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Multi-Commodity Multi-Source-Sinks Network Flow Interdiction Problem with Several Interdictors

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Abstract: The present study seeks to investigate the multi-commodity multi-source-sink network flow interdiction problem with several interdictors. A network user and interdictors are considered as two main problematic elements in this study. While the network user aims to maximize total flow from several sources to several sinks, the interdictors try to reduce the network user's maximum flow by blocking the arcs of the network. Arc blocking leads to some cost and there is limited resource for the required interdiction. To solve the problem, first, the user and interdictors problems were defined.

Key words: Interdiction problem, network flow, simplex method, duality, bi-level programming, multi-commodity, decomposition

INTRODUCTION

The present study seeks to investigate the multi-commodity multi-source-sink network flow interdiction problem with several interdictors. This study deals with two adverse elements: a network user/defender and multi-interdictors/attackers. The network user attempts to maximize the multi-commodity output flow coming from a number of sources which are a subset of a node set V in a directed network. The interdictors try to reduce the network user's maximum flow through the limited interdiction resource in order to block the arcs of the network.

Cormican *et al.* (1998) formulated and solved a stochastic version of the interdictor's problem. They minimized the expected maximum flow going through the network with interdiction variables being binary and random. Extensions were also made to handle uncertain arc capacities. Such stochastic integer programming problems can be used to interdict illegal drugs and to reduce the effectiveness of moving materials, troops, information, etc., in a military force, through a network in wartime.

Wollmer (1970) presented two algorithms for targeting strikes in a Lines-of-Communication (LOC) network. The LOCs were represented by a network of nodes and directed arcs. It was assumed that the user of the LOCs is attempting to achieve a circulation flow at a minimum cost. A very general goal was as special cases, maximizing the flow between two points, meeting the required flows between a source and a sink at a minimum cost and combinations of these two.

Almost, all studies prior to Wood (1993) are specific to the applications stated above and are not extendible to more general contexts. Wood was the first to adopt a mathematical programming model to solve the problem. He developed a min-max formulation of MFNIP and then converted it to an Integer-Programming Model.

Another category of the network-interdiction problem is that of maximizing the length of the shortest path where a set of network arcs are disabled in order to maximize the length of the shortest path between s and t through the usable portion of the network. Fulkerson and Harding (1977) have notably contributed to resolving this problem.

MULTI-COMMODITY MULTIPLE SOURCE-SINKS NETWORK INTERDICTION PROBLEM

Multi-commodity multiple source-sinks network interdiction problem is defined on the basis of capacitated and directed network, G(V, L) with node set V and arc set L consisting of ordered pairs of distinct nodes. Flow on each edge (i, j) can move from i to j. The total flow of each arc (i, j) is restricted by a positive integral capacity m_{ije} for the eth commodity.

The sets of sources and sinks are represented by $N = \{n_1, n_2, ..., n_n\}$ and $D = \{d_1, d_2, ..., d_m\}$, respectively.

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Each source provides the flow for several sinks. The set of sinks that are fed up by a certain source n_i is denoted by D_i, i.e., $D_i = \{d_j/d_j \in D$ and d_j is a sink for n$\}$. The sets D_i, i = 1, ..., n may have several joint nods. The set of middle nodes (which are neither a source nor a sink) is shown by $VP = V/(N \cup D)$. In this model, it has been assumed that sinks may not be middle nodes in another path.

Ditto, the network user aims to maximize multi-commodity total flow through network having several sinks and sources but on the other hand an interdictor aims to minimize the maximum flow achievable by the network user via arc blocking or arc destruction. We assume that the interdictor uses a single type of interdiction resource with a total amount of C units. Interdicting an arc $(i, j) \in L$ requires $c_{ij} \geq 0$ units of the resource C.

The main problem of multi-commodity multiple source-sinks network interdiction problem consists a network user and an interdictor. The user-interdictor or leader-follower relationship is similar to the one in a static Stackelberg game (Simaan and Cruz Jr., 1973) with the difference being that a more general Stackelberg game continues in alternating plays between the leader and the follower. Such, a game can be expressed mathematically as a bi-level programming problem (Dempe, 2002). In the aforesaid method, this problem is designed and analyzed as a bi-level min-max program. Later, it will be converted into a linear program. The problems of the user and the interdictor will be formulated in the following subsections. Let, x_{ijke} be the amount of flow out of the source node k of the eth commodity on arc (i, j). In this model, the interdictor's decision variable w_{ij} is removal rate then $w_{ij} \in [0, 1]$ and arc capacity will be $m_{ije}(1-w_{ij})$ for eth commodity.

