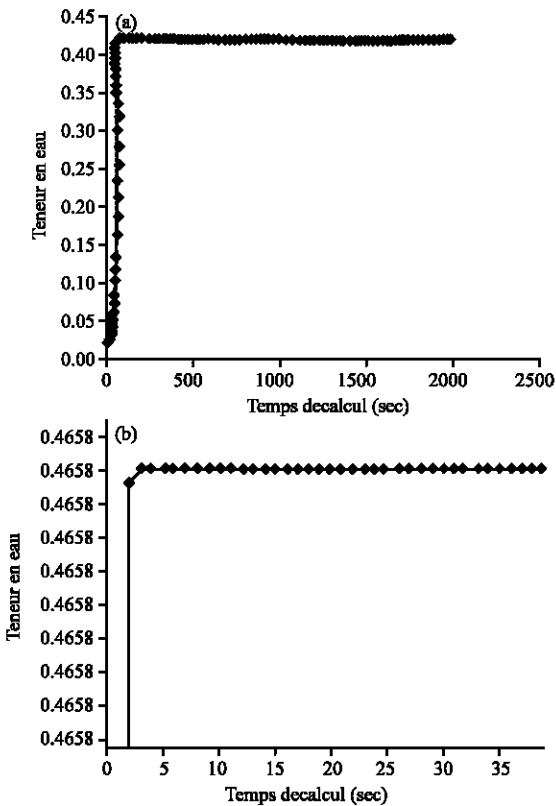


Fig. 4: The evolution of the tenure in water in: a) Limonous soils; b) Clays

researchers can say that the evolution of tenure in water in function to time s presented for estuary soils in Cameroon is really situated in the beach reserved for the different type of soil and is equal to 0.42 for luminous soils types and 0.46 for clays types (Fig. 4 and 5).

In this continuation, researchers have presented and described the behavior of fluids in the soils of the Cameroon estuary zone under the influence of pressure gradient and their behaviors in function to the tenure in water of soils. Researchers have come to conclusion results. The pressure gradient and the tenure in water of a soil being binds, researchers have in function of results obtained, determined the behavior of fluids under the influence of the pressure gradient in function to the tenure in water. In a clearly defined stitch, researchers have projected the graph in 3 dimensions 3D to more observe the influence of capillarity pressure on the behavior of the fluid, in variably saturated areas.

Researchers observe that the tenure in water volume is not separated in a homogeneous manner through the column Stoltz. This observation confirms that the capacity of a field is not unique and depends on the pressure capillary conditions. Due to these observations,

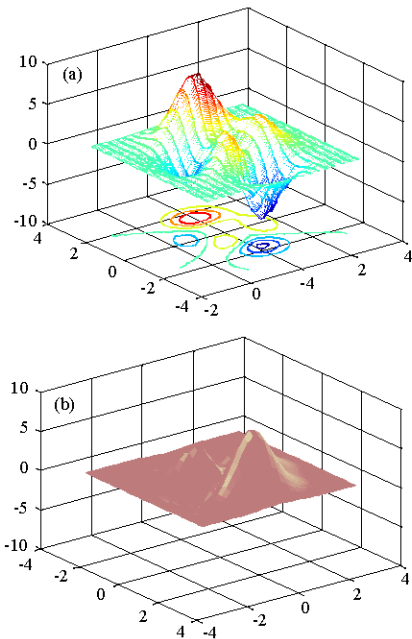


Fig. 5: Evolution of the pressure gradient in function of the tenure in water in limonous soils

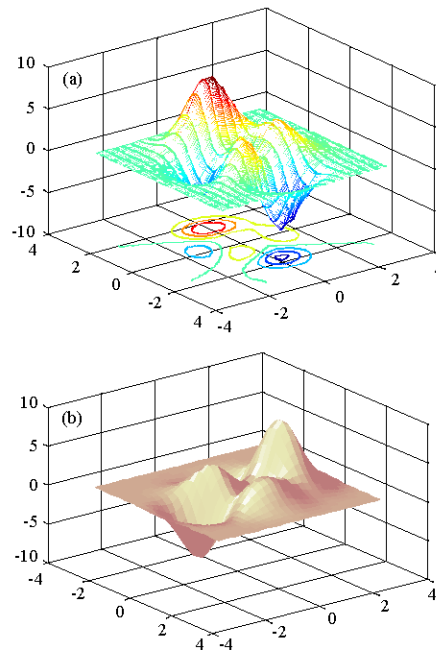


Fig. 6: Evolution of the tenure in water in function of the pressure gradient in clays soils

researchers can conclude that the capillarity forces vary equally with the height. The out line graph has been realized in the aim to show that the fluids particles highly developed following some parabolic paths (Fig. 6).