Model 1:

$$\min_{w} \max \sum_{e=1}^{E}\sum_{k=1}^{n}\left(\sum_{\substack{j:(i,j)\in L \\ i\in N \\ j\notin N}} x_{ijke} - \sum_{\substack{j:(i,j)\in L \\ j\in N \\ i\notin N}} x_{ijke} \right) \quad (1)$$

$$\text{s.t: } \sum_{k=1}^{n} x_{ijke} \leq m_{ije}(1-w_{ij}) \qquad \forall (i, j)\in L,\ e = 1, ..., E \quad (2)$$

$$\sum_{j:(i,j)\in L} x_{ijke} - \sum_{j:(j,i)\in L} x_{jike} = 0 \ \forall i \in VP,\ k = 1, ..., n,\ e = 1, ..., E \quad (3)$$

$$\sum_{j\in D_k}\left(\sum_{i:(i,j)\in L} x_{ijke} - \sum_{i:(j,i)\in L} x_{jike} \right) - \sum_{(n_k, j)\in L} x_{n_k jke} = 0 \quad (4)$$

k = 1, ..., n, e = 1, ..., E

$$\sum_{(i,j)\in L} c_{ij} w_{ij} \leq C \quad (5)$$

where, $0 \leq w_{ij} \leq 1$, $\forall (i, j)\in L$, $x_{ijke} \geq 0$, $\forall (i, j)\in L$, k = 1, ..., K, e = 1, ..., E. The above model is bi-level model which each level is solved with separate decision. In fact, the internal model maximizes total output flow of all sources while on the minimum model, interdictor minimizesinternal problem out-flow coming from all sources in N. Constraints 2 set all flows on $arc_{(i, j)}$ to zero when $w_{ij} = 1$ and at most to m_{ije} when $w_{ij} = 0$.

GENERALIZED PROBLEM

The network user aims to maximize total flow through network having several sinks and sources with multi-commodity but on the other hand interdictors aim to minimize the maximum flow achievable by the network user via arc blocking or arc destruction. We assume that each interdictor uses a single type of interdiction resource with a total amount of C_r units. Interdicting an arc $(i, j)\in L$ by rth interdictor requires $c_{ijr} \geq 0$ units of the resource. The interdictors' decision variable, w_{ijr} takes on the value of 1 if arc (i, j) is interdicted by the rth interdictor and 0 otherwise. Now, network user's multi-source-sink maximum flow interdiction problem with several interdictors is formulated as following:

Model 2:

$$\min_{w} \max \sum_{e=1}^{E}\sum_{k=1}^{n}\left(\sum_{\substack{j:(i,j)\in L \\ i\in N \\ j\notin N}} x_{ijke} - \sum_{\substack{j:(i,j)\in L \\ j\in N \\ i\notin N}} x_{ijke} \right) \quad (6)$$

$$\text{s.t: } \sum_{k=1}^{n} x_{ijke} \leq m_{ije}\left(1 - \sum_{r=1}^{R} w_{ijr}\right) \qquad \forall (i, j)\in L,\ e = 1, ..., E \quad (7)$$

$$\sum_{j:(i,j)\in L} x_{ijke} - \sum_{j:(j,i)\in L} x_{jike} = 0 \quad \forall i \in VP, \quad (8)$$

k = 1, ..., n, e = 1, ..., E

$$\sum_{j\in D_k}\left(\sum_{i:(i,j)\in L} x_{ijke} - \sum_{i:(j,i)\in L} x_{jike} \right) - \sum_{(n_k, j)\in L} x_{n_k jke} = 0 \quad (9)$$

k = 1, ..., n, e = 1, ..., E

$$\sum_{(i,j)\in L} c_{ij} w_{ijr} \leq C_r \qquad r = 1, ..., R \quad (10)$$

$$\sum_{r=1}^{R} w_{ijr} \le 1 \qquad \forall (i,j) \in L \tag{11}$$

where, $0 \le w_{ijr} \le 1$, $\forall(i,j) \in L$, $r = 1, ..., R$, $x_{ijke} \ge 0$, $\forall(i,j) \in L$, $k = 1, ..., K$, $e = 1, ..., E$. The above model minimizes the maximum flow achievable of all sources. For a fixed value of W, inner maximization is aimed to maximize the user's flow. In other words, the objective function minimizes the maximum out-flow coming from all sources in N. Note that, each arc are interdicted by R interdictors and each interdictor removes percentage of arc on network and arc capacity will be constrains 7. These constraints 7 set all flows on arc (i, j) to zero when sum of w_{ijr} be 1 and at most to m_{ije} for eth commodity when $\sum_{r=1}^{R} w_{ijr} = 0$ in otherwise flow on arc (i, j) is bounded be up $m_{ije}(1-\sum_{r=1}^{R} w_{ijr})$. Also, on interdictors opinion, an arc will be deleted completely then removal rate sum is <1 (constrain 11).

AN EQUIVALENT MODEL

The arcs and nodes of the network are divided into three groups including: S_1 includes all sources and arcs that connect sources together, S_2 all middle nodes (no source and no sink) and edges connecting middle nodes together and to the sources and the sinks, S_3 includes all sinks and edges that connect sinks together. Three sets S_1, S_2, S_3 may be formulated as following:

$$S_1 = N \cup \{(i,j): (i,j) \in L \text{ and } i, j \in N\}$$
$$S_2 = VP \cup \{(i,j): (i,j) \in L \text{ and } (i,j) \notin S_1 \cup S_3\}$$
$$S_3 = D \cup \{(i,j): (i,j) \in L \text{ and } i, j \in D\}$$

Now, we can assume S_1 as a pseudo-source and S_3 as a pseudo-sink. The user tries to maximize the network flow value from S_1-S_3 and the interdictors try to decrease this value as much as his accessible resource allows. According to the new definition of source and sink, the problem may be formulated as the following:

Model 3:

$$\min_{w} \max \sum_{e=1}^{E} V_e \tag{12}$$

$$\text{s.t.: } \sum_{k=1}^{n} x_{ijke} \le m_{ije}\left(1-\sum_{r=1}^{R} w_{ijr}\right) \quad \forall (i,j) \in L, e = 1, ..., E \tag{13}$$

$$\sum_{k=1}^{n}\left(\sum_{\substack{j:(i,j)\in L \\ i\in S_1 \\ j\notin S_1}} x_{ijke} - \sum_{\substack{j:(i,j)\in L \\ j\in S_1 \\ i\notin S_1}} x_{ijke}\right) = V_e \quad e = 1, ..., E \tag{14}$$

$$\sum_{j:(i,j)\in L} x_{ijke} - \sum_{i:(i,j)\in L} x_{jike} = 0 \quad \forall i \in S_2, k = 1, ..., n, e = 1, ..., E \tag{15}$$

$$\sum_{j\in D_k}\left(\sum_{i:(i,j)\in L} x_{ijke} - \sum_{i:(j,i)\in L} x_{jike}\right) - \sum_{(n_k,j)\in L} x_{n_k jke} = 0 \quad k = 1, ..., n, e = 1, ..., E \tag{16}$$

$$\sum_{(i,j)\in L} c_{ij} w_{ijr} \le C_r \quad r = 1, ..., R$$

$$\sum_{r=1}^{R} w_{ijr} \le 1 \qquad \forall (i,j) \in L$$

where, $0 \le w_{ijr} \le 1$, $\forall(i,j) \in L$, $r = 1, ..., R$, $x_{ijke} \ge 0$, $\forall(i,j) \in L$, $k = 1, ..., K$, $e = 1, ..., E$. We call this form of the problem as grouped multi-commodity multi-source-sinks interdiction network flow problem with several interdictors. The objective 12 minimizes the maximum of flow value from S_1-S_3. Constraints 13 are the interdicting constraints on arcs. Constraints 14 ensure that the total net flow exiting from all sources (and reaching all sinks) is equal to V_e. Constraints 16 enforce the sum of arrived flows to the nodes of D_k is equal to the output flow from the kth source.

Our method takes the dual of inner maximization by temporarily fixing Wand then releasing W to obtain a mixed linear "min-min" Model which is simply a minimization model.