CONCLUSION

In this study, researchers have determined numerically the hydrodynamics parameters of soils of the estuary zone of Cameroon, gulf of Guinea. Researchers have also show the influence of the capillary forces in such soils of luminous and clays types. The tenure in water stays more times in the clay soils type. The path of the fluids of this two types of soils (limonous and clays), develops in a parabolic way.

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NOMENCLATURE

Small letters:

- x = Longitudinal coordinate (m)
- y = Vertical coordinate (m)
- t = Time (sec)
- a, m, n et l = Parametres caracteristiques du milieu poreux
- a_p^{def} = Coefficient de la charge hydraulique h au centre de la maille
- a_E, a_W, a_N et a_S = Coefficient de la charge hydraulique h au centre des mailles voisines
- b = Terme source

Capital letters:

$$C(H_p) = \frac{\partial \theta}{\partial H_p} = - \frac{n\theta}{|\alpha|n} \times \frac{1}{|H_p|^{n+1}}^{def} =$$

Hydraulic conductivity:

$$K(H_p) = \frac{K_s}{|\alpha|^m} \times \frac{1}{|H_p|^m}^{def} =$$

- D = Cylinder diameter (m)
- P = Pressure (Pa)
- D_i = Hydraulic diffusivity ($m^2 \text{ sec}^{-1}$)
- C = Capillary capacity (m^{-1})
- u, v = Velocities components ($m \text{ sec}^{-1}$)
- Δt = Time difference between two nodes
- 0x = Longitudinal axis
- 0z = Vertical axis

Greek symbols:

- ν = Kinetic viscosity of air ($m^2 \text{ sec}^{-1}$)
- μ = Dynamic viscosity of air (Pa.s)
- ϵ = Dissipation ratio of the turbulent kinetic energy
- K = Hydraulic conductivity
- \vec{k} = Hydraulic conductivity matrix
- K_s = Hydraulic conductivity at saturation ($m \text{ sec}^{-1}$)
- θ_s = Tenure in water volume at saturation
- θ = Tenure in water residue
- ϵ = Tenure in volume of porous areas
- ρ = Volume mass ($m^3 \text{ sec}^{-1}$)
- β = Coefficient de compressibilite specifique du fluide et de la matrice poreuse

REFERENCES

Barenblatt, G.I., V.M. Entov and V.M. Ryzhik, 1990. Theory of Fluids Through Natural Rocks. Kluwer Academic Publishing, Dordrecht.

Belfort, B. and F. Lehmann, 2005. Comparison of equivalent conductivities for numerical simulation of one-dimensional unsaturated flow. *Vadose Zone J.*, 4: 1191-1200.

Bottero, S., S. Hassanizadeh, P. Kleingeld and A. Bezuijen, 2006. Experimental study of dynamic capillary pressure effect in two-phase flow in porous media. Proceedings of the 16th International Conference on Computational Methods in Water Resources, June 19-22, 2006, Copenhagen, Denmark.

Bottero, S., S.M. Hassanizadeh and P.J. Kleingeld, 2011. From measurements to an upscaled capillary pressure-saturation curve. *Transp. Porous Med.*, 88: 271-291.

Di Carlo, D.A., 2004. Experimental measurements of saturation overshoot on infiltration. *Water Resour.*, Vol. 40. 10.1029/2003WR002670

Di Carlo, D.A., 2005. Modeling observed saturation overshoot with continuum additions to standard unsaturated theory. *Adv. Water Resour.*, 28: 1021-1027.

Ginting, V., 2004. Computational upscaled modeling of heterogeneous porous media flow utilizing finite volume method. Ph.D. Thesis, Texas AM University.

Hassanizadeh, S.M. and W.G. Gray, 1987. High velocity flow in porous media. *Transport Porous Media*, 2: 521-531.

Killough, J.E., 1976. Reservoir simulation with history-dependent saturation functions. *Soc. Pet. Eng.*, 16: 37-48.

- Manzini, G. and S. Ferraris, 2004. Mass conservative finite volume methods on 2 unstructured grids for the Richards equation. *Adv. Water Resour.*, 27: 1199-1215.
- Mirzaei, M. and D.B. Das, 2007. Dynamic effects in capillary pressure-saturations relationships for two-phase flow in 3d porous media: Implications of micro-heterogeneities. *Chem. Eng. Sci.*, 62: 1927-1947.
- Silin, D. and T. Patzek, 2004. On Barenblatt's model of spontaneous countercurrent imbibitions. *Transport Porous Media*, 54: 297-322.
- Stauffer, F., 1978. Time dependence of the relationships between capillary pressure, water content and conductivity during drainage of porous media. *Proceedings of the IAHR Symposium On Scale Effects in Porous Media*, August 29-September 1, 1978, IAHR, Thessaloniki, Greece.