Model 4:

$$\min_{\alpha,\theta,w} \sum_{e=1}^{E} \sum_{(i,j)\in L} m_{ije}\left(1-\sum_{r=1}^{R} w_{ijr}\right)\theta_{ije}$$

$$\text{s.t: } \theta_{ije} - \gamma_{ke} \ge 0 \quad \forall (i,j) \in L, \forall i, j \in S_1, e = 1, ..., E \tag{17}$$

$$\theta_{ije} - f_e - \alpha_{ike} \ge 0 \quad \forall k = 1, ..., n, \forall (i,j) \in L, \forall i \in S_2 \text{ and } j \in S_1, e = 1, ..., E \tag{18}$$

$$f_e + \theta_{ije} + \alpha_{jke} - \gamma_{ke} \ge 0 \quad \forall (i,j) \in L, \forall i \in S_1 \text{ and } j \in S_2, e = 1, ..., E \tag{19}$$

$$\theta_{ije}+f_e+\gamma_{ke}\geq 0,\ \forall k=1,...,n,\ \forall(i,j)\in L,\ \forall i\in S_1 \text{ and } j\in S_3,\ e=1,...,E \quad (20)$$

$$\theta_{ije}-f_e-\gamma_{ke}\geq 0,\ \forall k=1,...,n,\ \forall(i,j)\in L,\ \forall i\in S_3 \text{ and } j\in S_1,\ e=1,...,E \quad (21)$$

$$\theta_{ije}-\alpha_{ike}+\gamma_{ke}\geq 0,\ \forall k=1,...,n,\ \forall(i,j)\in L,\ \forall i\in S_2 \text{ and } j\in S_3,\ e=1,...,E \quad (22)$$

$$\theta_{ije}-\gamma_{ke}+\alpha_{jke}\geq 0,\ \forall k=1,...,n,\ \forall(i,j)\in L,\ \forall i\in S_3 \text{ and } j\in S_2,\ e=1,...,E \quad (23)$$

$$
\begin{array}{ll}
-f_e\geq 1 & e=1,...,E\\
\sum_{(i,j)\in L} c_{ij}w_{ijr}\leq C_r & r=1,...,R\\
\sum_{r=1}^{R} w_{ijr}\leq 1 & \forall(i,j)\in L\\
0\leq w_{ijr}\leq 1 & \forall(i,j)\in L,\ r=1,...,R\\
\gamma_{ke}\ \text{free} & k=1,...,n,\ e=1,...,L\\
f_e\ \text{free} & e=1,...,E\\
\theta_{ije}\geq 0 & \forall(i,j)\in L,\ e=1,...,E\\
\alpha_{ike}\ \text{free} & i\in VP \text{ and } k=1,...,n,\ e=1,...,E
\end{array}
$$

The dual variables θ_{ije}, f_e, α_{ike} and γ_{ke} in the model above correspond to constraints 13-16, respectively.

Lemma 1: There is an optimal solution to model 4 such that: $-1\leq\alpha_{ike}\leq 0$, $\forall i\in S_2$, $k=1,...,n$, $e=1,...,E$ and $0\leq\theta_{ije}\leq 1$, $\forall(i,j)\in L$, $e=1,...,E$, $-1\leq\gamma_{ke}\leq 0$, $\forall k=1,...,n$, $e=1,...,E$.

Proof: Note that: the coefficients of θ_{ije} in the objective function are positive, so that making each θ_{ije} as small as the size permitted by the constraints decreases the value of the objective function; no two variables α_{ike} and $\alpha_{ik'e}$ with the same node index (i) and different source indices (k and k') appear in the same constraint. Accordingly, the restriction of a variable α_{ike} to the interval [-1, 0] does not affect any other α_{ike} for $k\neq k'$.

Constraints 17 and 23 imply that the variable θ_{ije} is bounded below by $\max_{k,e}\{-\gamma_{ke}, -\alpha_{jke}+\gamma_{ke}\}$. The restriction of the variables α_{ike} and γ_{ke} to the interval [-1, 0] implies that the lower bound on θ_{ije} is at most 1. Accordingly, restricting θ_{ije} to the interval [0, 1] for such arcs maintains feasibility without loss of optimality. This completes the proof.

Lemma 2: If $P_{ije}=(1-\Sigma_{r=1}^{R} w_{ijr})\theta_{ije}$ $\forall(i,j)\in L$, $e=1,...,E$, then $0\leq P_{ije}\leq 1$ and $p_{ije}\geq\theta_{ije}\Sigma_{r=1}^{R} w_{ijr}$ $\forall(i,j)\in L$, $e=1,...,E$.

Proof: Since, $\Sigma_{r=1}^{R} w_{ijr}\in[0,1]$, $1-\Sigma_{r=1}^{R} w_{ijr}$ is a value between 0 and 1. According to $P_{ije}=(\theta_{ije}-\theta_{ije}\Sigma_{r=1}^{R} w_{ijr})$ and $0\leq\theta_{ije}\leq 1$, we have $0\leq P_{ije}\leq 1$ and $P_{ije}\geq\theta_{ije}-\Sigma_{r=1}^{R} w_{ijr}$. Now using P_{ije}, Model 4 is written as:

Model 5:

$$\min_{\alpha,\theta,w,p}\sum_{e=1}^{E}\sum_{(i,j)\in L} m_{ije}p_{ije}$$

$$
\begin{array}{ll}
\text{s.t: } \theta_{ije}-\gamma_{ke}\geq 0 & \forall(i,j)\in L,\ \forall i,j\in S_1,\ e=1,...,E\\
\theta_{ije}-f_e-\alpha_{ike}\geq 0 & \forall k=1,...,n,\ \forall(i,j)\in L, \forall i\in S_2 \text{ and } j\in S_1,\ e=1,...,E\\
f_e+\theta_{ije}+\alpha_{jke}-\gamma_{ke}\geq 0 & \forall(i,j)\in L,\ \forall i\in S_1 \text{ and } j\in S_2,\ e=1,...,E\\
\theta_{ije}+f_e+\gamma_{ke}\geq 0 & \forall k=1,...,n,\ \forall(i,j)\in L,\ \forall i\in S_1 \text{ and } j\in S_3,\ e=1,...,E\\
\theta_{ije}-f_e-\gamma_{ke}\geq 0 & \forall k=1,...,n,\ \forall(i,j)\in L,\ \forall i\in S_3 \text{ and } j\in S_1,\ e=1,...,E\\
\theta_{ije}-\alpha_{ike}+\gamma_{ke}\geq 0 & \forall k=1,...,n,\ \forall(i,j)\in L,\ \forall i\in S_2 \text{ and } j\in S_3,\ e=1,...,E\\
\theta_{ije}-\gamma_{ke}+\alpha_{jke}\geq 0 & \forall k=1,...,n,\ \forall(i,j)\in L,\ \forall i\in S_3 \text{ and } j\in S_2,\ e=1,...,E\\
-f_e\geq 1 & e=1,...,E\\
\sum_{(i,j)\in L} c_{ijr}w_{ijr}\leq C_r & r=1,...,R\\
\sum_{r=1}^{R} w_{ijr}\leq 1 & \forall(i,j)\in L\\
0\leq w_{ij}\leq 1 & \forall(i,j)\in L\\
p_{ije}\geq\theta_{ije}-\sum_{r=1}^{R} w_{ijr} & \forall(i,j)\in L,\ e=1,...,E\\
-1\leq\gamma_{ke}\leq 0 & k=1,...,n,\ e=1,...,E\\
f_e\ \text{free} & e=1,...,E\\
0\leq\theta_{ije}\leq 1 & \forall(i,j)\in L,\ e=1,...,E\\
-1\leq\alpha_{ike}\leq 0 & i\in S_2 \text{ and } k=1,...,n,\ e=1,...,E\\
0\leq p_{ije}\leq 1 & \forall(i,j)\in L,\ e=1,...,E
\end{array}
$$

Lemma 3: The optimal value of p_{ije} is θ_{ije}.

Proof: If $\Sigma_{r=1}^{R} w_{ijr}=0$ is optimal in Model 2, the corresponding term in the objective function is equal to $m_{ije}\theta_{ije}$. If $\Sigma_{r=1}^{R} w_{ijr}=1$ is optimal, then the corresponding term in objective function is 0. Thus, to linearize the model, it must be true that $p_{ije}=0$ when $\Sigma_{r=1}^{R} w_{ijr}=1$ and $p_{ije}=\theta_{ije}$ when $\Sigma_{r=1}^{R} w_{ijr}=0$. When $\Sigma_{r=1}^{R} w_{ijr}=1$, constraints $p_{ije}\geq\theta_{ije}-\Sigma_{r=1}^{R} w_{ijr}$, $\forall(i,j)\in L$, e, ..., E are satisfied for $0 \leq \theta_{ije} \leq 1$. However, since setting p_{ije} to any value >0 increases the objective function, the value of p_{ije} must be zero. When $\Sigma_{r=1}^{R} w_{ijr} = 0$, constraints $p_{ije} \geq \theta_{ije} - \Sigma_{r=1}^{R} w_{ijr}$, $\forall(i, j) \in L$, e, ..., E are satisfied for $p_{ije} \geq \theta_{ije}$. However, due to the minimization goal $(\min_{\alpha, \theta, w, p} \Sigma_{e=1}^{E} \Sigma_{(i,j)\in L} m_{ije} p_{ije})$, it must be true that $p_{ije} = \theta_{ije}$.

COMPLEXITY

This study reveals that the above mentioned problem is NP-Hard. This claim was proved by using the equivalence of simple binary Knapsack and multi-commodity multi-source-sinks interdicted problems with several interdictors.

Binary knapsack problem: Consider a set of items, K with each $k \in K$ having a positive integer profit m'_k and a positive integer weight c'_k and two positive integers M' and C'. Does there exist a subset $K' \subset K$ such that $\Sigma_{k\in K} m'_k \geq M'$ and $\Sigma_{k\in K} c'_k \leq C'$? That is, does there exist a set of items whose total profit is at least M' and whose total weight is no more than C'?

Note that in the proposed algorithm, set S_1 is considered to be a hypothetical source that includes multi-sources and similarly set S_3 concludes all sinks.

Interdictor problem: Consider a directed graph G = (V, L) with distinguished sets S_1 and S_3, positive integer capacities m_{ij} for each arc $(i, j) \in L$, positive integer resource c_{ij} required for the deletion of any arc $(i, j) \in L$ and two positive integers M and C. Does there exist a subset of arcs $L' \subset L$ such that $\Sigma_{(i, j)\in L} c_{ij} \leq C$ and the maximum S_1-S_3 flow G-L' in is no more than M? In other word, does there exist a subset of arcs whose deletion consumes no more than C units of the resource and which leave behind a network where the maximum S_1-S_3 flow does not exceed M?

The knapsack problem was shown to be NP-Hard by Garey and Johnson (1979). Using this proof, Wood (1993) showed that the interdictor problem with one source and one sink is NP-Hard as well. In the following Theorem, we show that the above problem is NP-Hard too.

Theorem 1: Multi-commodity multi-source-sink network flow interdiction problem with several interdictors is NP-Hard.

Proof: First, this problem is NP-Hard in state of one commodity and one interdictor. Consider a knapsack problem as above which is NP-Hard. Now create a directed network G = (V, L) with two sets of nodes in S_1 and S_3 and for each item $k \in K$ in the knapsack problem create an arc $k \in L$ directed from S to T with capacities $m_{ke} = m'_{ke}$ and resource requirement $c_{ke} = c'_{ke}$.

Furthermore, consider C = C' and $M_e = \Sigma_k m_{ke} - M'_e$. Now, suppose there exists a subset $K' \subset K$ such that $\Sigma_{k\in K'} m'_{ke} \geq M'_e$ and $\Sigma_{k\in K'} c'_{ke} \leq C'_e$. Let, L' correspond to K'. Then, because of the simple topology of the network,the maximum flow in G-L' is at most $M_e = \Sigma_k m_{ke} - M'_e$ and trivially $\Sigma_{k\in L'} c_k \leq C'$. Conversely, suppose there exist a set of arcs L' in G such that the maximum S_1 and S_3 flow in G-L' is no more than M and $\Sigma_{k\in L'} c_k \leq C'$. Then, if K' in the Knapsack problem corresponds to L', it follows that $\Sigma_{k\in K'} m'_{ke} \geq \Sigma_k m_{ke} - M_e = M'_e$ and trivially $\Sigma_{k\in K} c'_k \leq C$. Now given the fact that the problem is clearly NP-HARD, the new variable V_e can be defined in the case of multi-commodity (constraint 14) and on problem with several interdictors, linear interdiction constraints are added to model, then problem is NP-Hard.

CONCLUSION

Considering such problems, the general mathematical model of the problem was developed. Besides, the model was transformed to a bi-level min-max program. Afterwards, in order to be able to compute a solution, the program was converted into a mixed-integer linear program.

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